

Comment by Per Pettersson-Lidbom*

This short comment concerns the robustness of the estimates from the RD kink design used by Lundqvist et al. (2014). Specifically, it addresses two issues: (i) the robustness to the choice of bandwidth and (ii) the polynomial specification of the forcing variable. Regarding robustness, they write: “It is clear from the table [Table 2] that all estimates are highly statistically significant, irrespective of order of polynomial and bandwidth. The magnitude of the estimates is around 3, although that differs somewhat across the different specifications”.¹ For convenience, I have reproduced their Table 2 below. In the following, I will show that their statement of the robustness of their results is completely misleading.

Regarding the choice of bandwidth, the recommended and by now firmly established procedure when analyzing a regression-discontinuity RD design (kink or “jump”) is first to compute a benchmark estimate with a bandwidth selector, such as Imbens and Kalyanaraman (2012), and then check the sensitivity of the benchmark estimate to other choices of the bandwidth, such as double and half of the original bandwidth (e.g., Lemieux and Imbens 2008 and Card et al. 2012). Had they followed this approach then it had been very clear that the estimates of the kink are extremely sensitive to the choice of bandwidth. For example, the Imbens and Kalyanaraman (2012) bandwidth selector yields a bandwidth of 4 while the selector proposed by Calonico et al. (2014) gives a bandwidth of 3.² Columns 6 and 7 in Table 1 show the results from these bandwidth selectors.³ When the bandwidth is 4 then the local linear kink estimate is 1.48 and when bandwidth is 3 the estimate is -1.13. In addition, all the estimates are negative and sometimes statistically significant from zero for the second and third order polynomial specifications. Moreover, the estimates for half or twice the size of the benchmark bandwidth are all very different since they are both positive as well as negative estimates. As a result, the estimates from the RD kink design are extremely sensitive to the choice of bandwidth.

Turning to the polynomial specification of the forcing variable, I discovered that the polynomials are misspecified for the second and third order specifications. For example, they have specified the second order polynomials as

$$(1) y = \beta_0 + \beta_1(x-c) + \beta_2 D(x-c) + \beta_3(x-c)^2$$

where x is the forcing variable, c is the cutoff and D is the indicator for being above the cutoff. Specification (1) is misspecified since it omits the second order interaction term $D(x-c)^2$. Similarly, the cubic polynomial specification is also misspecified since it does not include two interaction terms. Omitting these interaction terms imply that the second and third order derivatives of the function, $f(x-c)$, must be equal on both sides of the cutoff c . This type of functional form restriction therefore makes the RD kink design flawed. In Table 1, I have re-estimated all their specifications using the

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¹ Theoretically, the parameter should be equal to 1, which ought to have raised concerns about the potential of misspecification of the RD kinked design if the estimate turns out to be around 3.

² The exact numbers are 4.4 and 3.2 respectively

³ As can be seen from Table 2, the number of observations is not small for these bandwidth choices since there are 708 observations from 174 local governments in the smallest bandwidth.

correct polynomial specifications.⁴ Although, the estimates for second-order specification are broadly similar to their estimates (somewhat larger), the estimates from the third-order specification are completely different as can be seen by comparing the results from Columns 1 and 2 in Table 1 with the corresponding results from their table. Now the estimates are 0.84 and -1.44 instead of 3.35 and 4.08, respectively. This clearly illustrates that the estimates are extremely sensitive to the degree of the polynomial specification.

I have also discovered a number of peculiarities in their reporting style. First, four out of the total of 12 entries in their table are left out intentionally although these results could easily been reported. These results are however reported in columns 3 and 4 of Table 1. Had they reported these results it had been very clear that the estimates are very sensitive to both bandwidth and polynomial specification since the estimates are both positive and negative. Secondly, if one marginally reduces their smallest bandwidth of 5 to 4.9, Columns 5 in Table 1 shows that the estimate from the local linear specification is no longer statistically significant at the 5 percent level. Third, after their paper was accepted for publications in this Journal they have chosen to include footnote 25 where they acknowledge that “When we estimate the first stage using higher-order polynomials with the two narrowest bandwidths, or with bandwidths smaller than 5, the point estimate starts to vary a lot.” However, the footnote was only included after I discovered that their results were sensitive to the choice of bandwidth (Pettersson-Lidbom 2013). Finally, looking at their table, it is also noteworthy that there is hardly any change in the size of the samples between full sample (column 1) and those with a bandwidth of 15 (column 2) or 10 (Column 3). There are 2,511 observations in the full sample while there are 2,346 and 2,047 observations in the two other samples.⁵ The only stark difference in sample size is when the bandwidth is reduced from 10 to 5, which reduces the sample size to 1,240. However, in a RD design (kink or “jump”) it is imperative to include a large number of different bandwidths (including small ones) and show different polynomial specifications (including flexible ones) in order to assess the sensitivity to bandwidth choices and the functional form specification of the forcing variable. If they had displayed the results in Table 1, it had been very clear that the estimates from the RD kink design are extremely non-robust. To conclude, the results of their study do not survive the conventional set of robustness checks of a RD kink design.

References

Calonico, S., M. D., Cattaneo, and R., Titiunik (2014): “Robust Data-Driven Inference in the Regression-Discontinuity Design,” Working paper, University of Michigan

Card, D., D. S. Lee, Z. Pei, and A. Weber (2012): “Nonlinear Policy Rules and the Identification and Estimation of Causal Effects in a Generalized Regression Kink Design,” Working paper, University of California at Berkeley.

Imbens, G., and T. Lemieux (2008): “Regression Discontinuity Designs: A Guide to Practice,” *Journal of Econometrics*, 142(2), 615-635.

⁴ In Pettersson-Lidbom (2013), I show that I can replicate all of their results in their table.

⁵ The number of municipalities is also almost the same across these bandwidths as can be seen from Table 1.

Imbens, G. W., and K. Kalyanaraman (2012): "Optimal Bandwidth Choice for the Regression Discontinuity Estimator," *Review of Economic Studies*, 79(3), 933-959.

Pettersson-Lidbom, P. (2013): "Stimulating Local Public Employment: Do General Grants Work?": Comment," Working paper, Stockholm University.

TABLE 2—FIRST-STAGE ESTIMATES

	Full sample	$h = 15$	$h = 10$	$h = 5$
$p = 1$	4.174*** (0.684)	3.636*** (0.739)	3.988*** (0.744)	1.980** (0.961)
$p = 2$	3.176*** (0.761)	3.118*** (1.055)		
$p = 3$	3.350*** (1.036)	4.076*** (1.351)		
Observations	2,511	2,346	2,047	1,241

Notes: For different bandwidths, h , and order of polynomials, p , the table reports estimates of α_1 in the first-stage equation (3) on cost-equalizing grants. Standard errors clustered on municipality are in parentheses.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

Source: The SALAR

Table 1. First stage estimates for a range of bandwidths and polynomial specifications

	Full (1)	h=15 (2)	h=10 (3)	h=5 (4)	h=4.9 (5)	h=4 (6)	h=3 (7)	h=2 (8)	h=1 (9)
P=1 (linear)	4.17*** (0.88)	3.64*** (0.68)	3.99*** (0.74)	1.98** (0.96)	1.79* (1.05)	1.48 (1.47)	-1.13 (1.87)	-3.82 (2.54)	-10.08** (5.05)
P=2 (quadratic)	4.70*** (1.02)	3.91*** (1.24)	1.62 (1.49)	-3.42 (2.81)	-3.43 (2.72)	-5.39** (2.77)	-5.59* (3.03)	-5.67 (3.75)	-10.26 (7.69)
P=3 (cubic)	0.84 (1.47)	-1.44 (2.03)	-1.23 (2.54)	-5.04 (3.10)	-5.19* (3.16)	-4.00 (3.05)	-5.48 (3.40)	-4.72 (4.19)	-9.04 (13.52)
Number of local governments	279	272	258	213	210	196	174	157	134
Observations	2,511	2,346	2,047	1,241	1,210	1,019	708	521	265