Spatial versus social mismatch

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A B S T R A C T

The aim of this paper is to provide a new mechanism based on social interactions, explaining why distance to jobs can have a negative impact on workers’ labor-market outcomes, especially ethnic minorities. Building on Granovetter’s idea that weak ties are superior to strong ties for providing support in getting a job, we develop a model in which workers who live far away from jobs choose to have less connections to weak ties. Because of the lack of good public transportation in the US, it is costly (both in terms of time and money) to commute to business centers to meet other types of people who can provide other source of information about jobs. If distant minority workers mainly rely on their strong ties, who are more likely to be unemployed, there is then little chance for them of escaping unemployment. It is therefore the separation in both the social and physical space that prevents ethnic minorities from finding a job.

1. Introduction

There is ample evidence showing that distance to jobs is harmful to workers, in particular, ethnic minorities. This is known as the “spatial mismatch hypothesis”. Indeed, first formulated by Kain (1968), the spatial mismatch hypothesis states that, residing in urban segregated areas distant from and poorly connected to major centres of employment growth, black workers face strong geographic barriers to finding and keeping well-paid jobs. In the US context, where jobs have been decentralized and blacks have stayed in the central parts of cities, the main conclusion of the spatial mismatch hypothesis is that distance to jobs is the main cause of their high unemployment rates. Since Kain’s study, hundreds of others have been conducted trying to test the spatial mismatch hypothesis (see, in particular, the literature surveys by Ihlannfeldt and Sjoquist (1998), Ihlannfeldt (2006), and Zenou (2008)). The usual approach is to relate a measure of labor-market outcomes, typically employment or earnings, to another measure of job access, typically some index that captures the distance between residences and centres of employment. The general conclusions are: (a) poor job access indeed worsens labor-market outcomes, (b) black and Hispanic workers have worse access to jobs than white workers, and (c) racial differences in job access can explain between one-third and one-half of racial differences in employment.

Despite this huge empirical literature, few theoretical models have been proposed (for a survey on the theoretical literature, see Gobillon et al., 2007; Zenou, 2006b, 2009). The standard approach is to use a search model to show that distant workers tend to search less (due to lack of information about jobs or less opportunities to find a job) and thus stay longer unemployed (Coulson et al., 2001; Wasmer and Zenou, 2002).1

In the present paper, we propose an alternative explanation. Building on Granovetter’s (1973, 1974, 1983) idea that weak ties are superior to strong ties for providing support in getting a job,2 we develop a model in which workers who live far away from jobs tend to have less connections to weak ties. As underscored by Granovetter, in a close network where everyone knows each other, information is shared and so potential sources of information are quickly shaken down so that the network quickly becomes redundant in terms of access to new information. In contrast Granovetter stresses the strength of weak ties involving a secondary ring of acquaintances.

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1 See also Brueckner and Zenou (2003) for a model of spatial mismatch but without an explicit search model. In an efficiency wage model where, in equilibrium, no worker shirks, they show that housing discrimination can lead to adverse labor-market outcomes for black workers.

2 In his seminal papers, Granovetter (1973, 1974, 1983) defines weak ties in terms of lack of overlap in personal networks between any two agents, i.e. weak ties refer to a network of acquaintances who are less likely to be socially involved with one another. Formally, two agents A and B have a weak tie if there is little or no overlap between their respective personal networks. Vice versa, the tie is strong if most of A’s contacts also appear in B’s network.
who have contacts with networks outside ego's network and therefore offer new sources of information on job opportunities.\(^3\)

Our explanation of the spatial mismatch is that distant (black) workers live in neighborhoods with closed networks that are limited in getting information about possible jobs. Because of the lack of good public transportation in the US, it is costly (both in terms of time and money) to commute to business centers to meet other types of people (weak ties) who can provide other source of information about jobs. If distant (black) workers mainly rely on their (black) strong ties and if the latter are unemployed, there is then little chance of escaping unemployment and finding a job. In other words, in our framework, ethnic minorities experience higher unemployment rate because they are separated both in the urban and the social space.

This is the first aim of our paper. The second aim is to provide a unified theory linking the urban and the social space. Indeed, social interactions are a key aspect of everyday's life. People interact with each other to exert social activities, exchange information about jobs, etc. These interactions, in particular in the labor market, tend to be localized. For instance, using Census Tract data for Chicago in 1980 and 1990, Topa (2001) finds a significantly positive amount of social interactions across neighboring tracts, especially for areas with a high proportion of less educated workers and/or minorities. Bayer et al. (2008) also document that people who live close to each other, defined as being in the same census block, tend to work together, that is, in the same census block.\(^4\) In order to understand the interactions between the labor and the land market and the role of social networks, a model incorporating all these elements is needed. Indeed, households make trade-offs among the opportunity for social interaction, commuting costs, and housing costs in deciding residential location. This, in turn, affects their opportunities in the labor market. The second aim of this paper is therefore to develop a model where social interaction, labor and land market aspects are all explicitly taken into account.

To be more precise, we consider a dynamic model of the labor market in which dyad members do not change over time so that two individuals belonging to the same dyad hold a strong tie with each other. However, each dyad partner can meet other individuals outside the dyad partnership, referred to as weak ties or random encounters. By definition, weak ties are transitory and only last for one period. The process through which individuals learn about jobs results from a combination of a socialization process that takes place inside the family (in the case of strong ties) and a socialization process outside the family (in the case of weak ties).\(^5\) Thus, information about jobs is essentially obtained through strong and weak ties and thus word-of-mouth communication.\(^6\) Workers commute to a business center to work and to interact with other people. We find that housing prices increase with the level of social interactions in the city because information about jobs is transmitted more rapidly and, as a result, individuals are more likely to be employed and to be able to pay higher land rents.

We then extend this framework by endogenizing social interactions. We find that workers living far away from jobs pay lower housing prices but experience higher unemployment rates than those living close to jobs because they mainly rely on their strong ties to obtain information about jobs.

This last result is important because it allows us to provide a theoretical mechanism explaining why residents in certain neighborhoods may be stuck in high unemployment ‘traps’ since they mostly exchange information with their strong ties, who are themselves not likely to possess much useful information about job opportunities. Since most blacks in the United States tend to live further away from jobs (Ihlanfeldt and Sjoquist, 1998), then this model could explain why they have difficulty leaving unemployment. In our model, it is due to the fact that they mainly interact with their strong ties (other blacks) and very little with their weak ties (whites) so that their information about jobs is limited since blacks tend to be more unemployed and have poorer social networks than whites (see, e.g., Wial, 1991). This is related to Putnam (2007) who finds that higher levels of ethnic homogeneity are associated with higher level of trust.\(^1\) In other words, blacks will not interact with whites (and vice versa) because they do not trust each other. In our framework, they do not interact with each other because they are physically separated and, as a result, it is too costly for blacks to interact with whites (weak ties). Dawkins (2006) underscores this result by noticing that social networks may also influence the rate of residential mobility, if households are reluctant to move away from particular locations when local social ties are strong. We assume that strong ties are always of the same race (family, best friends) and there is no spatial costs of interacting with them because they tend to live in the same neighborhood. On the contrary, weak ties can be of either race and meeting them implies a commute to the center of activities. Our main result shows that a separation in the physical space (due, for instance, to housing discrimination) can have dramatic consequences for blacks’ outcomes. In other words, even if black and white workers are totally identical in terms of income, commuting costs, job-information rate, job-destruction rate, etc., then if blacks are separated from whites in the geographical space by living further away from jobs (spatial mismatch), they will also separated in the social space (social mismatch) and will therefore experience higher unemployment rates.

1.1. Related literature

There is a growing interest in theoretical models of peer effects and social networks (see e.g. Akerlof, 1997; Glaeser et al., 1996; Ballesler et al., 2006; Calvó-Armengol et al., 2009), especially in the labor market.\(^8\) However, few models of social networks in the labor market are dynamic. Montgomery (1994) and Calvó-Armengol et al. (2007) propose a dynamic model of weak and strong ties but the former focuses on inequality while the latter on the interaction between crime and labor markets. Zenou (2011) also develops a model of strong and weak ties but does not model the land/housing market. Calvó-Armengol and Jackson (2004) have a more general network analysis (since they can encompass any network structure) but do not model the urban space. To the best of our knowledge, there are nearly no theoretical papers in which social interactions in the labor market are embedded in an urban space.\(^9\) An exception is Selod and Zenou (2006) but there is no explicit analysis of the social network.

\(^{3}\) The existing empirical evidence lends some support to Granovetter's ideas. Yakubovich (2005) uses a large scale survey of hires made in 1998 in a major Russian metropolitan area and finds that a worker is more likely to find a job through weak ties than through strong ones. These results come from a within-agent fixed effect analysis, so are independent of workers’ individual characteristics. Using data from a survey of male workers from the Albany NY area in 1975, Lin et al. (1981) find similar results. Lai et al. (1998) and Marsden and Hurlbert (1988) also find that weak ties facilitate the reach to a contact person with higher occupational status, who in turn leads to better jobs, on average. See Topa (2011) for an overview and Patacchini and Zenou (2008) who find evidence of the strength of weak ties in crime.

\(^{4}\) See also Ioannides and Topa (2010).

\(^{5}\) This idea was first put forward by Bisin and Verdier (2000, 2001) in the context of the transmission of a trait like, for example, religion or identity.

\(^{6}\) Resorting to word of mouth and newspaper ads are two major job-search methods used by unemployed workers (see e.g. Holzer, 1987, 1988; Wahba and Zenou, 2005). Word of mouth, in particular, seems to be of crucial importance: almost 70% of the jobs obtained by white workers and almost 60% of those obtained by black workers are found by checking with relatives or friends or through direct application without referral (Holzer, 1987). For a summary of the evidence, see Ioannides and Loury (2004) and Topa (2011).

\(^{7}\) Other studies have also found that socioeconomic diversity is associated with lower level of trust (Alesina and La Ferrara, 2002; Glaeser et al., 2000). See the literature review by Costa and Kahn (2003).

\(^{8}\) See the excellent literature review by Ioannides and Loury (2004).

\(^{9}\) See Ioannides (2012, Chapter 5) who reviews the literature on social interactions and urban economics.
In the present paper, we use the basic framework of Calvó-Armengol et al. (2007) to incorporate the urban space. We also explicitly model the interactions between black and white workers in both the urban and the social space. All these new aspects are crucial to explain the stylized facts described at the beginning of Section 1. To the best of our knowledge, this is the first theoretical paper that explicitly models both the urban and the social space in a unified framework. It has to be emphasized that this combination is difficult because social networks consider a finite number of individuals (Vega-Redondo, 2007; Goyal, 2007; Jackson, 2008; Jackson and Zenou, in press) while the urban monocentric city model has a continuum of individuals (Fujita, 1989; Fujita and Thisue, 2002; Zenou, 2009). Our model can be seen as a first step towards this direction. One the one hand, the network is extremely simplified since we only consider dyads, i.e., individuals belong to mutually exclusive two-person groups. On the other, because of dyads, we can develop a dynamic model, an essential feature of labor markets.\footnote{There are some papers that combine social interactions and urban spatial structure (Helsey and Strange, 2007; Brueckner and Largey, 2008). However, in all these papers, the social network is not explicitly modelled. Social interactions are captured by externalities and only average effects are considered. There are also two recent papers (Helsey and Zenou, 2011; Ghiglino and Nocco, 2012) that explicit model the social network but the labor market is not taken into account.}

\section{2. The model}

Consider a population of individuals of size one.

### 2.1. Dyads

We assume that individuals belong to mutually exclusive two-person groups, referred to as dyads. We say that two individuals belonging to the same dyad hold a strong tie to each other. We assume that dyad members do not change over time. A strong tie is created once and for ever and can never be broken. Thus, we can think of strong ties as links between members of the same family, or between very close friends.

Individuals can be in either of two different states: employed or unemployed. Dyads, which consist of paired individuals, can thus be in three different states,\footnote{The inner ordering of dyad members does not matter.} which are the following:

(i) both members are employed – we denote the number of such dyads by \( d_2 \);

(ii) one member is employed and the other is unemployed (\( d_1 \));

(iii) both members are unemployed (\( d_0 \)).

#### 2.2. Aggregate state

By denoting the employment rate and the unemployment rate at time \( t \) by \( e(t) \) and \( u(t) \), where \( e(t), u(t) \in [0,1] \), we have:

\[
\begin{align*}
  e(t) &= 2d_2(t) + d_1(t) \\
  u(t) &= 2d_0(t) + d_1(t)
\end{align*}
\]

The population normalization condition can then be written as

\[
e(t) + u(t) = 1
\]

or, alternatively,

\[
d_2(t) + d_1(t) + d_0(t) = \frac{1}{2}
\]

#### 2.3. Social interactions

Time is continuous and individuals live for ever. We assume repeated random pairwise meetings over time. Matching can take place between dyad partners or not. At time \( t \), each individual can meet a weak tie with probability \( \omega(t) \) (thus \( 1 - \omega(t) \) is the probability of meeting his strong-tie partner at time \( t \)).\footnote{If each individual has one unit of time to spend with his friends, then \( \omega(t) \) can also be interpreted as the percentage of time spent with weak ties.} In Sections 2 and 3, we assume these probabilities to be constant and exogenous, not to vary over time and thus, they can be written as \( \omega \) and \( 1 - \omega \). We endogenize \( \omega \) in Sections 4 and 5 below. Observe that strong ties and weak ties are assumed to be substitutes, i.e. the more someone spends time with weak ties, the less he has time to spend with his strong tie.

We refer to matchings inside the dyad partnership as strong ties, and to matchings outside the dyad partnership as weak ties or random encounters. Within each matched pair, information is exchanged, as explained below. Observe that we assume symmetry within each dyad, that is if I meet a strong (or a weak) tie, then my strong (or weak) tie has to meet me. In the language of graph theory, this means that the network of relationships is undirected (Jackson, 2008).

### 2.4. Information transmission

Each job offer is taken to arrive only to employed workers, who can then direct it to one of their contacts (through either strong or weak ties). This is a convenient modelling assumption, which stresses the importance of on-the-job information.\footnote{There is strong evidence that firms rely on referral recruitment (Bartram et al., 1995; Barber et al., 1999; Mencken and Winfield, 1998; Pellizzari, 2010) and it is even common and encouraged strategy for firms to pay bonuses to employees who refer candidates who are successfully recruited to the firm (Berthiaume and Parsons, 2006).} The gist of the analysis would be preserved if this assumption is relaxed. To be more precise, employed workers hear of job vacancies at the exogenous rate \( \lambda \) while they lose their job at the exogenous rate \( \delta \). All jobs and all workers are identical (unskilled labor) so that all employed workers obtain the same wage. Therefore, employed workers, who hear about a job, pass this information onto their current matched partner, who can be a strong or a weak tie. Thus, information about jobs is essentially obtained through social networks.

This information transmission protocol defines a Markov process. The state variable is the relative size of each type of dyad. Transitions depend on labor market turnover and the nature of social interactions as captured by \( \omega \). Because of the continuous time Markov process, the probability of a two-state change is zero (small order) during a small interval of time \( t \) and \( t+dt \). This means, in particular, that both members of a dyad cannot change their status at the same time. For example, two unemployed workers cannot find a job at the same time, i.e. during \( t \) and \( t+dt \), the probability assigned to a transition from a \( d_0 \)-dyad to a \( d_2 \)-dyad is zero. Similarly, two employed workers (\( d_2 \)-dyad) cannot both become unemployed, i.e. switch to a \( d_0 \)-dyad during \( t \) and \( t+dt \). This applies to all other dyads mentioned above.

#### 2.5. Flows of dyads between states

It is readily checked that the net flow of dyads from each state between \( t \) and \( t+dt \) is given by:

\[
\begin{align*}
  \frac{d}{dt}d_2(t) &= h(e(t))d_1(t) - 2\delta d_2(t) \\
  \frac{d}{dt}d_1(t) &= 2g(e(t))d_2(t) - (\delta + h(e(t)))d_1(t) + 2\delta d_2(t) \\
  \frac{d}{dt}d_0(t) &= \delta d_1(t) - 2g(e(t))d_0(t)
\end{align*}
\]

where \( h(e(t)) = [1 - \omega + \omega e(t)] \) and \( g(e(t)) = \omega e(t) \).

Let us explain in details these equations. Take the first one. Then, the variation of dyads composed of two employed workers (\( d_2(t) \)) is equal to the number of \( d_1 \)-dyads in which the unemployed worker has found a job (through either his strong tie with
probability \((1 - \omega)\) or his weak tie with probability \(\omega e(t) \lambda\) minus the number of \(d_2\)-dyads in which one of the two employed workers has lost his job. In the second equation, the variation of dyads composed of one employed and one unemployed worker \((d_1(t))\) is equal to the number of \(d_0\)-dyads in which one of the unemployed workers has found a job (only through his weak tie with probability \(g(e(t))\)) since his strong tie is unemployed and cannot therefore transmit any job information) minus the number of \(d_1\)-dyads in which either the employed worker has lost his job (with probability \(i\)) or the unemployed worker has found a job with the help of his strong or weak tie (with probability \(h(e(t))\)) plus the number of \(d_2\)-dyads in which one of the two employed has lost his job. Finally, in the last equation, the variation of dyads composed of two unemployed workers \((d_0(t))\) is equal to the number of \(d_1\)-dyads in which the employed worker has lost his job minus the number of \(d_\sigma\)-dyads in which one of the unemployed workers has found a job (only through his weak tie, with probability \(g(e(t))\)). These dynamic equations reflect the flows across dyads (Fig. 1).

Observe that the assumption stated above that both members of a dyad cannot lose their status at the same time is reflected in the flows described by (4). What is crucial in our analysis is that members of the same dyad (strong ties) always remain together throughout their life. So, for example, if a \(d_2\)-dyad becomes a \(d_0\)-dyad, the members of this dyad are exactly the same; they have just changed their employment status.

Taking into account (3), the system (4) reduces to a two-dimensional dynamic system in \(d_2(t)\) and \(d_1(t)\) given by:

\[
\begin{align*}
\dot{d}_2(t) &= h(e(t))d_1(t) - 2\delta d_2(t) \\
\dot{d}_1(t) &= 2g(e(t))(1/2 - d_2(t) - d_1(t)) - [\delta + h(e(t))]d_1(t) + 2\delta d_3(t)
\end{align*}
\]

where, using (1):

\[
e(t) = 2d_2(t) + d_1(t)
\]

3. Steady-state equilibrium analysis

A steady-state equilibrium requires solving simultaneously two problems:

(i) (steady state) labor flows (referred to as a labor market equilibrium);

(ii) a location and rental price outcome (referred to as an urban land use equilibrium).

For convenience, we expose first the steady-state labor market equilibrium and then the urban land use equilibrium.

3.1. Labor-market equilibrium

In a steady-state \((d_2^*, d_1^*, d_0^*)\), each of the net flows in (4) is equal to zero. Setting these net flows equal to zero leads to the following relationships:

\[
\begin{align*}
d_2^* &= \frac{(1 - \omega + \omega e^*)}{2\delta} d_1^* \\
d_1^* &= \frac{2\omega e^* \lambda}{\delta} d_0^*
\end{align*}
\]

where

\[
\begin{align*}
d_0^* &= \frac{1}{2} - d_2^* - d_1^* \\
e^* &= 2d_2^* + d_1^* \\
w^* &= 1 - e^*
\end{align*}
\]

Definition 1. A steady-state labor market equilibrium is a four-tuple \((d_2^*, d_1^*, d_0^*, e^*, w^*)\) such that Eqs. (5)—(9) are satisfied.

Proposition 1.

(i) There always exists a steady-state equilibrium \(U\) where all individuals are unemployed and only \(d_\sigma\)-dyads exist, that is \(d_2^* = d_1^* = e^* = 0, d_0^* = 1/2\) and \(w^* = 1\).

(ii) If

\[
\frac{\delta}{\lambda} < \frac{\omega + \sqrt{(\omega(4 - 3\omega))}}{2}
\]

there exists a steady-state equilibrium \(Z\), where \(0 < e^* < 1\) is defined by

\[
e^* = B^2 \frac{d_0^*}{2d_0} - B - Z > 0.
\]

For convenience, we expose first the steady-state labor market equilibrium and then the urban land use equilibrium.
\[ e^* = \frac{\sqrt{2|\lambda + 4\beta (1 - \omega)|} - 2\delta + 2\delta \omega - \lambda}{2\delta \omega} \]  
\[ d_0^* = \frac{\delta^2}{K^* \omega + \lambda \omega \sqrt{2|\lambda + 4\beta (1 - \omega)|}} \]  

3.2. Urban land-use equilibrium

Consider a continuum of equally productive workers uniformly distributed along a linear and closed city. All land is owned by absentee landlords and all firms are exogenously located in the Business District (BD hereafter). The BD is a unique employment center located at one end of the linear city. In a centralized city, it corresponds to the Central Business District, whereas in a completely decentralized city, it represents suburban employment. Workers are risk neutral, optimally decide their place of residence between the BD and the other end of the city, and all consume the same amount of land (normalized to 1 for simplicity). Without loss of generality, the density of residential land parcels is taken to be unity, so that there are exactly \( n \) units of housing within a distance \( x \) from the BD. As stated above, the total population is normalized to 1.

Each individual is identified with one unit of labor. Each employed worker goes to the BD to work and incurs a fixed monetary commuting cost \( \tau \) per unit of distance. When living at a distance \( x \) from the BD, he also pays a land rent \( R(x) \), consumes 1 unit of land and earns a wage \( y \). The wage is assumed to be exogenous. For example, one could think of a minimum wage that is exogenously fixed by the government. The instantaneous (indirect) utility of an employed worker located at a distance \( x \) from the BD is equal to:

\[ V_1(x) = y - \tau x - R(x) \]  

We assume that unemployed workers commute less often to the BD than the employed workers because, for a given distance to jobs and because of their job, the employed workers have to go more often to the BD than the unemployed workers. As a result, we assume that the unemployed workers incur a commuting cost \( \tau x \) per unit of distance, where \( 0 < s \leq 1 \). The instantaneous (indirect) utility of an unemployed worker residing at a distance \( x \) from the BD is therefore equal to:

\[ V_0(x) = b - s\tau x - R(x) \]  

where \( b < y \) is the unemployment benefit. We assume that \( b \) is exogenously financed by taxpayers who reside elsewhere (for example absentee landlords).

We are now able to calculate the expected utility of each worker. To do that, as in Zenou (2006a), we assume perfect capital markets with a zero interest rate.\(^\text{15}\) As a result, workers engage in income smoothing as they cycle in and out of unemployment. Thus, workers save while employed and draw down their savings when out of work, with their consumption expenditure reflecting average income. This means that all workers have identical disposable incomes, equal to the average income over the job cycle. To compute this income, observe that a worker spends a fraction \( e'(\omega) \) of his time employed and a fraction \( 1 - e'(\omega) \) of his time unemployed. Therefore, the expected utility of a worker residing in \( x \) is given by:

\[ EV(x) = e'(\omega)V_1(x) + [1 - e'(\omega)]V_0(x) \]  

Using (19) and (20), this expected utility can be written as:

\[ EV(x) = e'(\omega)(y - \tau x) + [1 - e'(\omega)](b - s\tau x) - R(x) \]  

where \( e'(\omega) \) is given by (11). Observe that, in order to write this expected utility, we have implicitly assumed that, because workers are able to smooth their income over time, a worker's residential location remains fixed as he enters and leaves unemployment. Other models have assumed that changes in employment status involve changes in residential location (Zenou, 2009). Which assumption is more relevant may depend on the nature of the labor market considered. When unemployment and employment spells are short (i.e., a US style labor market), assuming that workers change their residential relocation as soon as they change job is not necessarily appealing. Indeed, even if residential mobility in the US is quite high,\(^\text{16}\) about 58% of the moves are within county (Rupert and Wasmer, 2009). In Table 2 of Rupert and Wasmer (2009), most of the intra-county mobility is house related (65.4%), and only a small fraction (5.6%) move within a county for job related reasons. This indicates that very few moves are job related and thus confirm the fact people do not change residence as soon as they change jobs. However, in a European context, long spells of employment and unemployment make it more likely that relocation and labor transitions coincide, in which case the assumption of absence of mobility costs would be relevant. In the present model, we have chosen the assumption of high-relocation costs because we have the US situation in mind.

Let us now solve the urban land use equilibrium. The timing is as follows. Assume that there is an initial situation when workers pick locations without knowing their initial employment status. They will not change location afterwards. Then, given zero discounting and income smoothing, people bid for rents given that they anticipate the time they will spend in each employment state. Thus, the whole structure of the analysis is: (i) initial period location determination; (ii) ensuing labor market shocks resulting in unemployment, wage, etc. In equilibrium, because of the competition in the land/housing market, all ex ante identical workers will obtain the same expected utility \( EV \). It should be clear that the presence of high-relocation costs means that there is no bidding after initial location decisions.

We now need to calculate the bid rent of workers \( \Psi(x, EV) \), which is defined as the maximum land rent that a worker is willing to pay at a given location \( x \) so as to reach a given level of utility \( EV \). By solving (21) in \( R(x) = \Psi(x, EV) \) for the utility level \( EV \), we easily obtain the following linear bid rent function:

\[ \Psi(x, EV) = e'(\omega)(y - \tau x) + [1 - e'(\omega)](b - s\tau x) - EV \]

with

\[ \frac{\partial \Psi(x, EV)}{\partial x} = -[e'(\omega) + [1 - e'(\omega)]s] \tau < 0 \]

Indeed, in this model, bid rents compensate workers for their expected commuting costs. Those who live close to jobs pay higher land rents because they have lower pecuniary costs whether they are employed or not while those who live far away from jobs have the reverse. By normalizing the agricultural land to zero and by noticing that the size of the city is equal to 1, we have the following definition:

**Definition 2.** An urban land-use equilibrium is a couple \( (EV, R(x)) \) such that:

\(^{15}\) Rosenthal (1988) shows that in the United States, the median renter moves roughly every 1–2 years, while the median homeowner moves every 6–7 years. Even if it is not explained why people move, this shows a high level of residential mobility in the United States, at least for renters. Also, Table 1 in Rupert and Wasmer (2009) reveals that 15.5% of American residents move yearly for one reason or another. In comparison, data for 15 European Union countries show that less than 5% of residents move yearly.
\[ \Psi(1, FV') = 0 \]  
(22)

\[ R'(x) = \max\{ \Psi(x, FV'), 0 \} \]  
(23)

The first equation guarantees that the land rent is continuous everywhere in the city while the second equation is such that absentee landlords allocate land to the highest bidders. Solving (22) and (23) gives:

**Proposition 2.** At the urban land use equilibrium, we have:

\[ FV' = e'(\omega)(y - \tau) + (1 - e'(\omega))(b - s\tau) \]  
(24)

and for \( 0 < x < 1 \),

\[ R'(x) = e'(\omega)(1 + (1 - e'(\omega))\tau)(1 - x) \]  
(25)

We can define the general equilibrium where both the steady-state interior labor and urban land-use equilibria are solved for simultaneously. Ignoring the equilibrium \( \mathcal{U} \), we have the following result:

**Proposition 3.** If (10) holds, then there exists an interior steady-state equilibrium where the endogenous variables \((\mu', e', d_0, d_1, d_2, FV', R'(x))\) are respectively determined by (9), (11)–(14), (24) and (25).

### 3.3. Social interactions

The most interesting results of this model is the impact of social interactions (captured by \( \omega \)) on the different endogenous variables. We have a first important result.

**Proposition 4.** Assume

\[ \frac{\delta}{\lambda} < \sqrt{\frac{\omega}{\delta}} \]  
(26)

and consider steady-state equilibrium \( \mathcal{I} \). Then, increasing the time spent with weak ties \( \omega \) decreases the number of \( d_0 \)-dyads and increases the employment rate \( e' \) in the economy, i.e.

\[ \frac{\partial d_0}{\partial \omega} < 0, \quad \frac{\partial e'}{\partial \omega} > 0 \]

The effects of \( \omega \) on \( d_1 \) and on \( d_2' \) are, however, ambiguous.

**Proof.** See Appendix A.

We show here that by increasing the probability of meeting new workers (i.e. weak ties), the steady-state employment rate increases. This is not a trivial result, since, by increasing \( \omega \), we have different and opposite effects on the job formation/destruction process. On the one hand, we increase the probability of getting out of unemployed dyads, while, on the other hand, we potentially give up the information of an employed partner in favor of a link with an unemployed one. This result is nontrivial since strong and weak ties are substitutes. However, in our model, it is better to meet weak ties because a strong tie does you no good in state \( d_0 \) since your best friends are all unemployed. But a weak tie can do you good in any state because that person might be employed. So there is an asymmetry that is key to the model and that will explain (see below) why some workers (blacks) may be stuck in poverty traps (i.e. \( d_0 \)-dyads) having little contact with weak ties that can help them leaving the \( d_0 \) dyad. This result is also interesting, since it solves a trade off between status quo relations and new relations. It also formally demonstrates the Granovetter’s informal idea of the strength of weak ties in finding a job.

Let us be more specific about this result. Here, individuals belong to mutually exclusive groups, the dyads, and weak tie interactions spread information across dyads. The parameter \( \omega \) measures the proportion of social interaction that occurs outside the dyad, the inter-dyad interactions. When \( \omega \) is high, the social cohesion between employed and unemployed workers is high and thus they are in close contact with each other. In this context, increasing \( \omega \) induces more transitions from unemployment to employment and thus \( e' \), the employment rate in the economy, increases. This is true if (26) holds.\(^{17}\) This condition (26) also guarantees that (10) holds, i.e. that an interior steady-state equilibrium \( \mathcal{I} \) exists (see the Appendix). Condition (26) states that the job-destruction rate \( \delta \) has to be sufficiently low while the job-contact rate \( \lambda \) and social interactions \( \omega \) have to be sufficiently large. As a result, we are in a “reasonable” economy where jobs are not destroyed too fast and jobs are created at the sufficient high rate (otherwise we will end up with the steady-state equilibrium \( \mathcal{U} \) where all workers are unemployed). Take our model and interpret the unit time as one quarter of a year. In the US, the sample average for the quarterly job destruction rate is 5.5% (Davis and Haltiwanger, 1992), thus \( \delta = 0.055 \). We know from most studies that \( \lambda = 4 \), which means that on average people hear from a job every 3 weeks. In that case, condition (26) is always satisfies even for very low values of \( \omega \), like e.g. \( \omega = 0.01 \).

Even though \( e' \) increases with \( \omega \), the effect on \( d_1' \) and \( d_2' \) is ambiguous. Indeed, from Fig. 1, individuals leave dyad \( d_1 \) and enters dyad \( d_2 \) at rate \( h(e) \equiv 1 - \omega \omega \). Since

\[ \frac{\partial e'}{\partial \omega} > 0, \quad \frac{\partial e'}{\partial \omega} > 0 \]

is ambiguous (because \( -1 + e < 0 \)), the effects mentioned above are also ambiguous. Now consider the effect of \( \omega \) on \( d_0 \). This is clearly negative. Indeed, from Fig. 1, one can see that individuals leave dyad \( d_0 \) at rate \( 2\omega e \). Since

\[ \frac{\partial (1 - \omega - \omega \omega \lambda)}{\partial \omega} = \left(-1 + e + \omega \frac{\partial e}{\partial \omega}\right) \lambda > 0 \]

then, when \( \omega \) increases, there are fewer \( d_0 \)-dyads.

**Proposition 5.** Assume (26) and consider steady-state equilibrium \( \mathcal{I} \). Then, increasing the percentage of weak ties \( \omega \) increases both the price of land (and housing) everywhere in the city and the utility level of all workers, i.e.

\[ \frac{\partial \lambda}{\partial \omega} > 0, \quad \forall x \in [0, 1] \]

\[ \frac{\partial \lambda}{\partial \omega} > 0 \]

Indeed, when the strength of weak ties \( \omega \) increases, people find jobs more easily and thus spend more time employed during their lifetime. As a result, there are able to bid more for land and thus the competition in the land market becomes fiercer. Consequently, the price of housing increases at each location in the city. Because the positive impact of \( \omega \) on employment is large enough to outweigh the negative effect of the land rent, the expected utility increases with an increase in \( \omega \). The effect of weak ties on the land rent is an interesting and new result. It is though simple and intuitive since it says that if there are more social interactions in an area, then information about jobs is transmitted more rapidly and, as a result, more people would be employed and land rents would be higher.\(^{18}\)

---

\(^{17}\) Even if (26) does not hold, it can still be true since (26) is a sufficient condition.

\(^{18}\) There is a paper by Fu (2005) who tests in some sense this result. Fu (2005) uses the 1990 Massachusetts census data and estimates an hedonic housing model with social amenities. He found that an increase in the percentage of new residents has significant positive effects on property values. He concludes that this is “probably due to the strength of weak ties”. Of course, it could also be consistent with other aspects such as, for example, gentrification. The results of a direct empirical test of the impact of social interactions on land rents will be very interesting and will help us to verify if the prediction of our model is correct.
In the broader context of the search literature, the results of Proposition 5 illustrate the fact that anything that reduces search frictions in the labor market is going to increase employment and hence expected incomes and land rents. The interesting feature here is that it makes this connection explicit in the specific context of labor market referrals.

4. Choosing social interactions

We would like now to extend the model so that \( \omega \) is chosen by individuals. We assume that there is some cost of interacting with weak ties. This cost is a function of both \( \omega \) and \( x \) (distance to jobs) and is denoted by \( C(\omega, x) \). We assume that, \( \forall x \in [0, 1] \),

\[
C'(\omega, x) = \frac{\partial C(\omega, x)}{\partial \omega} > 0, \quad C'_{\omega}(\omega, x) > 0, \quad C_{\omega}(0, x) = 0.
\]

and

\[
C_{\omega\omega}(\omega, x) = \frac{\partial^2 C(\omega, x)}{\partial \omega^2} > 0 \quad (27)
\]

Assumptions (27) are quite natural and will guarantee that an interior solution exists and is unique. Assumption (28) says that the farther away a worker lives from the BD, the higher is the marginal cost of interacting with weak ties. This captures some ideas developed in Section 1 that, because of the lack of good public transportation in the US, it is costly (both in terms of time and money) to commute to business centers to meet other types of people (weak ties) who can provide other source of information about jobs. This implies a more costly interaction with weak ties so that individuals in segregated communities tend not to interact with people outside their neighborhoods. This will become even clearer when we deal with black and white labor-market outcomes in the next section.

The expected utility of workers is still given by (21) but we need to add the interaction costs. We have:

\[
EV(\omega, x) = e'(\omega)[y - \tau x] + [1 - e'(\omega)][b - s\tau x] - R(x) - C(\omega, x)
\]

where \( e'(\omega) \) is defined by (17).

Each individual optimally chooses \( \omega \) that maximizes \( EV(\omega, x) \). The first-order condition yields:

\[
\frac{\partial e'(\omega)}{\partial \omega} (y - b - (1 - s)\tau x) - C_{\omega}(\omega, x) = 0 \quad (29)
\]

Observe that \( y - b - (1 - s)\tau x \geq 0, \forall x \in [0, 1] \), and we have seen (see Proposition 4) that if (26) holds, then \( \frac{C_{\omega\omega}(\omega, x)}{C_{\omega}(\omega, x)} > 0 \). We have the following result:

**Proposition 6.** Assume (26) and consider steady-state equilibrium \( \mathbf{I} \). Then there exists a unique interior \( \omega^* \) such that \( 0 < \omega^* < 1 \), which is given by (29). Furthermore,

(i) workers living further away from jobs will interact less with weak ties than those residing closer to jobs, i.e.

\[ \frac{\partial \omega^*}{\partial x} < 0 \]

(ii) higher wages or lower unemployment benefits will increase the interactions with weak ties, i.e.

\[ \frac{\partial \omega^*}{\partial y} > 0, \quad \frac{\partial \omega^*}{\partial b} < 0 \]

(iii) higher commuting costs will decrease the interactions with weak ties, i.e.

\[ \frac{\partial \omega^*}{\partial s} > 0, \quad \frac{\partial \omega^*}{\partial \tau} < 0 \]

Consider the wage \( y \) and the unemployment benefit \( b \). Then, a higher \( y \) or a lower \( b \) increases the value of employment and, since \( e'(\omega) \) and \( \omega \) are positively related, workers will interact more with weak ties. The same intuition applies for a lower commuting cost \( \tau \) or a higher \( s \), the fraction of BD trips for the unemployed.

Let us now focus on the negative relationship between \( \omega^* \) and \( x \) (distance to jobs). Workers want to interact with weak ties because it increases their probability of being employed (or, equivalently, the time they spend employed during their lifetime), i.e. \( \frac{\partial \omega^*}{\partial x} < 0 \). However, because it is always more expensive to commute to the business district when employed than when unemployed (i.e. \( \tau > s \)), the marginal gain of interacting with weak ties is higher for workers residing closer to jobs than for those locating further away. To be more precise, when \( x \) increases, the (spatial) cost of employment, \( (1 - s)\tau x \), increases while the gain of employment decreases at the margin since \( \frac{\partial e'(\omega)}{\partial \omega} < 0 \). Furthermore, we have to observe that \( \frac{\partial C_{\omega}(\omega, x)}{\partial \omega} > 0 \), i.e. the marginal cost of interacting with weak ties is higher, the farther away someone lives from the BD. As a result, people living further away from jobs find it optimal to interact less with weak ties and more with their strong ties.

Let us now provide some evidence on the negative relationship between \( \omega^* \) and \( x \). It should be clear that the relationships with strong ties are, in general, stronger closer to where people live while relationships with weak ties are more intense closer to business and shopping centers (see, for example, Ioannides and Topa, 2010). As expressed by Glaeser (2000), “social influences decay rapidly with distance”. For example, Topa (2001) and, more recently, Bayer et al. (2008) found evidence of significant social interactions operating at the block level.20 In our model, these are interactions between strong ties since they are repeated over time. On the contrary, having contact with weak ties, defined as relationships with random encounters that are not repeated over time, are more likely to take place in dense and animated areas, like the business district (BD) in our model.

In the proposition above, we have shown that \( \omega^*(x) < 0 \), meaning that there are more interactions in the BD with weak ties than further away from the BD.21 Sigelman et al. (1996), for the US, show that most superficial encounters occur while shopping, going to bars, and the like. So, basically, the closer people are from the business district, the more likely they interact with random encounters (weak ties). Similarly, Holland et al. (2007) show that public places located in the center of London (UK), are important areas of social interactions. In particular, social mixing takes place there where people of different income and ethnic groups tend to interact with each other. Also, Henning and Lieberg (1996) investigate the structure of networks and the content of ties in selected neighborhoods in Linköping, Sweden. Strong ties were those of importance to the respondent and which were characterized by regular contact. Weak ties consisted of nodding acquaintances and conversational contacts. Henning and Lieberg found that neighborhood where people live

---

19 In a previous version of the paper, we assumed that the cost of interacting with weak ties was not a function of \( x \), distance to jobs, i.e. \( C(\omega, x) = c \omega \). We still obtained the same results as in Proposition 6, in particular, the impact of \( x \) on \( \omega^* \) since

\[
\frac{\partial \omega^*}{\partial x} = \frac{C_{\omega}(\omega, x)}{C_{\omega}(\omega, x)} \frac{1}{y - b - (1 - s)\tau x} < 0
\]

---

20 See also Kan (2007) who shows social capital to be very local.

21 Using a different model, Helsey and Strange (2007) model social interactions in a more “extreme” way since all social interactions occur at a single location (the “center”) and are defined as the number of visits to the center.
was relatively unimportant in weak ties relationships for both white-collar and blue-collar residents – three quarters of contacts were outside the local area.

Let us provide more evidence on the fact that social interactions with strong ties tend to be localized while those with weak ties are not. For that, we put forward the principle of homophily. Homophily behavior or in essence “birds of a feather flock together,” has been firmly established by many empirical studies (McPherson et al., 2011). While we clearly tend to be friends with people who are like us, there are many situations where having a lot of friends like us is simply because we are stuck with people who are like us in the first place. For example if you are a millionaire and all your friends are millionaires, it might simply be because you were born into an elite family and live in an elite area so you only know millionaires in your life, even though you do not actively choose to be friend with millionaires over non-millionaires. A basic source of baseline homophily is the geographical space. As a matter of simple opportunity and/or the need to minimize efforts to form and maintain a social tie (Zipf, 1949), we can expect that we tend to form ties with those who are geographically close to us. Thus, intuitively, this creates a very strong constraint on our potential tie pool. In fact, there is ample empirical evidence that demonstrates this claim. The earliest studies of which we are aware of date back to Festinger et al. (1950) and Caplow and Forman (1950) both on student housing communities. The results showed that in these rather homogeneous communities, spatial arrangement of student rooms/units was an important factor in predicting whether two dwellers have at least weak ties. Many other network studies also reached similar results (for example, see Athanasiou and Yoshioka, 1973; Barrett and Campbell, 1999).

More recently, Wellman (1996) and Mok et al. (2007) re-analyzed Wellman’s earlier dataset on Toronto personal communities (Wellman et al. 1988; Wellman, 1996) and noted that most personal friendships were indeed “local,” contrary to the beliefs that recent technological advancements have freed us from spatial constraints. For instance, in Wellman et al. (1988), it was found that on average 42% of “frequent contact” ties live within a mere 1 mile radius of a typical person, while the rest of his ties could be directed to anywhere in the rest of the world.

Let us now close the model. All endogenous variables (i.e. $\rho$, $\rho'$, $d_0$, $d_1$, $d_2$ and $d_3$) are now a function of $x$ and not of $e(x)$ (and of course a function of all the other exogenous variables). In particular, this means that, if condition (26) holds, then:

$$\frac{\partial d_0'}{\partial x} = \frac{\partial d_0}{\partial x} \frac{\partial \rho}{\partial \rho} > 0 \quad (30)$$

$$\frac{\partial d_1}{\partial x} = \frac{\partial d_2}{\partial x} \frac{\partial \rho}{\partial \rho} < 0 \quad (31)$$

Indeed, for individuals living far away from jobs, it is less likely for them to meet weak ties who can provide information about jobs. So, for example, if someone is unemployed and belongs to a $d_0$-dyad, then the only persons who can provide information about jobs are weak ties. But if this person lives far away from the BD, he will not be very much in contact with weak ties, and therefore will have little information about jobs.

Let us now determine the urban-land use equilibrium. The expected utility is:

$$EV(x) = e'(x)(y - \tau x) + [1 - e'(x)](b - \tau stx) - R(x) - C(x)$$

and thus the bid rent function is given by:

$$\Psi(x, EV) = e'(x)(y - \tau x) + [1 - e'(x)](b - \tau stx) - C(x) - \bar{EV}$$

This bid rent is not anymore linear. We have:

$$\frac{\partial \Psi(x, EV)}{\partial x} = \frac{\partial e'(x)}{\partial x} (y - b - (1 - s)\tau x) - (1 - s)\tau e'(x) - \bar{EV}$$

$$- C_{x}(x) \frac{\partial \bar{e}'}{\partial x}(1 - s)\tau e'(x) - \bar{EV}$$

$$= \frac{\partial \bar{e}'}{\partial x} (y - b - (1 - s)\tau x) - (1 - s)\tau e'(x) - \bar{EV}$$

since $\frac{\partial \bar{e}'}{\partial x} = \frac{\partial e'}{\partial x}$.

Using (29), we have

$$\frac{\partial \Psi(x, EV)}{\partial x} = - (1 - s)\tau e'(x) - \bar{EV} - C_{x}(x) < 0 \quad (34)$$

The role of the land rent is now to compensate remote locations for both higher commuting costs, higher unemployment rates and higher social-interaction costs.

Adopting the same definition of equilibrium as in Definition 2, we obtain:

**Proposition 7.** At the urban land use equilibrium where social interactions are endogenously chosen, we obtain:

$$EV' = e'(x)(y - \tau x) + (1 - e'(1))(b - \tau s) - C(x)$$

and for $0 \leq x \leq 1$.

$$R'(x) = e'(x)(y - b - (1 - s)\tau x) - e'(1)(y - b - (1 - s)\tau x) + \tau s(1 - x) - C(x)$$

We can then solve the labor-market equilibrium as before. The dynamic system is given by:

$$\begin{cases}
\dot{d}_1(t) = h(e(t))d_1(t) - 2\bar{d}_2(t) \\
\dot{d}_2(t) = 2g(e(t))d_0(t) - [\delta + h(e(t))]d_1(t) + 2\bar{d}_2(t) \\
\dot{d}_3(t) = \delta d_1(t) - 2g(e(t))d_0(t)
\end{cases}$$

where $h(e(t)) = [1 - \bar{\omega} + e(t)]\lambda$ and $g(e(t)) = \bar{\omega} e(t)\lambda$ and where

$$\bar{\omega} = \int_0^1 \omega(x) dx$$

This is a well-defined object since we have shown in Proposition 6 that $e'(x)$ is unique and is between 0 and 1, whatever $x \in [0, 1]$. So the only difference with the previous sections is that, when calculating the aggregate flows in the labor market, we use the average time workers spend with their weak ties, that is $e(t)$. In Section 5, when we introduce black and white workers, we fully develop this approach.

To summarize, when the relationship to weak ties is decreasing with distance to jobs, individuals who are close to the BD obtain a lot of information about jobs, spend little time in a $d_0$-dyad (where both friends are unemployed) and experience low unemployment rate. On the contrary, those who live far away from jobs spend most of their time with strong ties and thus get little information from weak ties. This means that when they belong to a $d_0$-dyad, where their best friend is also unemployed, they have little chance of finding a job and are stuck in the unemployment state. This is why they experience higher unemployment rates.

5. **Spatial mismatch and the strength of weak ties**

We would like now to extend our model to incorporate black and white workers and to analyze the impact of segregation in the physical and social space on their labor-market outcomes.
5.1. The model

There is a continuum of black and white workers whose masses are given by $N_B$ and $N_W$, with $N_B + N_W = N$. As in the previous section, workers endogenously chose the fraction of their time they spend with weak ties, $\alpha_j^t$, $j = B,W$, which is given by (29).

As before, individuals belong to dyads. We assume that strong ties are always of the same race (family, best friends) and there is no spatial costs of interacting with them because they tend to live in the same neighborhood. On the contrary, weak ties can be of either race and meeting them implies a commute to the center of activities, here the BD. By denoting the employment level and the unemployment level of workers of type $j = B,W$ at time $t$ by $E_j(t)$ and $U_j(t)$, we have:

$$
\begin{align*}
E_j(t) &= 2d_j(t) + d_j^t(t) \\
U_j(t) &= 2d_j^t(t) + d_j(t)
\end{align*}
$$

The population condition can then be written as

$$
E_j(t) + U_j(t) = N_j
$$

As before, we denote the employment rate and the unemployment rate of workers of type $j = B,W$ at time $t$ by $e_j(t)$ and $u_j(t)$, where $e_j(t)$, $u_j(t) \in [0,1]$, $\forall j \in \{B,W\}$. We have:

$$
e_j(t) = \frac{E_j(t)}{N_j}, \quad u_j(t) = \frac{N_j - E_j(t)}{N_j}
$$

which means that

$$
u_j(t) = 1 - e_j(t)
$$
or, alternatively,

$$
d_j(t) + d_j^t(t) + d_j^s(t) = N_j / 2 \quad (37)
$$

As in the previous sections, each job offer is taken to arrive only to employed workers, who can then direct it to one of their contacts (through either strong or weak ties). Employed workers (black or white) hear of job vacancies at the exogenous rate $\lambda$ while they lose their job at the exogenous rate $\delta$.

Employed workers, who hear about a job, pass this information onto their current matched partner, who can be a strong or a weak tie. White (black) employed workers pass the job information to their white (black) strong tie and to any (white or black) weak tie.

We need to solve simultaneously the land/housing market and the labor market. In this section, it is more convenient to solve first the urban land use equilibrium and then the steady-state labor market equilibrium.

5.2. Urban land use equilibrium

We assume that blacks face housing discrimination in the housing market, as documented by Yinger (1986, 1997). To keep the model tractable, the analysis focuses on a situation where housing discrimination is so strong that landlords in the area close to the BD refuse to rent to blacks under any circumstances. In other words, blacks are prevented from living in the interval $[0,N_W]$ regardless of their willingness to pay for land in this area. The resulting “restricted” residential pattern and the associated bid-rent curves are shown in Fig. 2.

Using the same analysis as above, the expected utility of a worker of type $j = B,W$ can be written as:

$$
EV_j(x) = e_j^t(x)/(y - \tau x) + [1 - e_j^t(x)](b - \tau x - \lambda(x) - C(\alpha_j^t(x),x))
$$

As can be seen from this equation, blacks and whites are totally identical in terms of income, transport costs, housing consumption, etc. The only difference between the two types of workers is due to the fact that blacks are discriminated against in the housing market while whites are not. The bid rent function of a type $-j$ individual is then given by:

$$
\Psi_j(x, EV_j) = e_j^t(x)/(y - \tau x) + [1 - e_j^t(x)](b - \tau x - \lambda(x) - C(\alpha_j^t(x),x) - EV_j)
$$

As before, these bid rents are decreasing and non-linear. We can now define the urban land-use equilibrium as follows:

**Definition 3.** An urban-land use equilibrium with black and white workers is a 3-tuple $(EV_{W0}, EV_{W1}, R(x))$ such that:

$$
\begin{align*}
\Psi_{Wj}(N_W, EV_{W0}) &= R(x) = 0 \quad (38) \\
\Psi_{Wj}(N, EV_{W1}) &= R(x) = 0 \quad (39) \\
R(x) &= \max \{ \Psi_{Bj}(x, EV_{W0}), \Psi_{Bj}(x, EV_{W1}), 0 \} \quad \text{at each } x \in [0,N]
\end{align*}
$$

Eqs. (38) and (39) reflect the equilibrium conditions in the land market. In equilibrium, because of extreme housing discrimination, the two communities (black and whites) are totally separated so that there are two distinct housing markets (see Fig. 2). Eq. (38) says that, at the frontier $N_W$, the bid rent offered by white workers is equal to the agricultural land, which is normalized to zero. Similarly, Eq. (39) says that the bid rent of black workers at the city-fringe $N$ must be equal to the agricultural land, which is normalized to zero. Finally, as in the previous section, Eq. (40) defines the equilibrium land rent as the upper envelope of the equilibrium bid rent curves of all workers and the agricultural rent line.

By solving the first two equations, we obtain:

$$
\begin{align*}
EV_{W0} &= e_j^t(N_W)[y - b - (1 - s)\tau N_W] + b - \tau N_W - C(\omega_{Wj}(N_W), N_W) \\
EV_{W1} &= e_j^t(N)[y - b - (1 - s)\tau N] + b - \tau N - C(\omega_{Wj}(N), N)
\end{align*}
$$

Finally, by plugging these values into the bid rent function, we get:

$$
\begin{align*}
R(x) &= \left\{ \begin{array}{ll}
\omega_{Wj}(x) & \text{for } 0 \leq x < N_W \\
\omega_{Wj}(x) & \text{for } N_W < x < N \end{array} \right.
\end{align*}
$$

We have the following result:

**Proposition 8.** In the urban configuration described in Definition 3, whites spend more time with weak ties than blacks do, that is $\omega_{Wj}(x) > \omega_{Wj}(x')$, $\forall x \in [0,N_W]$ and $\forall x' \in [N_W,N]$. This is, of course, also true on average, i.e. $\tilde{\omega}_{Wj} > \tilde{\omega}_{Wj}$, where

$$
\tilde{\omega}_{Wj} = \frac{1}{N_W} \int_0^{N_W} \omega_j^t(x) dx \quad (41)
$$

$$
\tilde{\omega}_{Wj} = \frac{1}{N_W} \int_0^{N_W} \omega_j^t(x) dx \quad (42)
$$

22 Subscripts $B$ and $W$ stand for “Black” and “White”. Observe that, contrary to the previous sections, total population is not normalized to 1 but is fixed to $N$.

23 We could have assumed that $\lambda$ is race specific so that $\lambda_B > \lambda_W$, i.e. whites hear more about job opportunities than blacks because they are better connected ("old boy" networks). This will just reinforce our results.

24 This assumption has also been made by Brueckner and Zenou (2003).

25 In Fig. 2, we plot the bid rents as if they were linear. This is just for the ease of the exposition.

26 This urban-land use equilibrium is similar to that of Brueckner (1996) and Selod and Zenou (2001), for the case of South Africa under Apartheid.
This proposition is a consequence of housing discrimination and the fact that the time spent with weak ties is a decreasing function of the distance to jobs, i.e. $\alpha'(x) < 0$ (see Proposition 6). We know that $\alpha'(x) < 0$ is a result of a choice since the marginal gain of interacting with weak ties is higher for workers residing closer to jobs than for those locating further away. In other words, this proposition links the physical and the social space since it says that separation in the physical space (here through housing discrimination) leads to separation in the social space, i.e. individuals living far away from jobs will mainly interact with their strong ties. In the context of black and white workers, this proposition shows that housing discrimination by separating black and white workers in the physical space does also separate them in the social space since black workers will find optimal not to commute to the BD to interact with weak ties and thus with whites.

There is some evidence that residents of high poverty neighborhoods rely more on strong ties, and on more geographically concentrated networks and thus less on weak ties. Elliot (1999) shows that less educated workers in high poverty neighborhoods are twice as likely to have found a job through neighbors (local contacts in the same group of city blocks) than in low poverty areas; this is consistent with evidence on the geographic concentration of social networks of poorer individuals, as reported in Fischer (1982) and Kadushin and Jones (1992). Moreover, jobholders in high poverty areas are more likely to have found jobs through strong rather than weak ties than in low poverty places (73% versus 48%).

Observe that, as explained in Section 3.2, the BD (Business District) is the unique employment center located at one end of a linear city. In a centralized city, it corresponds to the Central Business District, whereas in a completely decentralized city, it represents suburban employment. In the United States, black families tend to live in the city center while whites are more likely to reside in the suburbs. Even if there are jobs in the center, the jobs that low-skill black workers need are mostly located in the suburbs. This is the situation we are capturing here where black workers live far away to the BD, i.e. the location where jobs are. For example, Raphael and Stoll (2002) have categorized all Metropolitan Statistical Areas (MSAs) in the US according to the severity of their spatial mismatch, which measures the spatial imbalance between jobs and residential locations using an index of dissimilarity. In their measure, the dissimilarity index ranges from 0 to 100, with higher values indicating a greater geographic mismatch between populations and jobs within a given metropolitan area. For instance, a dissimilarity index of 50 for blacks means that 50% of all blacks residing in the metropolitan area would have had to relocate to different neighborhoods within the metropolitan area in order to be spatially distributed in perfect proportion with jobs. Table 1 documents the spatial segregation of black workers in the US by giving the value of this dissimilarity index (denoted by SM) for both black and white families. It is easily seen that segregation/spatial mismatch of black families is very severe in the US, especially in big cities.27

5.3. Labor-market equilibrium

We would like now to analyze the consequence of the “double” separation of black workers (Proposition 8) on their labor-market outcomes. Let us write the flows of dyads between states for black and white workers. For a worker of type $j = B, W$, they are given by:

$$
\begin{align*}
\dot{d}_{2j}(t) &= h_j(t)d_{1j}(t) - 2\delta d_{2j}(t) \\
\dot{d}_{1j}(t) &= 2g_j(t)d_{0j}(t) - [\delta + h_j(t)]d_{1j}(t) + 2\delta d_{2j}(t) \\
\dot{d}_0(t) &= \delta d_{1j}(t) - 2g_j(t)d_{0j}(t)
\end{align*}
$$

Fig. 2. Urban-land use equilibrium with housing discrimination.

In Table 1, the following variables mean: % Pop: Percentage of (black or white) individuals in the population in the MSA or PMSA; SM: Measure of the Spatial Mismatch (for black or white) between people and jobs using the Raphael’s and Stoll’s dissimilarity index. % Un: Percentage of (black or white) male unemployed in the MSA or PMSA.
where $h_j(t) \equiv \left(1 - \omega_j\right) \hat{\lambda} + \omega_j \left[\frac{\lambda}{2} \epsilon_W^j(t) + \frac{\lambda}{2} \epsilon_W(t)\right] \hat{\lambda}$ and $g_j(t) \equiv \omega_j \left[\frac{\lambda}{2} \epsilon_W^j(t) + \frac{\lambda}{2} \epsilon_W(t)\right] \hat{\lambda}$, and with $\omega_j < \omega_W$ (Proposition 8). Indeed, when calculating the aggregate flows in the labor market for each type of workers, we use the average time workers of each type spend with their weak ties, that is $\omega_j$, which are defined by (41) and (42). Let us explain the first equation since the interpretation of the other equations is similar. The variation of dyads composed of two employed workers of type $j$ ($d_{ij}(t)$) is equal to the number of $d_{ij}$-dyads in which the unemployed worker of type $j$ has found a job through either his strong tie of type $j$ with probability $(1 - \omega_j) \hat{\lambda}$ or his weak tie with probability $\omega_j \left[\frac{\lambda}{2} \epsilon_W^j(t) + \frac{\lambda}{2} \epsilon_W(t)\right] \hat{\lambda}$ minus the number of $d_{ij}$-dyads in which one of the two employed workers has lost his job. It is important to understand why the probability of finding a job through a weak tie is $\omega_j \left[\frac{\lambda}{2} \epsilon_W^j(t) + \frac{\lambda}{2} \epsilon_W(t)\right] \hat{\lambda}$ for workers of type $j$. When a person of type $j = B, W$ who spends on average $\omega_j$ of his time with weak ties goes to the BD,28 he can meet a weak tie who is either an employed black worker who is aware of a job opportunity with probability $\frac{\lambda}{2} \epsilon_W^j(t) \hat{\lambda}$ or an employed white worker who is aware of a job opportunity with probability $\frac{\lambda}{2} \epsilon_W(t) \hat{\lambda}$. In other words, there is no meeting bias with weak ties but there is a strong meeting bias with strong ties (since a person of type $j$ only meets his strong tie belonging to the same race $j$ all his life). This is to capture the idea that people are born with a type $j$ and interacts with strong ties of the same type $j$ because they are either members of the family or very close friends met during the childhood. On the contrary, individuals meet weak ties randomly by going to bars, doing sport activities or shopping in the BD. In that case, they meet randomly other people of either race.

Taking into account (37), the system (43) reduces to a two-dimensional dynamic system in $d_{ij}(t)$ and $d_{ij}(t)$ given by:

\[
\begin{align*}
\dot{d}_{ij}(t) &= h_j(t)d_{ij}(t) - 2\delta d_{ij}(t) \\
\dot{d}_{ij}(t) &= 2g_j(t)[N_j/2 - d_{ij}(t) - d_{ij}(t)] - \delta h_j(t)d_{ij}(t) + 2\delta d_{ij}(t)
\end{align*}
\]

where $N_j\epsilon_j^j(t) = 2d_{ij}(t) + d_{ij}(t)$.

In a steady-state ($d_{ij}, d_{ij}, d_{ij}$), each of the net flows in (43) is equal to zero. Setting these net flows equal to zero leads to the following relationships for workers of type $j = B, W$:

\[
\begin{align*}
\frac{d_{ij}}{d_{ij}} &= \left(1 - \omega_j\right) \hat{\lambda} + \omega_j \left[\frac{\lambda}{2} \epsilon_W^j(t) + \frac{\lambda}{2} \epsilon_W(t)\right] \hat{\lambda} \\
\frac{d_{ij}}{d_{ij}} &= \frac{2\omega_j \left[\frac{\lambda}{2} \epsilon_W^j(t) + \frac{\lambda}{2} \epsilon_W(t)\right] \hat{\lambda}}{\delta} \\
\frac{d_{ij}}{d_{ij}} &= N_j \frac{1}{2} - d_{ij} - d_{ij} \\
\text{where} \\
N_j\epsilon_j^j = 2d_{ij} + d_{ij} \\
\epsilon_j^j = 1 - \epsilon_j^j.
\end{align*}
\]

The model is much more complicated now because $\epsilon_W$ and $\epsilon_W$ enter both in each dyad of each type $j$ of worker. As a result, we cannot analyze the steady-state equilibrium separately for black and white workers and we end up with eight equations to solve (Eqs. (44)–(47) for $j = B, W$) for eight unknowns ($d_{ij}, d_{ij}, d_{ij}, \epsilon_j$ for $j = B, W$).

We have the following result:

**Proposition 9.**

(i) There always exists a steady-state equilibrium $\nu$ where all individuals are unemployed and only $d_{ij}$-dyads exist, that is $d_{ij}^\nu = d_{ij}^\nu = 0$, $d_{ij}^\nu = N_j/2$ and $\epsilon_j^\nu = 1$ for $j = B, W$.

(ii) All the other steady-state equilibria are interior, that is $0 < \epsilon_W < 1$ and $0 < \epsilon_W < 1$. These equilibria are characterized by:

\[
\frac{N_j\epsilon_W + N_W\epsilon_W}{N} = \frac{N\delta^2 - 2\lambda^2 \omega_j\delta_{ij}^\nu(1 - \omega_j) - 2\lambda^2 \omega_j\delta_{ij}^\nu(1 - \omega_W)}{2\lambda^2 \omega_j \delta_{ij}^\nu(1 + \delta) + \omega_W \delta_{ij}^\nu(1 + \delta)}
\]

Furthermore, if the separation in the physical space for blacks is sufficiently high (meaning that $\omega_B \gg \omega_W$), i.e.

\[
\frac{\omega_W(1 - \omega_W)}{\omega_B(1 - \omega_B)} > \frac{N_W}{N_B}
\]
then the employment rate (unemployment rate) of black workers is lower (higher) than that of whites, i.e. \( e_B < e_W \) and \( u_B > u_W \).

This proposition formally proves the intuition developed earlier. If black workers are sufficiently separated in the physical space, then they will mainly interact with their black strong ties and will therefore have very little interaction with weak ties, especially whites. Weak ties are an important source of job information and when black individuals miss it, they end up having a higher unemployment rate than whites. This is a vicious circle since blacks experience a higher unemployment rate and mostly rely on their weak ties. As a result, when they found themselves in close contact with each other. Therefore, little interaction with weak ties induces more transitions from employment to unemployment. Weak ties are an important source of job information and therefore have very little interaction with weak ties, especially blacks who also experience a high unemployment rate, etc. Since jobs are mainly found through social networks via employed friends, black individuals are stuck in their location with no job. In particular, those residing far away from jobs, will mainly rely on their weak ties. As a result, when they found themselves in a black-dyad, they have nearly no chance of leaving it since the only way out is to meet an employed weak tie. As underscored by Granovetter (1973, 1974, 1983), in a close network where everyone knows each other, information is shared and so potential sources of information are quickly shaken down so that the network quickly becomes redundant in terms of access to new information. In contrast Granovetter stresses the strength of weak ties involving a secondary ring of acquaintances who have contacts with networks outside ego’s network and therefore offer new sources of information on job opportunities. To summarize, when the time spent with weak ties is low, the social cohesion between employed and unemployed workers is also low and thus they are not in close contact with each other. Therefore, little interaction with weak ties induces more transitions from employment to unemployment and thus the unemployment rate increases.

Fernandez and Fernandez-Mateo (2006) elaborate the various network mechanisms by which minorities can be isolated from good job opportunities. Using unique data from one employer, they examine multiple steps in the chain of network referral processes originating from employees to the applicants for entry-level jobs at this employer. They study access to a set of desirable jobs that examine multiple steps in the chain of network referral processes and therefore have very little interaction with weak ties, especially the fact that whites are more likely to be isolated from minorities. This means that they mostly meet other black workers. As can be seen in this table, the only (ex ante) difference between blacks and whites is their location in the geographical space. Since housing consumption has been normalized to 1, the city size is equal to \( N = 1000 \). The area where whites live is between \( x = 0 \) and \( x = 800 \) and the area where blacks reside is between \( x = 800 \) and \( x = 1000 \). As a result, because blacks reside far away from jobs, they optimally choose to spend on average only 1.5% of their time with weak ties while this number is 17.74% for white workers. Fig. 3 displays the optimal decision of \( \omega_j \) (solution of (29)) for all workers in the city by plotting the relationship between \( \omega_j \) (vertical axis) and distance to jobs \( x \) (horizontal axis). One can see that whites who live very close to the business district BD spend a lot of their time with their weak ties. On the contrary, even blacks who live the closest from the job center (i.e. at \( x = N_B = 800 \)) spend already less than 20% of their time with weak ties while those who live at the city fringe spend most of their time with their strong ties. This means that they mostly meet other black workers.

At the steady-state equilibrium, this initial difference in \( \omega_j \) translates into large differences in labor-market outcomes between these black and white workers (Table 2). First, the unem-

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Black workers</th>
<th>White workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_j ) (%)</td>
<td>1.50</td>
<td>17.74</td>
</tr>
<tr>
<td>( u_j ) (%)</td>
<td>58.80</td>
<td>18.87</td>
</tr>
<tr>
<td>( d_{ij} )</td>
<td>54.21</td>
<td>37.70</td>
</tr>
<tr>
<td>( 2d_{ij}/N_j )</td>
<td>0.542</td>
<td>0.094</td>
</tr>
<tr>
<td>( d_{ij} )</td>
<td>9.191</td>
<td>75.61</td>
</tr>
<tr>
<td>( 2d_{ij}/N_j )</td>
<td>0.092</td>
<td>0.189</td>
</tr>
<tr>
<td>( d_{ij} )</td>
<td>36.60</td>
<td>286.69</td>
</tr>
<tr>
<td>( 2d_{ij}/N_j )</td>
<td>0.366</td>
<td>0.717</td>
</tr>
</tbody>
</table>

5.4. Numerical simulations

Take our model and interpret the time period as one quarter of a year. The job destruction rate is equal to \( \delta = 0.1 \), that is, workers keep on average their job for 2 years and 6 months. Remember that these are unskilled jobs. The job information rate is equal to \( \lambda = 0.8 \), which means that an employed worker is aware about a job every 3 months (and 20 days). We fix the total population to \( N = 1000 \) with 20% blacks and 80% whites, i.e. \( N_B = 200 \) and \( N_W = 800 \). The difference between wages and unemployment benefits are set to 50 (i.e. \( y - b = 50 \)), the commuting cost per unit of distance is set to \( \tau = 0.05 \) and \( s = 0.1 \) (the employed workers commute 10 times more than the unemployed workers). It is easily verified that \( y - b - (1 - s)\tau x > 0, \forall x \) since, for \( x = 1000 \), \( y - b - (1 - s)\tau x = 5 \).

Concerning social interactions, each individual \( j = B,W \) maximizes his expected utility. The optimal \( \omega_j \) is given by (29) and indicates the time spent with weak ties. The average optimal time spent with weak ties, \( \omega_j \) is given by (41) for \( j = W \) and by (42) for \( j = B \). For the simulations, we assume the following cost function \( C(\omega_j, x) = \omega_j s x \), which has the properties required, in particular \( C_{\omega j}(\omega_j, x) > 0 \). Using (44)-(48), for each \( j = B,W \), we obtain:

\[
\begin{align*}
\rho_j' &= \frac{2\omega_j \left( N_W e_W + N_B e_B \right) \lambda (N_B e_B + N_W e_W) \left( N_B e_B + N_W e_W - 1 \right) + \lambda + 2\delta_j + 2\delta^2}{N_j \delta^2} \\
\rho_j &= \frac{2\omega_j \left( N_W e_W + N_B e_B \right) \lambda (N_B e_B + N_W e_W) \left( N_B e_B + N_W e_W - 1 \right) + \lambda + 2\delta_j + 2\delta^2}{N_j \delta^2} \\
\rho_j' &= \frac{2d_{ij} + d_{ij}'}{N_j} \\
\rho_j' &= 1 - \epsilon_j
\end{align*}
\]

where \( \frac{N_B e_B + N_W e_W}{N_j} \) is given by (49).

The results of the simulations are displayed in Table 2.
employment rate of black workers is nearly three times higher than that of white workers (58.80% versus 18.87%). As showed above, this is because blacks choose to spend most of their time with strong (black) ties, which implies that they have a higher chance of being unemployed (since there is a negative relationship between employment and $\omega$; see Proposition 4). Furthermore, since they mostly interact with black strong ties, who themselves are more likely to be unemployed, they have little chance of escaping unemployment since they cannot rely on their strong ties to provide them with job information. If we look at Table 2, we see that black families spend more than 54% of their time in a $d_0$ dyad because, once they are in this state, they are stuck there for a long time because none of the worker in the $d_0$ dyad can help his strong tie. On the contrary, whites spend only 9.4% of their time in the $d_0$ dyad because they spend a lot of time meeting weak ties who can help them find a job. This is why the unemployment rate of black workers is so high. Interestingly, because $i$ is relatively high (employed workers of either race has a job opportunity every 3 months), black workers do also spend a lot of time in a $d_2$ dyad (36%) where both strong ties are employed.

This first result highlights the large different outcomes between blacks and whites mainly due to the separation of the former in the physical space and the fact that distance to jobs (spatial mismatch) induces blacks to optimally choose to mostly interact with their (black) strong ties (social mismatch). We would like now to see more closely how the increase in segregation affects unemployment outcomes. For that, we vary $N_W$, which measures the size of the area where whites live, and analyze its impact on the labor market. Notice that by increasing $N_W$ and by keeping $N$ fixed, we reduce the size of the black population but, more importantly, we decrease both $\omega_B$ and $\omega_W$ (see Fig. 3). Assume for example that $N_W = N_B = 500$. We obtain the following equilibrium steady-state values:

Now, because black workers are on average much more closer to the BD (average distance of 900 in Table 2 and of 750 in Table 3), they interact more with weak ties (4.45% of their time) and thus meet more whites who can help them find a job. As a result, they get less stuck in a $d_0$ dyad (“only” 28.8% of their time) and their unemployment rate is sharply reduced (from 58.80% to 35.17%). Interestingly, the effect on whites’ unemployment rate is very small because, on the one hand, they choose to interact more with weak ties (they are on average closer to jobs), which reduces unemployment but, on the other hand, they meet more black weak ties, who are more likely to be unemployed and thus cannot help them find a job.

We can continue to vary segregation and analyze its impact on the labor-market outcomes of black and white workers. Fig. 4 displays the results for the unemployment rates. In this figure, the solid line corresponds to outcomes of black workers while the dashed line to that of white workers.

It can be seen that increasing segregation is mainly detrimental to blacks, which experience a drastic increase in their unemployment rate from 18.8% (when there are mostly blacks in the population, i.e. $N_B = 990$ and $N_W = 10$ and $\omega_B = 0.15$, $\omega_W = 0.90$) to more than 73% when there is a majority of whites (i.e. $N_B = 100$ and $N_W = 900$ and $\omega_B = 0.05$, $\omega_W = 0.55$). Interestingly, for whites, the unemployment rate is nearly unaffected by the increase in segregation since it increases from 13% to 13.35%. All the results can be explained by the time spent in a $d_0$ dyad. When blacks are very close to jobs ($N_W = 10$), their unemployment rate is 18.8%, the average time spent with weak ties is 15.4% and their time spent in a $d_0$ dyad is only 9.5% over their lifetime. On the contrary, when they reside very far away from jobs ($N_W = 900$), then their unemployment rate is 73.6%, the time spent with weak ties is 0.71% and the time spent in a $d_0$ dyad is 70.67%!

To sum up, our model highlights the fact that minority workers, especially blacks, are both cut off from employment opportunities (because of distance to jobs) and are embedded in the “wrong” network, that is, they tend to overuse networks (i.e. strong ties) that lead to no job at all. Using data from the UK Quarterly Labour Force Survey, Battu et al. (2011) examine the job finding methods of different ethnic groups in the UK. They show that, though personal networks are a popular method of finding a job for the ethnic minorities, they are not necessarily the most effective either in terms of gaining employment or in terms of the level of job achieved. Kasinitz and Rosenberg (1996) argue that poor, black residents of the Red Hook section of Brooklyn are cut off from good jobs on the waterfront. Since people tend to be hired into these jobs only through connections to union members who already work there, and since few African Americans are currently employed on the waterfront, they argue that African Americans are missing the connection to these jobs. Similarly, Newman (1995, 1999) in her studies of Harlem’s low-wage service workers argues that black youth also rely on networks in their job finding, but that these networks tend to lead them to low-paying jobs.

6. Discussion and policy implications

Because of the results of the previous section, our model can provide a mechanism explaining why black workers, who tend to live far away from jobs in the United States, experience high unemployment rates. Our explanation of the spatial mismatch is that distant (black) workers live in neighborhoods based on closed net-

---

Table 3

<table>
<thead>
<tr>
<th></th>
<th>Black workers</th>
<th>White workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_B$ (%)</td>
<td>4.45</td>
<td>22.51</td>
</tr>
<tr>
<td>$\gamma_W$ (%)</td>
<td>35.17</td>
<td>17.10</td>
</tr>
<tr>
<td>$d_{x_B}$</td>
<td>55.77</td>
<td>22.10</td>
</tr>
<tr>
<td>$2d_{x_B}/N_B$</td>
<td>0.288</td>
<td>0.074</td>
</tr>
<tr>
<td>$d_{x_W}$</td>
<td>29.12</td>
<td>58.38</td>
</tr>
<tr>
<td>$2d_{x_W}/N_W$</td>
<td>0.146</td>
<td>0.195</td>
</tr>
<tr>
<td>$d_{x_B}$</td>
<td>115.11</td>
<td>219.52</td>
</tr>
<tr>
<td>$2d_{x_B}/N_B$</td>
<td>0.576</td>
<td>0.732</td>
</tr>
</tbody>
</table>

Fig. 3. Relationship between $\omega_B$ and $x$.

---

29 For the interpretation of the results, it is better to use $2d_{x_B}/N_B$ than $d_{x_B}$ since the former is normalized and gives the time spent in a $d_0$ dyad. The same applies for $d_{x_W}$ and $d_{x_B}$. 
works, which are limited in getting information about possible jobs. Because of the lack of good public transportation in the US, it is costly (both in terms of time and money) to commute to business centers to meet other types of people who can provide other source of information about jobs. If distant (black) workers mainly rely on their strong ties and if the latter are unemployed, there is little chance to escape unemployment and to find a job.30

Our result is also related to that of Calvó-Armengol and Jackson (2004). Contrary to the present model where only a very specific network structure (i.e., the dyad) is assumed, they explicitly model a social network (which can have any possible structure) where information flows between individuals having a link with each other. They show that an equilibrium with a clustering of workers with the same status (or distance to low-income families, the MTO programs help them relocate. By giving housing assistance to low-income families, the MTO programs help them relocate to better and richer neighborhoods.32

In light of our results, our model predicts that, relative to the ‘control’ group, displaced workers (from low- to high-rental-housing areas) should improve their social network and therefore their labor outcomes. If labor market participation is a good ‘proxy’ for labor outcomes, then the findings of Rosenbaum and Harris (2001) confirm some of the predictions of our model. Indeed, using the survey data from the MTO program in Chicago, the findings of these authors, based on interviews an average of 18 months after families moved from public housing to higher rental housing areas, show an increase in labor force participation and employment. More precisely, Rosenbaum and Harris (2001) show that: “After moving to their new neighborhoods, the Section 8 respondents were far more likely to be actively participating in the labor force (i.e., working or looking for a job), while for MTO respondents, a statistically significant increase is evident only for employment per se.”

Another way of reducing the unemployment rate of minorities in the context of our model is to observe that institutional connections can be engineered to create connections between job seekers and employers in ways that parallel social network processes. For example, scholars like Granovetter (1973) and Wilson (1996) have called for poverty reduction programs to “create connections” between employers and people and disadvantaged job seekers. While labor market intermediaries of all types aim to place workers with employers, especially with respect to poor populations, there is some disagreement about how these linkages work. Although strengthening connections being poor job seekers and employers

30 Even if this is beyond the scope of this paper, our model could also explain the emergence of a “black culture” in areas far away from jobs since distance to jobs induces the black population to rely mostly on strong ties. See Sáez-Martí and Zenou (2012) for an analysis on this issue.

31 See e.g. Wellman (1996) who finds that 42% of yearly contacts in individual networks took place with neighbors that lived less than 1 mile away. See also Guest and Lee (1983), Otani (1999), Conley and Tora (2002) and Bayer et al. (2008). They find that a one standard deviation increase in access to jobs (the spatial mismatch hypothesis) is associated with a 3.6% increase in annual hours worked. Similarly, Card and Rothstein (2007) find that, holding constant family background and other factors, a shift from a fully segregated to a completely integrated city closes about one-quarter of the raw black–white gap in SAT scores.

Our analysis offers interesting policy implications. We have shown that the neighborhood and distance to jobs are crucial in understanding labor-market outcomes of ethnic minorities. In that case, neighborhood regeneration policies are the right tool to use. Such policies have been implemented in the US and in Europe through the enterprise zone programs (Papke, 1994; Boarnet and Bogart, 1996; Mauer and Ott, 1999; Bondonio and Engberg, 2000; Bondonio and Greenbaum, 2007) and the empowerment zone programs (Busso et al., in press). For example, the enterprise zone policy consists in designating a specific urban (or rural) area, which is depressed, and targeting it for economic development through government-provided subsidies to labor and capital. The aim of the empowerment zone program is to revitalize distressed urban communities and it represents a nexus between social welfare policy and economic development efforts. By implementing these types of policies, one brings jobs to people and thus facilitates the flows of job information in depressed neighborhoods.

Policies that promote social integration and thus increase the interracial interactions between weak ties would also have positive effects on the labor-market outcomes of minority workers. Such policies, like the Moving to Opportunity (MTO) programs (Katz et al., 2001; Rosenbaum and Harris, 2001; Kling et al., 2005), have been implemented in the United States. By giving housing assistance to low-income families, the MTO programs help them relocate to better and richer neighborhoods.32

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32 See also Beaman (2012) and Eddin et al. (2003) who both exploit natural experiments, consisting of refugee resettlement programs in the US and Sweden, respectively, to try to disentangle social network referral effects from sorting or correlation in unobservable attributes. Beaman (2012) finds that a one standard deviation increase in the number of network members in a given year lowers the employment probability of someone arriving 1 year later by 4.9% points. She also finds that a greater number of tenured network members improves the probability of employment and raises the hourly wage. Eddin et al. (2003) find similar positive results.
is often seen as desirable, past research has questioned whether labor market intermediaries actually perform this function for those most in need. Recently, Autor and Houseman (2010) have argued that in the low wage sector temporary services can help workers in the short term, but is not helpful in the longer-term because temporary employment weakens workers’ search efforts for direct hire jobs. On the employer’s side, a number of studies have shown that employers often stigmatize low wage workers who are sent to them by public and private labor market intermediaries (e.g., Laufer and Winship, 2004). In general, employers are concerned that intermediaries will be adversely selected, constituting the labor market “left-overs” who could not find a job through other means (Autor, 2009; Burtless, 1985; Van Ours, 1994). While low-wage employers generally stigmatize job-seekers sent to them from labor market intermediary organizations, Fernandez (2010) shows how it is that such biases can be overcome. To the degree that intermediary organizations can help the firm address its recruitment problems, “created connections” can serve as functional substitutes for social network processes in matching people to jobs. Actors will choose to work with brokers to the extent that brokers provide goods or services that are of greater value than those available through alternative means.

To conclude, we believe that weak ties generate ‘bridging’ social capital. Bridging social capital refers to ties across networks that may make the resources exist in one network accessible to a member of another. These social relationships enable members to ‘get ahead’. These are needed to extend beyond family to connect to a broader range of resources and opportunities that exist in networks to which they are otherwise not connected. If black workers do not have access to weak ties (especially whites), in particular because they are segregated and separated from business centers, then their main source of information about jobs will be provided by their strong ties. But if the latter are themselves unemployed, the chance of escaping unemployment will be very low.

Acknowledgments

I’m grateful to Vernon Henderson, Joan de Martí and Pierre M. Picard as well as two anonymous referees for very helpful comments.

Appendix A

Proof of Proposition 1. We establish the proof in two steps. First, Lemma 1 characterizes all steady-state dyad flows. Lemma 2 then provides conditions for their existence. □

Lemma 1. There exists at most two different steady-state equilibria:
(i) a full-unemployment equilibrium \(U\) such that \(e^* = 0\) and \(u^* = 1\),
(ii) an interior equilibrium \(I\) such that \(0 < e^* < 1\) and \(0 < u^* < 1\).

Proof. By combining (5)-(8), we easily obtain:
\[
e^* = \left(1 - \omega + \omega e^*\right)\lambda + \frac{2\omega e^*}{\delta}d_0
\]
(51)
We consider two different cases.

(i) If \(e^* = 0\), then Eq. (51) is satisfied. Furthermore, using (5) and (6), this implies that \(d_1 = d_2 = 0\) and, using (7) and (9), we have \(d_0 = 1/2\) and \(u^* = 1\). This is referred to as steady-state \(U\) (full unemployment).

(ii) If \(e^* > 0\), then solving Eq. (51) yields:
\[
e^* = \frac{1}{\delta} \left(\frac{\delta^2}{2\omega d_0} - \delta\right) - \frac{(1 - \omega)}{\omega}
\]
Define \(Z = (1 - \omega)/\omega\), \(B = \delta/(\lambda \omega)\). This equation can now be written as:
\[
e^* = \frac{B^2}{2d_0} - B - Z
\]
(52)
Moreover, by combining (5) and (6), we obtain:
\[
d_1^* = \frac{2e^*/B}{d_0}, \quad d_2^* = \frac{(Z + e^*/B)d_0}{B^2}
\]
• Let us first focus on the case where \(e^* = 1\). In that case, it has to be that only \(d_2\)-dyads exist and thus \(d_0^* = d_2^* = 0\), which, using (53) implies that: \(d_2^* = 0\). So this case is not possible.
• Let us now focus on the case: \(0 < e^* < 1\) (which implies that \(0 < u^* < 1\)).

By plugging (52) and (53) in (7) and after some algebra, we obtain that \(d_0^*\) solves \(\Phi(d_0^*) = 0\) where \(\Phi(x)\) is the following second-order polynomial:
\[
\Phi(d_0) = -\frac{Z}{B^2} < 1 (which implies that \(0 < u^* < 1\)).
\]
(54)

Lemma 2.

(i) The steady-state equilibrium \(U\) always exists.
(iv) The steady-state equilibrium \(I\) exists when \(\delta < \lambda\omega + \sqrt{\omega(4 - \omega)}\).

Proof.

(i) In this equilibrium \(e^* = 0\), which implies that \(h(e) = (1 - \omega)/\lambda\)
and \(q(e) = 0\). There are only \(d_0\)-dyads so all workers are unemployed and will never receive a job offer since \(q(e) = 0\). So when a \(d_0\)-dyad is formed it is never destroyed and thus this equilibrium is always sustainable.

(ii) We know from Lemma 1 that a steady-state \(I\) exists and that \(e^* \neq 1\). We now have to check that \(e^* > 0\) and \(0 < d_0^* < 1/2\). Let us thus verify whether there exists some \(0 < d_0^* < 1/2\) such that \(\Phi(d_0^*) = 0\), where \(\Phi(\cdot)\) is given by (54). We have \(\Phi(0) = B^2 > 0\) and \(\Phi(0) = (1 + Z)/2 < 0\). Therefore, (54) has a unique positive root smaller than \(1/2\) if and only if
\[
\Phi(1/2) = \frac{1}{4} \left(\frac{B^2 - (1 + Z)}{Z}\right) = \frac{1}{4} \left(1 + \frac{1}{B}\right)(B^2 - B - Z) < 0.
\]

The unique positive solution to \(x^2 - x - Z = 0\) is \(1 + \sqrt{(4 - 3\omega)/\omega}\). Then, \(d_0^* < 1/2\) if and only if \(B < [1 + \sqrt{(4 - 3\omega)/\omega}]\), equivalent to:
\[
\frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4 - 3\omega)}}{2}
\]
Observe that \(d_0^* < 1/2\) guarantees that \(e^* > 0\). □

Proof of Proposition 4.

(i) By totally differentiating (12), we obtain:
\[
\frac{\partial d_0^*}{\partial \omega} = \frac{2d_0^* + \frac{d_0^* - d_0^*}{2d_0^*}}{2} - \frac{1}{2d_0^*}
\]
and thus
\[ \text{sgn} \frac{\partial d_0}{\partial \omega} = \text{sgn} \left[ \frac{\dot{d}_0^2 + \frac{1}{2 \omega^2} d_0 - \frac{\delta^2}{2 \omega^2 \omega^3}}{d_0 - \frac{\delta^2}{2 \omega^2 \omega^3}} \right] \]

Let us study
\[ \Phi(d_0) \equiv \frac{\dot{d}_0^2 + \frac{1}{2 \omega^2} d_0 - \frac{\delta^2}{2 \omega^2 \omega^3}}{d_0 - \frac{\delta^2}{2 \omega^2 \omega^3}} \]
\[ \Phi(0) = -\frac{\delta^2}{2 \omega^2 \omega^3} \leq 0 \]
\[ \Phi'(0) = \frac{\dot{d}_0 + \frac{1}{2 \omega^2} d_0}{d_0 - \frac{\delta^2}{2 \omega^2 \omega^3}} > 0 \text{ when } d_0 \geq 0 \]
\[ \Phi''(d_0) = \frac{\dot{d}_0^2}{d_0 - \frac{\delta^2}{2 \omega^2 \omega^3}} > 0 \]

We have a quadratic function that crosses only once the positive orthant. Let us calculate \( d_0 \) the value for which \( \Phi(d_0) \) crosses the \( d_0 \)-axis. For that, we have to solve: \( \Phi(d_0) = 0 \). It is easy to verify that:
\[ d_0 = \frac{\delta^2}{4 \omega^2} \left( 1 + \frac{8 \omega^2}{\delta} - 1 \right) > 0 \]

It should be clear that if \( d_0 < 1/2 \), then \( \Phi(d_0) < 0 \) for \( 0 < d_0 < 1/2 \) and thus \( \frac{\partial d_0}{\partial \omega} < 0 \). Let us thus check that \( d_0 < 1/2 \), which is equivalent to:
\[ \Omega \left( \frac{\delta}{\lambda} \right) \equiv 2 \left( \frac{\delta}{\lambda} \right)^3 - \omega \frac{\delta}{\lambda} - \omega^3 < 0 \]

We have:
\[ \Omega(0) = -\omega^3 < 0 \]
\[ \Omega' \left( \frac{\delta}{\lambda} \right) = 6 \left( \frac{\delta}{\lambda} \right)^2 - \omega \]

with
\[ \Omega' \left( \frac{\delta}{\lambda} \right) < 0 \iff \frac{\delta}{\lambda} < \frac{\omega}{\sqrt{6}} \]

As a result, when \( \lambda < \sqrt{6} \), \( d_0 < 1/2 \) and thus \( \frac{\partial d_0}{\partial \omega} < 0 \). Since we are in equilibrium \( \mathcal{I} \), condition (10) has to hold, i.e.
\[ \frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4 - 3 \omega)}}{2} \]

Let us show that
\[ \frac{\omega}{\sqrt{6}} < \frac{\omega + \sqrt{\omega(4 - 3 \omega)}}{2} \]

This inequality is equivalent to:
\[ 4 + 2\sqrt{\omega(4 - 3 \omega)} > \frac{2}{3} + 2\omega \]

which is always true since \( \omega < 1 \) and thus \( 4 > \frac{2}{3} + 2\omega \). Consequently, when condition (26) holds, i.e. \( \lambda < \sqrt{6} \), \( \frac{\partial d_0}{\partial \omega} < 0 \), then condition (10) is always satisfied.

(ii) By totally differentiating (11), we obtain:
\[ \frac{\partial d_0}{\partial \omega} = \frac{\partial b}{\partial \omega} \left[ \frac{B}{2} \right] - \frac{B^2}{4} \frac{1}{\omega^2} \frac{\partial d_0}{\partial \omega} + \frac{1}{\omega^2} \frac{\partial Z}{\partial \omega} \]
\[ = \frac{-\delta^3}{4 \omega^2 \omega^3} \frac{1}{\omega^2} \frac{\partial d_0}{\partial \omega} + \frac{1}{\omega^2} \]
\[ = \frac{\delta^3}{4 \omega^2 \omega^3} \frac{1}{\omega^2} \frac{\partial d_0}{\partial \omega} + \frac{1}{\omega^2} \]
\[ = \frac{\delta^3}{4 \omega^2 \omega^3} \frac{1}{\omega^2} \frac{\partial d_0}{\partial \omega} + \frac{1}{\omega^2} \]
\[ = \frac{\delta^3}{4 \omega^2 \omega^3} \frac{1}{\omega^2} \frac{\partial d_0}{\partial \omega} + \frac{1}{\omega^2} \]

Thus, we have:
\[ \frac{\partial e^*}{\partial \omega} > 0 \iff \frac{\delta^3}{4 \omega^2 \omega^3} \frac{1}{\omega^2} \frac{\partial d_0}{\partial \omega} + 1 > \frac{\delta^3}{2 \omega^4} \]

Since \( \frac{\partial d_0}{\partial \omega} < 0 \) when (26) holds, then it suffices to show that:
\[ \frac{\delta^3}{3 \omega^4} > 1 \]

which is always true if
\[ \frac{\delta^3}{2 \omega^4} < 1 \]

This is equivalent to:
\[ \frac{\delta}{\lambda} < \sqrt{2} \omega \]

But since
\[ \sqrt{\frac{\omega}{6}} < \sqrt{2} \omega \]

is always true, then condition (26) guarantees that both
\[ \frac{\partial d_0}{\partial \omega} < 0 \text{ and } \frac{\partial e^*}{\partial \omega} > 0 \]

Since \( e^* - 1 - \alpha^*, \frac{\partial e^*}{\partial \omega} < 0 \iff \frac{\partial e^*}{\partial \omega} < 0 \). To summarize, when condition (26) holds, i.e. \( \lambda < \sqrt{6} \), we have: \( \frac{\partial d_0}{\partial \omega} < 0 \), \( \frac{\partial e^*}{\partial \omega} < 0 \), and condition (10) is always satisfied.

Finally, from (13) and (14), it is easy to see that \( \frac{\partial \omega}{\partial \omega} \) and \( \frac{\partial \omega}{\partial \alpha^*} \) cannot be signed. \( \square \)

**Proof of Proposition 6.**

(i) Let us first prove that the optimal \( \omega^* \) is unique and is such that \( 0 < \omega^* < 1 \).

The second order condition is:
\[ \frac{\partial^2 e(\omega, \alpha^*)}{\partial \omega^2} [y - b - (1 - s) \alpha^*] - e''(\omega^*, \alpha^*) \]

Since \( C''(\omega^*, \alpha^*) \geq 0 \) (see (27)), we need to show that \( \frac{\partial^2 e(\omega, \alpha^*)}{\partial \omega^2} < 0 \).

By differentiating (17), we obtain:
\[ 2 \lambda \omega^2 \frac{\partial e}{\partial \omega} = \frac{(\lambda + 2 \delta) \sqrt{\lambda + 4 \delta(1 - \omega)}}{\sqrt{\lambda + 4 \delta(1 - \omega)}} - \frac{(\lambda + 2 \delta) \sqrt{\lambda + 4 \delta(2 - \omega)}}{\sqrt{\lambda + 4 \delta(2 - \omega)}} \]

By differentiating this equation, we obtain:
\[ 2 \lambda \omega^2 \frac{\partial^2 e}{\partial \omega^2} = \frac{(\lambda + 2 \delta) \sqrt{\lambda + 4 \delta(1 - \omega)}}{\sqrt{\lambda + 4 \delta(1 - \omega)}} \cdot \frac{1}{\omega^2} \]
\[ - \frac{(\lambda + 2 \delta) \sqrt{\lambda + 4 \delta(2 - \omega)}}{\sqrt{\lambda + 4 \delta(2 - \omega)}} \cdot \frac{2}{\omega^3} \]

Since (see (29))
\[ \frac{\partial e}{\partial \omega} = \frac{(\lambda + 2 \delta) \sqrt{\lambda + 4 \delta(1 - \omega)}}{\sqrt{\lambda + 4 \delta(1 - \omega)}} \cdot \frac{1}{\omega^2} > 0 \]
then
\[ - \frac{(\lambda + 2 \delta) \sqrt{\lambda + 4 \delta(1 - \omega)}}{\sqrt{\lambda + 4 \delta(2 - \omega)}} \cdot \frac{2}{\omega^3} = \frac{2 \lambda \omega^2 \frac{\partial e}{\partial \omega}}{\omega^2} < 0 \]

As a result, to show that \( \frac{\partial^2 e}{\partial \omega^2} < 0 \), it suffices to show that
\[\delta \left( \frac{\left((\lambda + 2\delta) \sqrt{\lambda[2 + 4\delta (1 - \omega)]} \right) - \lambda [2 + 4\delta (2 - \omega)] \right)}{\lambda [2 + 4\delta (1 - \omega)]} \right) < 0\]

We have

\[\frac{\partial}{\partial \kappa} \left( \frac{\left((\lambda + 2\delta) \sqrt{\lambda[2 + 4\delta (1 - \omega)]} \right) - \lambda [2 + 4\delta (2 - \omega)] \right)}{\lambda [2 + 4\delta (1 - \omega)]} \right) = \frac{2\delta \left[ 1 - \frac{(\lambda + 2\delta)}{\sqrt{\lambda[2 + 4\delta (1 - \omega)]}} \right] \sqrt{\lambda[2 + 4\delta (1 - \omega)]} - \lambda [2 + 4\delta (2 - \omega)]}{\lambda [2 + 4\delta (1 - \omega)]} \]

Since

\[\lambda[2 + 4\delta (1 - \omega)] > 0\]

let us show that

\[2\delta \left[ 1 - \frac{(\lambda + 2\delta)}{\sqrt{\lambda[2 + 4\delta (1 - \omega)]}} \right] \sqrt{\lambda[2 + 4\delta (1 - \omega)]} - \lambda [2 + 4\delta (2 - \omega)] < 0\]

This inequality is equivalent to:

\[\sqrt{\lambda[2 + 4\delta (1 - \omega)]} - \frac{\lambda [2 + 4\delta (2 - \omega)]}{\sqrt{\lambda[2 + 4\delta (1 - \omega)]}} < 0\]

which is equivalent to:

\[\omega < 2\delta\]

which is always true. As a result, we have shown that

\[\frac{\partial \omega}{\partial \delta^2} < 0\]

and thus the second order condition (55) is always satisfied.

Let us now show that \(\omega^*\), the solution to (29), is strictly positive and less than 1. The optimal \(\omega^*\) is given by:

\[\frac{\partial \omega (\omega^*)}{\partial \kappa} \left[ y - b - (1 - s)\tau x \right] = C_{\omega^*}(\omega^*, x)\]

Because of the above, we have shown that \(\frac{\partial^2 \omega}{\partial \delta^2} < 0\), and we know that \(\frac{\partial \omega}{\partial \delta}\) is a decreasing function and that, using (56), we have

\[\lim_{\omega \to 0} \frac{\partial \omega}{\partial \delta} = \frac{1}{\omega^2} \left( \frac{\lambda + 2\delta - \sqrt{\lambda^2 + 4\delta^2}}{2\delta} \right) = +\infty\]

since \(\lambda + 2\delta > \sqrt{\lambda^2 + 4\delta^2}\). We also have

\[\lim_{\omega \to 1} \frac{\partial \omega}{\partial \delta} = 0\]

Furthermore, because of (28), \(C_{\omega^*}(0, x) = 0\) and \(C_{\omega^*}(\omega^*, x)\) is an increasing function of \(\omega\). As a result, there is a unique \(\omega^*\), which is between 0 and 1.

(ii) Comparative statics. By differentiating (29), we obtain:

\[\frac{\partial \omega^*}{\partial \kappa} = \frac{\partial \omega (\omega^*)}{\partial \kappa} \left[ y - b - (1 - s)\tau x \right] - C_{\omega^*}(\omega^*, x) < 0\]

The denominator is the second-order condition (55), which is strictly negative. The numerator is strictly positive because \(\frac{\partial \omega}{\partial \delta} > 0\) and \(C_{\omega^*}(\omega^*, x) > 0\) (see (28)).

By differentiating (29), we obtain:

\[\frac{\partial \omega^*}{\partial \kappa} = -\frac{\partial \omega(\omega^*)}{\partial \kappa} \left[ y - b - (1 - s)\tau x \right] - C_{\omega^*}(\omega^*, x) > 0\]

\[\frac{\partial \omega^*}{\partial b} = -\frac{\partial \omega(\omega^*)}{\partial b} \left[ y - b - (1 - s)\tau x \right] - C_{\omega^*}(\omega^*, x) < 0\]

\[\frac{\partial \omega^*}{\partial \kappa} = -\frac{\partial \omega(\omega^*)}{\partial \kappa} \left[ y - b - (1 - s)\tau x \right] - C_{\omega^*}(\omega^*, x) > 0\]

\[\frac{\partial \omega^*}{\partial \kappa} = -\frac{\partial \omega(\omega^*)}{\partial \tau} \left[ y - b - (1 - s)\tau x \right] - C_{\omega^*}(\omega^*, x) < 0\]

This proves all the results of this proposition. □

Appendix B. Proof of Proposition 9:

Lemma 3. There exist two types of steady-state equilibria: (i) a full-unemployment equilibrium \(\ell\) such that \(e^*_1 = 0\) and \(u^*_1 = 1\), (ii) an interior equilibrium \(I\) such that \(0 < e^*_1 < 1\) and \(0 < u^*_1 < 1\), \(\forall j = B, W\).

Proof. By combining (44)–(48), we easily obtain:

\[e^*_j = \left( 1 - \eta_j \right) \lambda + \eta_j \lambda N_0 \left( \frac{N_1 - N_{0W} e^*_W}{N_0} + \delta \right) 2\eta_j \lambda \left( \frac{N_1 - N_{0W} e^*_W}{N_0} + \delta \right) \int d\theta_j \left( N_0 e^*_W + N_0 W e^*_W \right) d\theta_j\]

which is equivalent to:

\[e^*_j = \left( \lambda + \delta \right) N - \eta_j \lambda N_0 e^*_W - N_0 N_1 e^*_W \int d\theta_j \left( N_0 e^*_W + N_0 W e^*_W \right) d\theta_j\]

(57)

We consider the following different cases:

(i) If \(e^*_B = e^*_W = 0\), then Eq. (57) is satisfied. We have that \(d^*_B = d^*_W = 0\) and \(e^*_W = 0\) or \(e^*_B = 0\). This is referred to as steady-state \(\ell\) (full unemployment).

(ii) If \(e^*_B = 0\) and \(e^*_W > 0\), then solving Eq. (51) yields for blacks:

\[0 = (1 - \eta_1) N + \eta_1 \lambda N_0 e^*_W + \delta N_1 \eta_1 N_0 W e^*_W d\theta_j\]

The only way this equation can hold is that \(e^*_W = 0\) (indeed \(d^*_W = 0\), and thus \(d^*_W = 0\)). We are back in case (i) where \(e^*_W = 0\) and steady-state \(\ell\) prevails.

(iii) If \(e^*_W = 0\) and \(e^*_B > 0\), then by a similar reasoning as in case (i), we end up with \(e^*_W = 0\) and steady-state \(\ell\) prevails.

Let us assume that \(e^*_B > 0\) and \(e^*_W > 0\). Let see if it is possible to have either \(e^*_B = 1\) or \(e^*_W = 1\) or both. If either \(e^*_B = 1\) or \(e^*_W = 1\) or both \(e^*_B = e^*_W = 1\), then it is easily verified that \(d^*_B = d^*_W = d^*_1 < 0\), which is impossible. As a result, if \(e^*_B > 0\) and \(e^*_W > 0\), then it has to be that \(e^*_B < 1\) and \(e^*_W < 1\). We call this steady-state equilibrium \(I\) because it is an interior equilibrium for which \(0 < e^*_1 < 1\) and \(0 < e^*_1 < 1\). □

Let us now focus on the case \(0 < e^*_1 < 1\) and \(0 < e^*_W < 1\) (which implies that \(0 < u^*_1 < 1\) and \(0 < u^*_W < 1\)) and prove Proposition 9. Using (44)–(48), we have:

\[d^*_B = \left( \lambda + \delta \right) N - \eta_1 \lambda N_0 e^*_W - N_0 N_1 e^*_W \int d\theta_j \left( N_0 e^*_W + N_0 W e^*_W \right) d\theta_j\]
As a result, this is equivalent to:

\[ \lambda \left( \frac{N - N_{f_i^*} - N_{w^*} e_w}{N} \right) \left( \frac{\bar{\omega}_{d_{i8}^*} d_{i8}}{N_{f_i}} - \frac{\bar{\omega}_{d_{i8}^*} d_{i8}}{N_{f_i}} \right) < (\lambda + \delta) \left( \frac{\bar{\omega}_{d_{i8}^*} d_{i8} N_{f_i} - \bar{\omega}_{d_{i8}^*} d_{i8} N_{f_i}}{N_{f_i}} \right) \]

which is equivalent to:

\[ \frac{\lambda}{(\lambda + \delta)} \left( \frac{N - N_{f_i^*} - N_{w^*} e_w}{N} \right) \left( \bar{\omega}_d d_{i8} N_{f_i} - \bar{\omega}_d d_{i8} N_{f_i} \right) < \bar{\omega}_d d_{i8} N_{f_i} - \bar{\omega}_d d_{i8} N_{f_i} \]

(58)

Observe that, since, by Proposition 8, \( \bar{\omega}_d < \bar{\omega}_d \), then

\[ \frac{\bar{\omega}_d}{\bar{\omega}_d} > \frac{\bar{\omega}_d}{\bar{\omega}_d} \]

(i) Assume \( \bar{\omega}_d d_{i8} N_{f_i} > \bar{\omega}_d d_{i8} N_{f_i} \), which implies that

\[ \bar{\omega}_d d_{i8} N_{f_i} > \bar{\omega}_d d_{i8} N_{f_i}, \]

since \( \frac{\bar{\omega}_d}{\bar{\omega}_d} \left( \frac{N - N_{f_i^*} - N_{w^*} e_w}{N} \right) < 1 \), a sufficient condition for this inequality to be true is

\[ \bar{\omega}_d d_{i8} N_{f_i} - \bar{\omega}_d d_{i8} N_{f_i} > 1 \]

which is equivalent to

\[ \bar{\omega}_d d_{i8} N_{f_i} > \bar{\omega}_d d_{i8} N_{f_i}, \]

Thus by assuming (59), \( \bar{\omega}_d d_{i8} N_{f_i} > \bar{\omega}_d d_{i8} N_{f_i} \) is automatically satisfied and so is (58).

(ii) Assume that

\[ \frac{\bar{\omega}_d}{\bar{\omega}_d} < \frac{d_{i8} N_{f_i}}{d_{i8} N_{f_i}} < \frac{\bar{\omega}_d}{\bar{\omega}_d} \]

which implies that

\[ \bar{\omega}_d d_{i8} N_{f_i} < \bar{\omega}_d d_{i8} N_{f_i} \]

and

\[ \bar{\omega}_d d_{i8} N_{f_i} > \bar{\omega}_d d_{i8} N_{f_i} \]

The inequality (58) is never satisfied, thus this is not possible.

(iii) Assume finally that

\[ \frac{\bar{\omega}_d}{\bar{\omega}_d} < \frac{d_{i8} N_{f_i}}{d_{i8} N_{f_i}} \]

which implies that

\[ \bar{\omega}_d d_{i8} N_{f_i} < \bar{\omega}_d d_{i8} N_{f_i} \]

and

\[ \bar{\omega}_d d_{i8} N_{f_i} > \bar{\omega}_d d_{i8} N_{f_i} \]
\( \frac{\partial N}{\partial q_N} - \frac{\partial q_N}{\partial q_W} < 0 \)

Inequality (58) can be written as
\[
\frac{\lambda}{\lambda + \delta} \left( \frac{N - N W e^s - N W e^w}{N} \right) \left( \frac{\partial^2 q_N}{\partial q_W^2} - \frac{\partial^2 q_N}{\partial q_W \partial q_N} \right) > \frac{\partial q_N}{\partial q_W} - \frac{\partial q_N}{\partial q_W} \frac{\partial q_N}{\partial q_N}
\]

which is equivalent to:
\[
\frac{\lambda}{\lambda + \delta} \left( \frac{N - N W e^s - N W e^w}{N} \right) > \frac{\partial q_N}{\partial q_W} - \frac{\partial q_N}{\partial q_W} \frac{\partial q_N}{\partial q_N}
\]

For this inequality to be true, since the left-hand side is less than 1, it has to be that
\[
\frac{\partial q_N}{\partial q_W} - \frac{\partial q_N}{\partial q_W} \frac{\partial q_N}{\partial q_N} < 1
\]

which is equivalent to:
\[
\frac{\partial q_N}{\partial q_W} \left( 1 - \frac{\partial q_N}{\partial q_W} \right) < \frac{\partial q_N}{\partial q_N}
\]

This is impossible because we have assumed
\[
\frac{\partial q_N}{\partial q_W} < \frac{\partial q_N}{\partial q_N}
\]

which implies
\[
\frac{\partial q_N}{\partial q_W} \left( 1 - \frac{\partial q_N}{\partial q_W} \right) < \frac{\partial q_N}{\partial q_N} \frac{\partial q_N}{\partial q_W} < \frac{\partial q_N}{\partial q_N}
\]

To summarize, only case (i) is possible and condition (59) guarantees that \( e^s < e^w \). Condition (59) corresponds to (50) in the proposition. \( \square \)

References


