

1. Short questions:

- a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
- (i)  $v(c) = 10c + 3$
  - (ii)  $v(c) = c^3 + 3c$
  - (iii)  $v(c) = e^{4c}$
  - (iv)  $v(c) = 1 - e^{-c}$
- b) Explain what "moral hazard" is. Describe how it arises in the insurance market.
- c) What is the "paradox of power"? Explain how it arises.
- d) Hen faces uncertainty regarding the state of the world tomorrow. The prior probabilities that Hen attributes to the two possible states (1 and 2) are given by  $\pi_1 = \frac{1}{4}$  and  $\pi_2 = \frac{3}{4}$ . Hen has the possibility to use a message service, which sends message  $m_1$  with probability  $q_1 = \frac{1}{2}$  and message  $m_2$  with probability  $q_2 = \frac{1}{2}$ . The information matrix of the message service is given by

		State	
		State 1	State 2
Message	$[q_{m.s}]$		
	$m_1$	$q_{1.1} = 1$	$q_{1.2} = \frac{1}{3}$
$m_2$		$q_{2.1} = 0$	$q_{2.2} = \frac{2}{3}$

State Bayes' Theorem. Apply Bayes' Theorem to calculate Hen's conditional (posterior) probabilities.

2. Consider a market with a monopsonist employer. There are two types of workers. Type 0 has marginal product  $\Theta_0 = 1$  and an outside opportunity wage of  $w_0(\Theta_0) = \frac{3}{2}$ . Type 1 has marginal product  $\Theta_1 = 3$  and an outside opportunity wage of  $w_0(\Theta_1) = 2$ . The cost of education  $z$  is given by  $C(z, \Theta_0) = \frac{z}{\Theta_0} = z$  for type 0 and  $C(z, \Theta_1) = \frac{z}{\Theta_1} = \frac{z}{3}$  for type 1. A worker's utility function is defined by  $U(w, z, \Theta) = w - C(z, \Theta)$ . The share of type 1 workers is given by  $\frac{1}{3}$ . Workers know their own type but the employer cannot tell the high from the low productivity workers.
- a) In the absence of any educational screening, will there be adverse selection in the market?
- b) Show that the monopsonist will profit from screening. What contract will the monopsonist offer? For simplicity assume that, if two contracts yield a worker the same level of utility, the worker prefers the one with less education. Illustrate your answer in a diagram with wage on the y-axis and the amount of education on the x-axis.
- c) Contrast this with the competitive (Nash) equilibrium. Illustrate your answer in the same diagram.
- d) For what range of outside opportunities  $w_0(\Theta_1)$  is it possible to screen for type 1 workers?
- e) Now suppose that the reservation wage of type 0 workers decreases to  $w_0^*(\Theta_0) = \frac{1}{2}$ , while it remains the same for type 1 workers ( $w_0(\Theta_1) = 2$ ). Consider only the case with a single employer. What contract will the monopsonist offer? Illustrate your answer in the same diagram as before. Is this outcome more or less efficient than the one in b)? Motivate your answer.

3. There are two states of the world, state 1 and state 2. The probability for state 1 occurring is  $\pi = \frac{1}{3}$  (and the probability for state 2 occurring is  $1 - \pi = \frac{2}{3}$ ). It is not possible to directly trade in state claims.

However, there exists a complete asset market, where two assets, asset  $A_1$  and asset  $A_2$ , can be traded. The price of  $A_1$  is given by  $P_1^A = 6$ , and the price of  $A_2$  is given by  $P_2^A = 6$ . The following yield matrix indicates how much each asset yields in each state (e.g.  $A_1$  yields  $z_{11} = 4$  in state 1):

	State 1	State 2
Asset $A_1$	$z_{11} = 4$	$z_{12} = 1$
Asset $A_2$	$z_{21} = 2$	$z_{22} = 2$

The individual's preference-scaling function is given by  $v(c) = \sqrt{c}$ , and the individual is endowed with  $\bar{q}_1 = 10$  units of  $A_1$  and  $\bar{q}_2 = 5$  units of  $A_2$ .

- What are the implicit prices of state claim 1 ( $P_1$ ) and state claim 2 ( $P_2$ )? (Hint:  $P_1^A = z_{11}P_1 + z_{12}P_2$  and  $P_2^A = z_{21}P_1 + z_{22}P_2$ .)
- What will the individual's portfolio of assets (i.e. the endowment  $\bar{q}_1$  and  $\bar{q}_2$ ) yield in the two different states? (That is, what is the individual's implicit endowment of state claims  $\bar{c}_1$  and  $\bar{c}_2$ ?)
- Set up the von-Neumann-Morgenstern expected utility function.
- To obtain the optimal amounts of implicit state claims, two conditions need to be satisfied. State these two conditions. Then calculate the optimal amounts of state claims.
- Given the optimal amounts of state claims, what are the optimal amounts of assets  $A_1$  and  $A_2$ ?