Agglomeration, City Size and Crime

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January 13, 2014

Abstract

This paper analyzes the relationship between crime and agglomeration where the land, labor, product, and crime markets are endogenously determined. Our main theoretical findings are: (i) better accessibility to jobs decreases crime in the short run but may increase crime in the long run; (ii) per-capita crime rate increases with city size; (iii) when allowing for endogenous policing, lower commuting costs make the impact of police on crime more efficient.

Key words: New economic geography, crime, agglomeration, policies.

JEL Classification: K42, R1.

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1 Introduction

Assuming that individuals are rational decision-makers who engage in either legal or illegal activities according to the expected utility from each activity, the economic literature shows that different crime-fighting urban policies can be implemented. First, more police resource may decline the crime rate by reducing the net benefits of crime because of a higher risk of detection and punishment. Some empirical studies reveal that an increase in the urban police force produced a 3% to 10% long-term decline in crime rates (Levitt, 1997).1 Second, a better access to legal labor market may raise the opportunity cost of illegal activity. Third, institutions may influence aversion to illegal activities of individuals.

The analysis of crime-reduction strategy fails, however, to take into account the effects of space (i.e. location of jobs and people) on criminal activities. Crime is an important social problem but also an urban phenomenon. It is well documented that there is relatively more crime in big than in small cities (Glaeser and Sacerdote, 1999; Kahn, 2010). For example, the rate of violent crime in cities with more than 250,000 population is 346 per 100,000 inhabitants whereas in cities with less than 10,000 inhabitants, the rate of violent crime is just 176 per 100,000 (Glaeser, 1998). Similar figures can be found for property crimes or other less violent crimes. Agglomeration creates a multiplier effect on crime rate through two channels. On the one hand, as mentioned in Glaeser and Sacerdote (1999), a higher city size induces greater expected pecuniary returns because criminals face a higher number of potential victims and a lower probability to be arrested. On the other hand, more workers in a city increase land rents or commuting costs, diminishing the opportunity cost of illegal activity.

The aim of this paper is to propose a model that captures some of the stylized facts observed in real-world cities and to analyze policies aiming at reducing crime in a spatial context. Our model delivers a full analytical solution that captures in a simple way how interactions between the land, product, crime and labor market yield agglomeration and criminal activity.

For that, we develop a model where city size and the type of activities (crime and

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1See also the natural experiment of Di Tella and Schargrodsky (2004) who look at a redeployment in Buenos Aires that followed the bombing of a Jewish center in 1994. Car thefts fell by 75 percent on the blocks where the extra police were stationed, and did not rise elsewhere. Klick and Tabarrok (2005) for Washington, D.C., Poutvaara and Priks (2009) for Stockholm, and Draca et al. (2011) for London find similar results.
job) are endogenous within a full-fledged general equilibrium model. The individuals are freely mobile between and within the cities. We consider four different markets in each city: land, labor, product, and crime. The land market is assumed to be competitive and land is allocated to the highest bidders in each city. Land is owned by absentee landlords. The labor market is also competitive and wage are determined by free entry. Monopolistic competition prevails in the product market, which implies that each firm has a monopoly power on her variety. Finally, the crime market is competitive and the mass of criminals is determined by a cost-benefit analysis for each person. Hence, a land market, spatial frictions, and agglomeration economies are introduced in our general equilibrium model.

Let us be more precise. We first develop a framework where the city size in population is exogenous (Section 3). Our model takes into account the following fundamental aspects of urban development: larger cities are associated with higher nominal wages (Baum-Snow and Pavan, 2011), more varieties (Handbury and Weinstein, 2014), higher housing and commuting costs (Fujita and Thisse, 2013) and higher crime rate (Glaeser and Sacerdote, 1999). Individuals are heterogeneous in their incentives to commit crime. They freely choose their location within the city and decide whether to become a criminal or a worker. In Section 4, we show the following results: (i) Higher commuting costs or equivalently worse job access lead to more criminal activities in the city; (ii) The impact of commuting costs on criminal activities is higher when the city size increases; (iii) Criminal activities increase more than proportionally when the city size rises.

Most of these results are empirically documented. For example, concerning (i), using 206 census tracts in city of Atlanta and Dekalb county and a state-of-the-art job accessibility measure, Ihlanfeldt (2002, 2006, 2007) demonstrates that modest improvements in the job accessibility of male youth, in particular blacks, cause marked reductions in crime, especially within category of drug-abuse violations. He found an elasticity of 0.361, which implies that 20 additional jobs will decrease the neighborhood’s density of drug crime by 3.61%. If we now consider (iii), then this is true in most cities in the world. Glaeser and Sacerdote have shown that this was true for the United States. If we look at European cities, the same pattern emerges. Table 1 documents this pattern for France.
Table 1: Relationship between the number of crimes reported by the police in France and city size

<table>
<thead>
<tr>
<th>Type of city</th>
<th>Crime rate per 1,000 inhabitants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris</td>
<td>109</td>
</tr>
<tr>
<td>Cities larger than 100,000 inhabitants (without Paris)</td>
<td>94</td>
</tr>
<tr>
<td>50,000 to 100,000 inhabitants</td>
<td>76</td>
</tr>
<tr>
<td>25,000 to 50,000 inhabitants</td>
<td>69</td>
</tr>
<tr>
<td>Cities less than 25,000 inhabitants</td>
<td>65</td>
</tr>
</tbody>
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Source: Direction Centrale de la Police Judiciaire, 2008, Authors’ calculations.

Nevertheless, we have to discuss the robustness of results by considering that households have two options in response to crime risk: they can vote for anti-crime policies or they can vote with their feet (Linden and Rockoff, 2008). In Section 5, we introduce an urban government that levies a tax on workers in order to finance policy resources to fight criminal activities. We show that there is a U-shaped relationship between the crime rate in the economy and the tax rate so that increasing resources to fight crime financed by taxes on workers can backfire if the tax rate is too high. We also show that lower commuting costs or better job access make the impact of police on crime more efficient.

These are short-run effects. In the long run, the individuals can migrate to avoid the high levels of crime in the large city. In Section 6, we extend our model by considering a system of two cities where the city size is endogenous because of the mobility of individuals between the two cities. This extension implies that, even if cities are ex ante identical, the ex post differences in crime rate, economic structure and population size across cities emerge as the unintentional outcome of a myriad of decisions made by firms and households pursuing their own interest. We totally characterize the three different stable equilibria as a function of commuting costs. When the commuting costs are high enough, the population of workers and criminals is evenly distributed between the two cities. When the commuting costs take intermediate values, there is a large and a small city where in the former there are more criminals and workers than in the latter. When commuting costs are low enough, there is a single city. This means that, when the commuting costs take intermediate values, even though the level of criminal activity creates a negative externality on workers, agglomeration takes place and the larger city attracts the majority of workers and has the higher share of criminals.

This framework allows us to study the effects of lower commuting costs in each city on criminal activity in the economy. If a decrease in commuting costs (or better access)
reduces crime in the short-run (when the city size is unchanged) because urban costs experienced by workers decline, this is no longer true in the long run when agents are perfectly mobile between cities. Indeed, a reduction in commuting costs induces more people and jobs to move to the larger city. In that case, a decrease in commuting costs has an ambiguous effect on crime since, in bigger cities, people earn higher wages but also experience higher urban costs and obtain higher proceeds from crime. We show that criminal activity increases in the economy when commuting costs decline only if the size of the agglomeration is high enough. This implies that the improvement of transportation will decrease crime only in very large cities where agglomeration is important but the global effect is in general ambiguous. As noted above, Ihlanfeldt (2002, 2006, 2007) shows that, for the city of Atlanta in the United States, an improvement in job access (lower commuting costs) always decreases crime. Studying the city of Bogotá, Columbia, Olarte Bacares (2014) shows that this effect is not always positive and depends on the type of crime committed. Indeed, in 2000, Bogotá implemented an ingenious and innovative Bus Rapid Transport (BRT) system called Transmilenio (TM) but only affected some areas of the city. Olarte Bacares (2014) shows that the presence of the Transmilenio system in an area increases the number of “thefts from people” as compared to areas without a TM system while it decreases house breaking. This means that policy makers should be careful in implementing a transportation policy because it may backfire on crime. Our paper provides an explanation of why this might be the case.

2 Related literature

To our knowledge, three types of theoretical models have integrated space and location in criminal behavior. First, social interaction models, which state that the individual criminal behavior not only depends on individual incentives but also on the behavior of peers and neighbors, are a natural way of explaining the concentration of crime by area. An individual is more likely to commit crime if her peers commit than if they do not commit crime (Glaeser et al., 1996; Calvó-Armengol and Zenou, 2004; Ballester et al., 2006, 2010; Calvó-Armengol et al., 2007; Patacchini and Zenou, 2012). This explanation is backed up by several empirical studies that show that indeed neighbors matter in explaining crime behaviors. Case and Katz (1991), using the 1989 NBER survey of young living in low-income, inner-city Boston neighborhoods, found that residence in a neighborhood in which many other youths are involved in crime is associated with an increase in an individual’s probability of committing crime. Exploiting a natural
experiment (i.e. the Moving to Opportunity experiment that has assigned a total of 614 families living in high-poverty Baltimore neighborhoods into richer neighborhoods), Ludwig et al. (2001) and Kling et al. (2005) find that this policy reduces juvenile arrests for violent offences by 30 to 50 percent, relative to a control group. This also suggests very strong social interactions in crime behaviors. Bayer et al. (2009) consider the influence that juvenile offenders serving time in the same correctional facility have on each other’s subsequent criminal behavior. They also find strong evidence of learning effects in criminal activities since exposure to peers with a history of committing a particular crime increases the probability that an individual who has already committed the same type of crime recidivates that crime. Finally, Patacchini and Zenou (2012) find that peer effects in crime are strong, especially for petty crimes.2

Second, Freeman et al. (1996) provide a theoretical model that explains why criminals are concentrated in some areas of the city (ghettos) and why they tend to commit crimes in their own local areas and not in rich neighborhoods. Their explanation is based on the fact that, when criminals are numerous in an area, the probability to be caught is low so that criminals create a positive externality for each other. In this context, criminals concentrate their effort in (poor) neighborhoods where the probability to be caught is small.3 This explanation has also strong empirical support (see e.g. O’Sullivan, 2000).

Finally, Verdier and Zenou (2004) show that prejudices and distance to jobs (legal activities) can explain crime activities, especially among blacks. If everybody believes that blacks are more criminal than whites -even if there is no basis for this- then blacks are offered lower wages and, as a result, locate further away from jobs. Distant residence increases even more the black-white wage gap because of more tiredness and higher commuting costs. Blacks have thus a lower opportunity cost of committing crime and become indeed more criminal than whites. In this model, beliefs are self-fulfilling.4

Our contribution is different since we focus on the impact of inter-city mobility, city size and agglomeration effects on criminal behavior. We believe this is the first model that integrates crime and agglomeration economics in a unified framework by modeling the labor, crime, land and product market. In particular, our model enables us to (i) reproduce different stylized facts observed in real-world cities by showing that there exist big cities with both a high number of workers and criminals and (ii) study the efficiency of

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2 See also Helsley and Zenou (2014) who show that the interaction between the social and geographical space can “amplify” the behavior of individuals.

3 See also Deutsch et al. (1987).

4 For more detailed surveys on the spatial aspects of crime, see Zenou (2003), Raphael and Sills (2005) and O’Flaherty and Sethi (2015).
different policies aiming at reducing crime in an urban context. One of our main results is to show that better accessibility to jobs decreases crime in the short run but may increase it in the long run.\footnote{Using a different model, O’Sullivan (2005) shows that gentrification results from a decrease in crime or an increase in the frequency of travel to the center.}

Our model is also related to the economic geography literature and can be viewed as a contribution to the theory of agglomeration with heterogeneous agents (for a recent survey, see Behrens and Robert-Nicoud, 2015). Indeed, while much attention has been devoted to the impact of commuting costs and pollution on the spatial organization of the economy, rather little consideration has been given to the interactions between crime and urbanization. Yet, crime as pollution and congestion is an important factor that discourages the growth of cities (Tolly, 1974; Kahn, 2010).\footnote{Observe that, contrary to the cross-city quality-of-life literature (see, for example, Desmet and Rossi-Hansberg, 2013), we here endogenize the “production” of city amenities.}

3 The model

Our model describes a city where each individual chooses where to locate and whether to become a criminal or not. Each individual can either be a worker or a criminal but not both. We consider four different markets: land, labor, product, and crime. As it is standard in the urban literature (Zenou, 2009; Fujita and Thisse, 2013), the land market is assumed to be competitive and land is allocated to the highest bidders in the city. Land is owned by absentee landlords. The labor market is also competitive and wage are determined by free entry. Monopolistic competition prevails in the product market, which implies that each firm has a monopoly power on her variety. Finally, the crime market is competitive and the mass of criminals is determined by a cost-benefit analysis exerted by each individual.

The city is formally described by a one-dimensional space and is described in Figure 1. It can accommodate firms, criminals and workers. The masses of criminals and workers are denoted by $C$ and $L$, respectively. The city has a Central Business District (CBD) located at $x = 0$ where all firms are established.\footnote{See the survey by Duranton and Puga (2004) for the reasons explaining the existence of a CBD.} Individuals live between the CBD and the city fringe. Without loss of generality, we focus on the right-hand side of the city, the left-hand side being perfectly symmetric. Distances and locations are expressed by the
same variable $x$ measured from the CBD.

[Insert Figure 1 here]

Each individual consumes a residential plot of fixed size (normalized to 1), regardless of her location and her status (criminal or worker). Let $\lambda = C + L$ denote the population residing in the city so that the size of the right-hand side of this city is given by $\lambda/2$.

### 3.1 Preferences and budget constraints

Individuals are heterogeneous in their incentives to commit crime. They have different aversions to crime, denoted by $c$, so that higher $c$ means more aversion towards crime. We assume that $c$ is uniformly distributed on the interval $[0, 1]$. The parameter $c$ could be interpreted in numerous other ways (ability to commit crime, disutility of working, etc.).

The individuals consume two types of goods: a homogenous good and non-tradeable goods (where trade costs are prohibitive), which are horizontally differentiated by varieties. One can think of a bundle of services locally produced like, for example, restaurants, retail shops, theaters, etc. (Glaeser et al, 2001). Preferences are the same across individuals and, for $v \in [0, n]$, the utility of a consumer is given by:

$$
U(q_0; q(v)) = \alpha \int_0^n q(v)dv - \gamma \left( \int_0^n q(v)dv \right)^2 - \frac{\beta - \gamma}{2} \int_0^n [q(v)]^2dv + q_0
$$

where $q(v)$ is the quantity of variety $v$ of services and $q_0$ the quantity of the homogenous good, which is taken as the numéraire. All parameters $\alpha$, $\beta$ and $\gamma$ are positive; $\gamma > 0$ measures the substitutability between varieties, whereas $\beta - \gamma > 0$ expresses the intensity for the love for variety. Equation (1), which has been extensively used in the economic geography literature (see, e.g. Ottaviano et al., 2002; Tabuchi and Thisse, 2002; Melitz and Ottaviano, 2008; Gaigné and Thisse, 2014), is a standard quasi-linear utility function with a quadratic sub-utility, symmetric in all varieties. Although this modeling strategy gives our framework a fairly strong partial equilibrium flavor, it does not remove the interaction between product, labor, land and crime markets, thus allowing us to develop a full-fledged model of agglomeration formation, independently of the relative size of the service sector. Note that the utility (1) degenerates into a utility function that is quadratic in total consumption $\int_0^n q(v)dv$ when $\beta = \gamma$.

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Each worker commutes to the CBD and pays a unit commuting cost per unit of distance of \( t > 0 \), so that a worker located at \( x > 0 \) bears a commuting cost equals to \( tx \). The budget constraint of a worker residing at \( x \) is then given by:\(^9\)

\[
\int_{0}^{n} p(v)q(v)dv + q_0 + R^w(x) + tx = I - \xi C + \bar{\eta}_0
\]

where \( p(v) \) is the price of the service good for variety \( v \), \( R^w(x) \) is the land rent paid by workers (superscript \( w \)) located at \( x \) and \( I \) is the income of a worker. The homogenous good is available as an endowment denoted by \( \bar{\eta}_0 \). In this formulation, \( C \) is the mass of criminals while \( \xi \) is a lump-sum amount stolen by each criminal. In other words, we assume that there are negative externalities of having criminals so that the higher is the number of criminals, the higher are these negative externalities. On average, the stolen amount per worker increases with the mass of criminals in the city. In this formulation, each worker is “visited” by \( C \) criminals who each takes \( \xi \). By the law of large numbers, this means that, on average, a worker meets \( C \) criminals. This also implies that each criminal “robs” \( L \) workers and takes \( \xi \) from each worker so that her average proceeds from crime is \( \xi L \).

Each worker chooses her location so as to maximize her utility (1) under the budget constraint (2).

The budget constraint of a criminal residing at \( x \) is given by:

\[
\int_{0}^{n} p(v)q(v)dv + q_0 + R^c(x) = \xi L + \bar{\eta}_0
\]

where \( L \) is the mass of workers and \( R^c(x) \) is the land rent paid by criminals (superscript \( c \)) located at \( x \). Observe that individuals are here specialized so that workers only work and do not commit crime while criminals only commit crime. As mentioned above, from equation (3), we see that the proceeds from crime \( \xi L \) are increasing in the number of workers \( L \) in the city. For simplicity and to be consistent with (2), each criminal is assumed to steal a fraction \( \xi \) from these workers. Hence, there is no direct competition between criminals. However, as we will show below, they are indirectly connected via the mass of workers in a city. Indeed, more criminals induce less workers and, in turn, lower pecuniary returns per criminal. In other words, the income of each criminal depends on the mass of criminals in each city.

\(^9\)Note that housing and commuting costs account for a large share of consumers’ expenditures. Housing accounts on average for 26% of household budgets in the United-States (23% in France) while 18% of total expenditures (15% in France) is spent on car purchases, gasoline, and other related expenses which do not include the cost of time spent in traveling (Bureau of Labor Statistics, 2012).
The assumption that criminals do not bear a commuting cost is just made for simplicity. This assumption could be viewed as inconsistent with the assumption that each criminal steals a given amount $\xi$ from each worker, which implies that criminal activity should involve commuting. One easy way to justify this assumption is to assume that, from time to time, workers go the “criminal area” (the city boundary between $L/2$ and $N/2$; see Figure 1) and are robbed there. A more convincing way is to assume that all crimes occurred in the CBD. Then, since all criminals reside in the same area of the city, we could assume that, within this area, the commuting costs are constant and equal to $\Theta$ so that each criminal have to pay a commuting cost equals to $\Theta + t'L/2$ to commute to the CBD and commit her crimes there, where $t'$ is the commuting cost per unit of distance for each criminal. This term would have to be added to the left-hand side of equation (3). Within this framework, we can show that all our results hold if $t'$ (criminal’s commuting cost) is lower than $t$ (worker’s commuting cost), which can be justified by the fact that criminals indeed use cheaper commuting modes or commute less frequently than workers.

3.2 Technology and market structure

Each variety of services $v$ is supplied by a single firm and any firm supplies a single differentiated service under monopolistic competition. Labor is the only production factor. The fixed requirement of labor needed to produce variety $v$ is denoted by $\phi > 0$ and the corresponding marginal requirement is constant. Without loss of generality, this marginal requirement is set equal to zero. Note that a lower value of $\phi$ means a higher labor productivity. Hence, the profit made by a service firm $v$ is given by:

$$\pi(v) = p(v)q(v)\lambda - \phi I$$

where $p(v)$ is the price quoted by a service firm $v$ and $I$ is the wage paid by a service firm to her workers. Consistent with (2) and (3), formulation (4) means that both criminals and workers consume all goods. As we will see below, workers and criminals consume the same level of each variety because, given our utility function, the demand for differentiated services does not depend on income.

3.3 Services market, equilibrium prices and consumer’s surplus

The maximization of utility (1) under the budget constraint (2) or (3) leads to the demand for a service $v$ given by:

$$q(v) = \frac{\alpha}{\beta} - \frac{p(v)}{\beta - \gamma} + \frac{\gamma}{\beta(\beta - \gamma)} \frac{P}{n}$$
where the price index $P = \int_0^n p(v)dv$ is defined over the range of services produced in the city because this good is non-tradeable. Since the demand for each differentiated product does not depend on the net income (wage minus land rent and commuting costs) of each individual, it does not matter if the budget constraint is (2) or (3).

Each service firm determines its price by maximizing (4), using (5) and treating the price index $P$ as a parameter. Solving the first-order conditions yield the equilibrium prices of a non-tradeable service for a variety in the city, given by:

$$p^* = \frac{\alpha(\beta - \gamma)}{\beta + (\beta - \gamma)}$$  \hspace{1cm} (6)

which is the same for all varieties. Hence, the consumer surplus generated by any variety at the equilibrium market price $p^*$ is equal to:

$$S^*(v) = \frac{q^*(v) [p(0) - p^*]}{2} = \frac{(\alpha - p^*)^2}{\beta} = \frac{\alpha^2 \beta^2}{[\beta + (\beta - \gamma)]^2 + \beta}$$

where $p(0)$ is the inverse demand when $q(v) = 0$. Note that the consumer surplus $S^*(v)$ for any variety is the same because all varieties are available at the same price. However, the consumer surplus generated by all varieties available in the city, i.e. $\sum S^*$, changes with the supply of varieties in the city. Without loss generality, we then set $\sum S^* = 1$. This assumption does not affect qualitatively the properties of the spatial equilibria but greatly simplifies the algebra.

### 3.4 Urban labor market and equilibrium wages

Because labor is the only factor of production, the number of varieties available in each city is proportional to the mass of individuals living and working in this city. More precisely, the labor market-clearing conditions imply

$$n = L/\phi$$  \hspace{1cm} (7)

Urban labor markets are local and the equilibrium wage is determined by a bidding process in which firms compete for workers by offering them higher wages until no firm can profitably enter the market. In other words, operating profits are completely absorbed by the wage bill. This is a free-entry condition that sets profits (4) equal to zero so that, using (5) and (6), we find that the equilibrium wage paid by service firms established in the city is equal to:

$$I^* = \varphi \lambda$$  \hspace{1cm} (8)
where
\[ \varphi \equiv \left( \frac{p^*}{\phi} \right)^2 = \frac{1}{\phi} \left[ \frac{\alpha (\beta - \gamma)}{\beta + (\beta - \gamma)} \right]^2 \]

Observe that \( \varphi \) corresponds to the real labor productivity. In accordance with empirical evidence, the equilibrium wage \( I^* \) increases with population size \( (\lambda = L + C) \). However, the equilibrium wage is unaffected by the residential location of each worker within the city. It is also worth stressing that the equilibrium wage rises with product differentiation (low \( \gamma \)) and labor productivity (low fixed requirement in labor \( \phi \)).

### 3.5 Land market and equilibrium land rents

Let us first determine the equilibrium land rent for the workers. From the budget constraint (2), we obtain:

\[ q_0 = I - R^w(x) - tx - \xi C + \bar{q}_0 - \int_0^n p(v)q(v)dv \]

By plugging this value and the equilibrium quantities and prices (5) and (6) into the utility (1), we obtain:

\[ V^w(x) = n + I^* - R^w(x) - tx - \xi C + \bar{q}_0 \]

where \( I^* \) is defined by (8). Because of the fixed lot size assumption (normalized to 1), the value of the consumption of the nonspatial goods \( \int_0^n q(v)p(v)dv + q_0 \) at the residential equilibrium is the same regardless of the worker’s location. Using (2), this implies that the total urban costs, \( UC^w(x) \equiv R^w(x) + tx \), borne by a worker living at location \( x \) in the city, is constant whatever the location \( x \).

Since criminals do not commute to the CBD, which implies that their utility does not depend on location \( x \), we have: \( R^c(x) = R^c \). In equilibrium, since it is costly for workers to be far away from the CBD, they will bid away criminals who will live at the city fringe, paying the opportunity cost of land \( R^a \) so that \( R^c = R^a \). Without loss of generality, the opportunity cost of land is normalized to zero, i.e. \( R^a = 0 \).

For workers, given \( V^w(x) \), the equilibrium land rent in the city must solve \( \partial V^w(x)/\partial x = 0 \) or, equivalently, \( \frac{\partial R^w(x)}{\partial x} + t = 0 \), whose solution is \( R^w(x) = r_0 - tx \), where \( r_0 \) is a constant.\(^{10}\) Because the opportunity cost of land \( R^a \) is equal to zero, it has to be that

\(^{10}\) We could easily extend the model to take into account the fact that workers residing further away from criminals experience lower negative externalities. For example, if we assume that \( \xi(x) = \xi_0 + \xi_1 x \) so that the criminals steal less to workers residing closer to the center, we can show that the results remain qualitatively the same.
$R^w(L/2) = 0$ (see Figure 1) so that $r_0 = L/2$. As a result, the equilibrium land rent for workers is equal to:

$$R^w(x) = t \left( \frac{L}{2} - x \right)$$  \hspace{1cm} (11)

and the urban costs borne by a worker are given by:

$$UC^w = t \frac{L}{2}.$$  \hspace{1cm} (12)

As expected, the urban costs increases with commuting costs and the mass of workers.

## 4 Criminal activities and city size

Let $\theta = C/\lambda$ be the share of criminals in the city so that

$$C = \theta \lambda \quad \text{and} \quad L = (1 - \theta) \lambda$$

In this section, $\theta$ is endogenously determined for any given population size $\lambda$. An individual becomes criminal if and only if $V^c - V^w > 0$, where $V^c$ and $V^w$ are the utility of a criminal and a worker at the equilibrium prices. Plugging the equilibrium land rent (11) into (10), we obtain:

$$V^w = n + \bar{I}^s - \xi C - tL/2 + \bar{q}_0$$  \hspace{1cm} (13)

From the budget constraint of criminals, (3), we obtain:

$$q_0 = \xi L - R^a + \bar{q}_0 - \int_0^n p(v)q(v)dv$$

By plugging this value and the equilibrium quantities and prices (5) and (6) into the utility (1) and adding the cost $c$ of committing crime, we obtain:

$$V^c = n + \xi L - c + \bar{q}_0$$  \hspace{1cm} (14)

Thus, the value of $c$ making a marginal individual indifferent between committing a crime and working is $\bar{c}$ and is given by

$$\bar{c} = (\xi - \varphi) \lambda + \frac{t(1 - \theta)}{2} \lambda$$  \hspace{1cm} (15)

where $\varphi$ is defined by (9). Hence, because of the uniform distribution of $c$, the fraction of criminals is $\theta = \bar{c}$. The equilibrium share of criminals $\theta^*$ is thus given by:

$$\theta^* = \frac{\lambda t + 2\lambda(\xi - \varphi)}{\lambda t + 2}$$  \hspace{1cm} (16)
It is easily verified that \( \theta^* < 1 \) if and only if \( \lambda (\xi - \varphi) < 1 \). A sufficient condition for \( \theta^* < 1 \) is \( \xi < \varphi \). We thus assume throughout that:

\[
\xi < \varphi
\]  

(17)

Moreover, \( \theta^* > 0 \) if and only if \( t > 2(\varphi - \xi) \). In this context, it is easily checked that \( \partial \theta^*/\partial t > 0 \) as soon as \( \theta^* < 1 \), which is guaranteed by (17). We have the following comparative statics results.

**Proposition 1** Assume (17) and \( t > 2(\varphi - \xi) \). Then,

(i) **Worse job access (or higher commuting costs) leads to more criminal activities in the city;**

(ii) **The impact of commuting costs on criminal activities is higher when the city size increases;**

(iii) **The more differentiated the products are or the lower labor productivity is, the higher is the mass of criminals in the city;**

(iv) **Criminal activities increase more than proportionally when the city size rises.**

The first result (i) is due to the fact that, when commuting costs increase, the total urban cost also increases so that the net wages of workers is reduced. This, in turn, leads to a larger fraction of individuals committing crime. This implies that a transport policy that aims at improving job access (lower \( t \)) would reduce criminal activities in the short run. We will investigate in more detail this issue in the next section. In addition, because \( \partial^2 \theta^*/\partial \lambda \partial t > 0 \), the impact of commuting costs on criminal activities is higher when the city size increases (result (ii)). This is because urban costs are positively correlated with population size and thus the effect of commuting costs on land rents is higher in larger cities. Finally, \( \partial \theta^*/\partial \gamma > 0 \), which means that the mass of criminals decreases with more differentiated products (lower \( \gamma \)). Indeed, when \( \gamma \) decreases, \( \varphi \) increases, meaning that the revenue per worker is higher for firms because there is less price competition and thus workers obtain higher wages, which deter criminality. A similar effect can be found for the labor productivity \( (1/\phi) \) since \( \partial \theta^*/\partial \phi > 0 \) (result (iii)).

If we now look at the effect of city size on criminal activities, we see that

\[
\frac{\partial \theta^*}{\partial \lambda} = \frac{t - 2(\varphi - \xi)}{1 + t\lambda/2} > 0
\]

14
as long as $\theta^* > 0$, which is result (iv). Thus, a larger population in a city triggers more criminals in this city. This is because, in bigger cities, people are more induced to be criminals since they experience higher urban costs (land rents and commuting costs) and obtain higher proceeds from crime (see (15)). However, they also obtain a higher wage. We show that the former effect dominates the latter and thus crime per capita increases with larger cities. This gives a microfoundation to the empirical result found in Glaeser and Sacerdote (1999). Even if the amount of stolen wealth by each criminal $\xi(1 - \theta)\lambda$ is proportional to the city size, there is proportionally more crime in bigger cities than in smaller cities.

5 Police resources and crime

In this section, we check whether our main results hold when the city government implements a policy fighting criminal activities. We now consider that the number of active criminals in the city is given by $(1 - a)C$, where $a \in [0, 1]$ is the share of criminals in jail or equivalently the individual probability of being caught (by the law of large number). This share of active criminals depends on the resources affected by the local government of the city to fight criminal activity. We consider that the probability of arresting a criminal is an increasing function of the per capita public resources, denoted by $T$. These public expenditures are financed by a local head tax $\tau$ paid by workers. More precisely, we assume that $a \equiv f(T)$, where $T = \tau L/\lambda$, which are the total resources per capita invested in police for the local government. For a same level of tax revenue, the probability to be arrested is lower in a larger a city. Increasing the resources devoted to police (higher $\tau$ or/and $L$) and decreasing the population size ($\lambda$) raises the probability of arresting a criminal.

In addition, we assume, quite naturally, that $f(0) = 0$, $f'(T) > 0$ and $f''(T) < 0$. We also assume that Because $L = (1 - \theta)\lambda$, we have $T = \tau(1 - \theta)$. As a result, if $\theta = 1$, then $f(T) \to 0$, i.e. the probability of being arrested tends to zero when there is no worker because there is no public resource. In contrast, if $\theta = 0$, then $T = \tau$, which is its maximum value. In that case, we assume that $f(\tau) \to 1$ when $T \to \tau$, i.e. the probability of being arrested is close to 1 when a worker becomes a criminal if there is no criminal in the city.

The timing is as follows. In the first stage, the local government chooses its tax policy $\tau$. In the second stage, types (or honesty parameters $c$) are revealed and individuals decide to commit crime or not while product, land and labor markets clear. As usual, the game
is solved by backward induction.

5.1 Taxation and share of criminals

Let us solve the stage where crime is decided for a given taxation level \( \tau \). Using (14), the indirect utility of a criminal is now given by

\[
V^c = n + \left[1 - f(T)\right] \xi L - c + \eta_0
\]  

(18)

with \( T = \tau L/\lambda = \tau(1 - \theta) \) whereas, using (13), we have:

\[
V^w = n + \bar{I}^* - \left[1 - f(T)\right] \xi C - tL/2 + \eta_0 - \tau
\]  

(19)

which is the indirect utility of a worker. Indeed, the number of active criminals is \( 1 - a \) since \( a \) represents the fraction of criminal in jail (incapacity effect). Note that the effect of the local head tax \( \tau \) on \( V^w \) is ambiguous since there is a direct negative effect (the net income of workers diminishes) and an indirect positive effect through \( f(T) \) (more criminals are in jail inducing a higher net income for workers). For the criminals, we assume, for simplicity, that only the consumption of the numeraire is affected if she is arrested. Using (18) and (19) and the fact that \( L = (1 - \theta) \lambda, L + C = \lambda, \) and \( \bar{I}^* = \varphi \lambda, \) the value of \( c \) making an individual indifferent between committing a crime and working is now given by

\[
\bar{c} = (\xi - \varphi) \lambda + \frac{t(1 - \theta)}{2} \lambda + \tau - \xi f(T) \lambda \equiv \Omega (\theta)
\]  

(20)

Compared to the case with no policy (see (15)), there is now an additional term, \( \tau - \xi f(T) \lambda \), which is endogenous and depends on \( T = \tau(1 - \theta) \). As before, because of the uniform distribution of \( c \), the fraction of criminals is \( \theta = \bar{c} \). The equilibrium share of criminal \( \theta^* \) is now implicitly defined by (20) where \( \theta^* = \bar{c} \). Observe first that, when \( \theta = 0 \), we want \( \Omega (0) > 0 \), that is:

\[
\left( \frac{t}{2} - \varphi \right) \lambda + \tau > 0
\]  

(21)

Since the left-hand side of this equation is the 45-degree line, then it suffices to show that the \( \partial \Omega (\theta) / \partial \theta < 0 \). This gives the following condition:

\[
\frac{t}{2} > \tau \xi f'(T)
\]  

(22)

So far, we have shown that there exists a unique \( \theta^* > 0 \). Furthermore, for \( \theta^* < 1 \), it suffices to show that \( \Omega (1) < 1 \), which is equivalent to:

\[
(\xi - \varphi) \lambda + \tau < 1
\]  

(23)
We can combine conditions (21), (22) and (23) into:
\[
\lambda \left( \varphi - \frac{t}{2} \right) < \tau < \min \left\{ 1 + \lambda \varphi - \xi \lambda, \frac{t}{2\xi f'(T)} \right\}
\]  
(24)

Thus, if (24) holds there exists a unique equilibrium share of criminals 0 < \theta^* < 1, given by:
\[
\theta^* \left[ 1 + t\lambda/2 \right] + \xi f(\tau(1-\theta^*)) \lambda - (t/2 + \xi - \varphi) \lambda - \tau = 0
\]  
(25)

We have:

**Lemma 2** Assume (24). There is an U-shaped relationship between \theta^* and \tau and \theta^* reaches its minimum value when \tau = \tau^c > 0, which is unique minimum and (implicitly) defined by
\[
\frac{\partial \theta^*}{\partial \tau} = -\xi \lambda f'(\tau^c(1-\theta)) (1-\theta^*) + 1 \\
1 + t\lambda/2 - \xi \lambda f'(\tau^c(1-\theta)) \tau^c = 0
\]  
(26)

**Proof.** See Appendix 1.

A rise in the tax rate has an ambiguous effect on the share of criminals in each city. On the one hand, it increases the probability of arresting a criminal so that less individuals have an incentive to become a criminal. On the other hand, it directly reduces the legitimate net income for all workers, making the criminal activity more attractive. Hence, there is an U-shaped relationship between \theta^* and \tau (see Figure 2) and \theta^* reaches its minimum value when \tau = \tau^c. Indeed, starting from low levels of tax rate, higher tax burden reduces the share of criminals in the city. Above the critical value of tax rate (\tau^c), criminal activities raise with tax burden.

Furthermore, by differentiating (25), we obtain:
\[
\frac{\partial \theta^*}{\partial \lambda} = \frac{[1 - \theta^*(\tau)] t/2 + (1 - \alpha) \xi - \varphi}{1 + t\lambda/2 - \xi \lambda f'(\tau)} > 0 \\
\frac{\partial \theta^*}{\partial t} = \frac{(1 - \theta^*) \lambda/2}{1 + t\lambda/2 - \xi \lambda f'(T) \tau} > 0
\]

which are both positive when (24) holds. As expected, regardless of tax rates, the share of criminals in the city is reduced when there is a decrease in commuting cost (or better job access) and in the city size. These two results are largely supported by empirical evidence (see Glaeser, 1998; Glaeser and Sacerdote, 1999; Ihlanfeldt, 2002).
5.2 Tax Policy, police and criminal activity

Let us now solve the first stage where the local (urban) government sets a tax rate \( \tau \) that maximizes the welfare of the representative worker (or, equivalently, the median voter), given by (19). Note that the specification of the government’s objective is a controversial issue in our case because individuals can either work or be criminals. We assume that the local government disregards the indirect utility of criminals and only maximizes the welfare of the representative worker. At the end of this section, we discuss an alternative objective.

Raising tax rate leads to two opposite effects on the welfare of workers: it increases it by reducing the share of criminals in jail but also decreases the welfare by raising the tax burden and land rents of workers. In addition, remember that, if the fraction of criminals in jail increases with \( \tau \), its effect on the share of criminal in the city is ambiguous. Using \( C^* = \theta^* \lambda, L^* = (1 - \theta^*) \lambda, n^* = L^* \) and \( I^* = \varphi \lambda \) as well as \( V^w = V^c(\bar{\epsilon}) \), (19) can be written as

\[
V^w = (1 - \theta^*) \lambda - \theta^* + \xi [1 - f(T^*)] (1 - \theta^*) \lambda
\]

where \( T^* = (1 - \theta^*) \lambda \). The first order condition of the government maximization is then given by:

\[
\frac{dV^w}{d\tau} = -\{ \lambda + 1 + \xi [1 - f(T^*)] \lambda \} \frac{\partial \theta^*}{\partial \tau} - \xi \lambda (1 - \theta^*) f'(T^*) \frac{dT}{d\tau}
\]

The equilibrium tax rate is denoted by \( \tau^d \) and is such that \( dV^w / d\tau = 0 \). Since \( \partial \theta^* / \partial \tau = 0 \) implies that \( \xi \lambda f'(T^*)(1 - \theta^*) = 1 \), we have

\[
\frac{dV^w}{d\tau} \bigg|_{\tau = \tau^c} = -\frac{dT}{d\tau} \bigg|_{\tau = \tau^c} = -(1 - \theta^*) < 0
\]

As a result, \( \tau^d < \tau^c \) and, when \( \tau = \tau^d \), \( \partial \theta^* / \partial \tau < 0 \). Hence, for a given city size \( \lambda \), the tax rate that maximizing the utility of the median voter is lower than the tax rate inducing the minimum value of the share of criminals (see Figure 2) because, at \( \tau^c \), taxes are too high. We can also conclude that \( dT / d\tau > 0 \) when \( \tau = \tau^d \). Hence, the equilibrium tax rate is in the upward-sloping portion of the curve.

In addition, there exists an interior solution which is positive if and only if

\[
\left. \frac{\partial V^w}{\partial \tau} \right|_{\tau = 0} = -(1 + \lambda + \xi \lambda) \left. \frac{\partial \theta^*}{\partial \tau} \right|_{\tau = 0} - \xi f'(0)(1 - \theta^*)^2 \lambda > 0
\]

where \( \partial \theta / \partial \tau < 0 \) when \( \tau = 0 \) and \( \xi f'(0)(1 - \theta^*) \lambda > 1 \). As a consequence, \( \tau^d > 0 \) if and only if

\[
\frac{\xi \lambda^2 f'(0) [1 - (\xi - \varphi) \lambda] (1 + 2\xi - \varphi)}{1 + \lambda(1 + \xi)} > 1 + \frac{\lambda}{2}
\]
where $1 - (\xi - \varphi) \lambda > 0$. Hence, the local urban government is more likely to levy taxes to fight criminal activities when commuting costs are low enough. Furthermore lower commuting costs impact directly on the share of criminals in each city and through a change in tax policy. Indeed, we have

$$\frac{d\theta^*}{dt} = \frac{\partial\theta^*}{\partial t} + \frac{\partial\theta^*}{\partial \tau} \frac{d\tau^d}{dt} = \frac{\partial\theta^*}{\partial t} + \frac{\partial\theta^*}{\partial \tau} \left( -\frac{\partial^2 V^w}{\partial t \partial \tau} \frac{\partial^2 V^w}{\partial \tau^2} \right)$$

Because $\frac{\partial^2 V^w}{\partial \tau^2}$ is highly non linear, we perform some numerical simulations to study the sign of $\frac{d\tau^d}{dt}$. Figure 3 displays the results of the simulations. From our simulations, it appears that $d\tau^d/dt < 0$ and, in turn, $d\theta^*/dt > 0$. Hence, the local government adjusts downward its tax rate when commuting costs decline, leading to less criminals in each city. This means that, when commuting costs are low (or job access is good), the optimal tax rate $\tau^d$ chosen by the local government is closer to $\tau^c$, the tax rate that minimizes total crime. The intuition of this result is straightforward. Since, when commuting costs are reduced, the incentives of becoming a criminal are lower and the workers’ welfare increase, the local government can increase its tax rate so that the share of criminal is reduced.

To summarize,

**Proposition 3.** Lower commuting costs or better job access make the impact of police on crime more efficient.

We could consider the case where the government aims at maximizing the probability of arresting a criminal ($a$). In that case, the tax rate that maximizes the probability of arresting a criminal $\tau^a$ is implicitly given by

$$\frac{da}{d\tau} = f'(.) \frac{dT}{d\tau} = f'(.) \left( 1 - \theta^* - \tau \frac{\partial\theta^*}{\partial \tau} \right) = 0$$

and is identical to the tax rate maximizing the tax revenue. Starting from low tax rate, higher tax rates raise public resources per capita and, in turn, increase the probability of arresting a criminal (see Figure 3). Beyond $\tau^a$, a rise in tax rate reduces the revenue from the tax because the number of taxpayers (workers) reaches low values (a variant of the Laffer curve). It is straightforward to check $da/d\tau > 0$ when $\tau = \tau^c$ so that

---

11 We consider that $f(T) = \sqrt{\tau(1 - \theta^*)}$ as well as $\varphi = 2\xi = 2$ and $\lambda = 0.8$.
12 We performed many more numerical simulations.
13 Note that $d^2 a/d\tau^2$ when $\tau = \tau^a$. 

19
\( \tau^a > \tau^c \). Hence, a local government maximizing the public resources to fight criminal activity induces more criminals and tax burden than a local authority minimizing the number of criminals.

6 Criminal activities, migration and urbanization

In this section, we extend our model to two cities and verify whether our main results hold when individuals can migrate between cities. There are two cities: city A with a population size given by \( \lambda_A \) and city B with a population size given by \( \lambda_B \). Without loss of generality, we assume that \( \lambda_A + \lambda_B = 1 \) (the total population in the economy is normalized to one) and \( \lambda_A \geq \lambda_B \). To ease the notation, we set \( \lambda_A = \lambda \) and \( \lambda_B = 1 - \lambda \), with \( \lambda \geq 1/2 \).

Let us now endogeneize the location choice of all individuals (\( \lambda \)). For simplicity, we go back to the benchmark model with no police so that \( a = 0 \) in each city. The timing of the model is now as follows. In the first stage, households choose in which city they will reside without knowing their type \( \epsilon \) but anticipating (with rational expectations) the average total population of criminals. The assumption that types are revealed only after location choices has been made to take into account the relative inertia of the land market compared to the crime and labor markets (see Verdier and Zenou, 2004, for a similar assumption). In the second stage, types (or honesty parameters) are revealed and individuals decide whether or not to commit crime. In the third stage, goods are produced, workers participate in the labor market while criminals participate in the crime market and all consume the two types of goods. Observe that in the second stage, workers are stuck in their initial locations (decided in the first stage) and cannot relocate to another city. They then decide whether to become criminal or not. Since we have solved the two last stages in the previous sections, let us determine the location choice of individuals.

6.1 Location choices

The location of individuals is driven by the inter-city difference in their expected utility. Before knowing their \( \epsilon \), the expected utility of living in city \( r = A, B \) is given by (using (13) and (14)):

\[
\text{EV}_r = \int_0^{\tilde{\epsilon}_r} V_r^c dc + \int_{\tilde{\epsilon}_r}^1 V_r^w dc = \theta_r^2/2 + V_r^w(\theta_r)
\]  

(27)
where \( \bar{c}_r = \theta_r \) and, according to (16), \( \theta_r \) is given by

\[
\frac{\theta_r^*}{\theta_r} = \frac{\lambda_r t + 2 \lambda_r (\xi - \varphi)}{\lambda_r t + 2}
\]

A sufficient condition to obtain \( \theta_r^* < 1 \) is (17). Observe that the expected utility (27) is based on \( \theta_r \), the average proportion of criminals in city \( r \). Observe also that, even though the individual demands of product variety \( v \) (5) are unaffected by income, the migration decision takes income into account. Indeed, everything else equal, workers are drawn by the city with the higher wage. When the population becomes larger, the local demand for services raises, which attracts additional firms. In addition, households are attracted by larger cities to have access to more varieties (taste for diversity; see (1)). On the other hand, the competition for land among workers raises land rent and commuting costs, which both increase with population size. In addition, the location of households is affected by the level of criminal activity (in accordance with empirical evidence, see Cullen and Levitt, 1999). These different mechanisms interact with the decision to become a criminal and, in turn, the level of agglomeration.

Hence, the spatial difference in the expected utility \( \Delta EV_A - EV_B \equiv \Delta EV \) is given by:

\[
\Delta EV(\lambda, \theta_A, \theta_B) = \left( \lambda - \frac{1}{2} \right) \Gamma(\lambda, t)
\]

where

\[
\Gamma(\lambda, t) \equiv \frac{-t(1 + \xi)(\xi - \varphi)(1 - \lambda)\lambda + 2[1 - \xi(1 + \xi + \varphi) + 2\varphi]}{(1 + \lambda t/2)[1 + (1 - \lambda)t/2]} + \frac{2(\xi - \varphi + t/2)[t(1 - \lambda)\lambda + 1]}{(1 + \lambda t/2)^2[1 + (1 - \lambda)t/2]^2}
\]

We would like now to analyze the equilibrium of this economy, which is defined so that no individual (worker or criminal) has an incentive to change location (or city).

**Definition 4**

(i) An equilibrium arises at \( 0 < \lambda^* < 1 \) when the utility differential \( \Delta EV[\lambda^*, \theta_r(\lambda^*)] = 0 \), or at \( \lambda^* = 1 \) when \( \Delta EV[1, \theta_r(1)] > 0 \) or at \( \lambda^* = 0 \) when \( \Delta EV[0, \theta_r(0)] < 0 \).

(ii) An interior equilibrium is stable if and only if the slope of the indirect utility differential \( \Delta EV \) is strictly negative in a neighborhood of the equilibrium, i.e., \( d\Delta EV[\lambda^*, \theta_r(\lambda^*)]/d\lambda < 0 \) at \( \lambda^* \).

(iii) A fully agglomerated equilibrium (i.e. when \( \lambda^* = 1 \) or \( \lambda^* = 0 \)) is stable whenever it exists.
It is well-known that new economic geography (NEG) models typically display several spatial equilibria (Combes et al., 2008; Fujita and Thisse, 2013). In such a context, it is convenient to use stability as a selection device since an unstable equilibrium is unlikely to happen. This is what is exposed in Definition 4 where an interior equilibrium is stable if, for any marginal deviation away from the equilibrium, the incentive system provided by the market brings the distribution of individuals back to the original one. In (ii), we give the conditions for which the equilibrium is stable.

The analysis of existence and stability equilibrium is reported in Appendix 2. We show that full dispersion is more likely to occur when (i) commuting costs are high enough (like in NEG models, see Gaigné and Thisse, 2014) or (ii) when the amount stolen by criminals ($\xi$) is high enough $\xi \geq \bar{\xi} \equiv \sqrt{3 + 4 \varphi + \varphi^2 - 1}$ (the negative externality created by criminal activity is high). In contrast, full agglomeration occurs when commuting costs reach low values if $t < \ell(\xi)$, where

$$\ell(\xi) \equiv 2 \left[ -\xi^2 + \xi \varphi + \varphi + (1 + \varphi - \xi) \sqrt{1 + (1 + \xi)^2} \right]$$

(29)

In Figure 4, we have depicted $\ell(\xi)$, which is a non-linear curve that increases and then decreases up to $\ell(\xi) = 0$. It appears that a same set of parameters may yield two stable spatial equilibria ($\lambda^*_a = 1/2$ or $\lambda^*_w = 1$ when $t < 7$ and $\xi \geq \bar{\xi}$ for example). In other words, different levels of criminal activity may emerge for the same economic conditions.

Let us now study partial agglomeration ($\lambda^* \in (1/2, 1)$), which occurs when $\Gamma(\lambda^*) = 0$. This is the case when $t > \ell(\xi)$. We show, in Appendix 2, that an asymmetric spatial configuration emerges when commuting costs take intermediate values. It appears that $\ell(\xi) > 2 (\varphi - \xi)$ for all $t, \xi > 0$, which guarantees that $\theta^*_w > 0$ where there is no full agglomeration. In addition, under this spatial configuration, we have

$$L^*_A - L^*_B = \left( \lambda^* - \frac{1}{2} \right) \Lambda(\lambda^*)$$

where

$$\Lambda \equiv \frac{2(1 - \xi + \varphi) + t(\varphi - \xi)(1 - \lambda^*)\lambda^*}{(1 + t\lambda^*/2)[1 + t(1 - \lambda^+)/2]}$$

and where $L^*_A = L^*_B$ when $\lambda^* = 1/2$ and $L^*_A > L^*_B$ when $\lambda^* > 1/2$ because $\partial \Lambda(\lambda)/\partial \lambda > 0$. In other words, the large city hosts more workers and more criminals. It is also worth stressing that, ex post, workers living in the smaller city are better off than workers living
in the larger city (ex ante they all have the same expected utility). Indeed, because \( \theta^*_A > \theta^*_B \) and \( \Delta EV(\lambda) \) = 0 when \( 2 < \lambda^* < 1 \), then \( V^w_A < V^w_B \).

The following proposition summarizes all our main findings whereas Figure 4 displays the spatial equilibria in the \( t - \xi \) space.

**Proposition 5** There are three stable spatial equilibria with respect to commuting costs (or job access):

(i) If \( \Gamma(1/2,t) < 0 \), i.e. commuting costs are high enough, there are two identical cities in population size, \( \lambda^*_A = \lambda^*_B = 1/2 \), and in share of criminals, \( \theta^*_1(1/2) = \theta^*_2(1/2) \).

(ii) If \( \Gamma(1/2,t) > 0 \) and \( t > \tau \), i.e. commuting costs take intermediate values, there is a large city and a small city where the former has more criminals and more workers than the latter, \( 1/2 < \lambda^*_r < 1 \), \( L_1 > L_2 \) and \( \theta^*_1 > \theta^*_2 \).

(iii) If \( t < \tau \), i.e. commuting costs are low enough, there is a single city.

### 6.2 Impact of commuting costs on criminal activities

We now analyze the impact of commuting costs (\( t \)) on the criminal activity when there is free mobility of workers between the two cities and see if our previous results hold. As above, a decrease in \( t \) can be interpreted as a more efficient transport policy or a better access to jobs. We can now define the mass of criminals in the economy as a function of the relative size of cities \( \lambda \). It is given by:

\[
C(\lambda) = \lambda \theta^*_A + (1 - \lambda) \theta^*_B = \frac{[t + 2(2 - \xi)] [(4 - t)\lambda^2 - (4 - t)\lambda + 2]}{(2 + \lambda t)(2 + (1 - \lambda)t)}
\]  \( (30) \)

It is easily verified that \( \partial C(\lambda)/\partial \lambda > 0 \) as long as \( \lambda \geq 1/2 \). This means that, when the size of the population in the first city is more than 50 percent, then the total crime in the economy increases with \( \lambda \). There is a \( U \)-shape relationship between total crime \( C \) and \( \lambda \) as illustrated in Figure 5. In our model agglomeration is defined by \( \lambda \neq 1/2 \) and the farther away is \( \lambda \) from 1/2, the more there is agglomeration.

[Insert Figure 5 here]

We have seen in Section 4 that the impact of a reduction in commuting costs (better job access) on total criminal activities was positive when the location choice of individuals was exogenous (i.e. when \( \lambda \) was given). This is not true anymore when individuals
choose location and, in fact, the total effect is ambiguous. Indeed, at any given location of households, lower commuting costs reduce the number of criminals in each city \( (\partial C_r/\partial t > 0) \) because \( \partial \theta_r/\partial t > 0 \) regardless of city \( r \). On the other hand, the location of individuals adjusts in the long run to a change in commuting costs. More precisely, falling commuting costs promotes agglomeration \( (\partial \lambda/\partial t < 0) \) and, in turn, more crimes are committed in the larger city \( (\partial C_A/\partial \lambda > 0) \) while the number of crime in the small city shrinks \( (\partial C_B/\partial \lambda < 0) \). As a result, the long-run effect associated with a decrease in commuting costs on criminal activity is ambiguous. Even though lower commuting costs induce higher legitimate net income for all workers, they also promote higher levels of agglomeration. This is a new and interesting result that shows that the impact of job access or transportation policy on crime differs between the short and long run.

Let us investigate this issue in more detail. Consider first that the economy shifts from full dispersion to full agglomeration due to lower commuting costs. Under these spatial configurations, we have

\[
C(1/2) = \frac{t/2 + \xi - \varphi}{2 + t/2} \quad \text{for } \Gamma(1/2, t) < 0 \quad \text{and} \quad C(1) = \frac{t/2 + \xi - \varphi}{1 + t/2} \quad \text{for } t < \underline{t}.
\]

For example, it appears that \( C(1/2, t = \bar{t}_2) > C(1, t = \bar{t}) \) if and only if \( \varphi > \bar{\varphi} \) where

\[
\bar{\varphi} = \frac{2(5 + 3\xi)\sqrt{2\xi + \xi^2 + 2} - 5\xi^2 - 15\xi - 14}{1 + \xi}
\]

and \( \bar{\varphi} \) is positive and increases with \( \xi \). Hence, a shift from dispersion to agglomeration due to lower commuting costs may give rise to a decline in criminal activity. The final effect is that there are less criminal activities in the economy (the former effect dominates the latter effect).

In addition, \( C(\lambda) \) reaches its minimum value when \( t \leq t_{\min} \) (\( C = 0 \)). It is easily verified that \( t_{\min} < \underline{t} \) so that \( C = 0 \) may occur when \( \lambda^* = 1 \) and not when when \( \lambda^* = 1/2 \). Therefore, improving access to jobs by reducing commuting costs can be a relevant policy tool in reducing crime.

When \( t \) decreases when partial agglomeration occurs, the degree of agglomeration \( (\lambda^*) \) increases gradually so that the relationship between \( C(\lambda^*) \) and \( t \) is ambiguous when \( \bar{t} > t > \underline{t} \). Because \( \lambda^* \) is highly nonlinear, we need to perform some numerical simulations. These simulations reveal a \( U \)-shaped relationship between \( C(\lambda^*) \) and \( t \). There exists a threshold value \( \hat{t} \) such that \( C(\lambda^*) \) decreases with a reduction in commuting costs when \( \bar{t} > t > \hat{t} \). However, whether criminal activity may fall in the economy when \( t \) moves from

\[\text{Indeed, } \Delta(1) = 1 + \xi \text{ when } t = t_{\min} \text{ and } d\Delta EV(\lambda^*)/d\lambda > 0 \text{ at } \lambda^* = 1/2 \text{ when } t = t_{\min}.\]
crime increases occur in the larger city due to a larger population size. Figure 6 displays the relationship between crime and commuting costs.

To summarize,

**Proposition 6** When there is no mobility between cities (short run), decreasing commuting costs (or improving job access) always increases total crime. When there is free mobility between the two cities (long run), a negative relationship between commuting costs and total crime is more likely to occur if $\varphi$ is low and $\xi$ high ($\varphi < \bar{\varphi}$).

Indeed, when $t$ decreases, there will be more agglomeration, which leads to two opposite effects. On the one hand, the urban costs in the big city will increase compared to the small city and thus more people decide to become criminal. On the other hand, real wages increases in the big city because of a bigger market size, which reduces the number of criminals. The net effect is thus ambiguous. When $\varphi$ is high and $\xi$ low, the latter effect dominates the former for a large range of commuting costs while, we have the reverse result, when $\varphi$ is low and $\xi$ is high. Remember that the incentive to become a criminal is relatively strong when real labor productivity ($\varphi$) is low or crime productivity ($\xi$) is high.

### 7 Concluding remarks

This paper provides the first model of agglomeration and crime in a general equilibrium framework. We first develop a one-city model where individuals are heterogeneous in their incentives to commit crime, freely choose their location within the city and decide whether to become a criminal or a worker. We show that higher commuting costs leads to more criminal activities in the city and that the impact of commuting costs on criminal activities is higher when the city size increases. We also show that the mass of criminals decreases with more differentiated products and with higher labor productivity and that criminal activities increase more than proportionally when the city size rises. We also consider an urban government that levies a tax on workers in order to finance policy resources to fight criminal activities. We show that there is a U-shaped relationship between the crime rate in the economy and the tax rate so that increasing resources to fight crime financed by taxes on workers can backfire if the tax rate is too high. We also show that lower commuting costs or better job access make the impact of police on crime more efficient.
We then extend our framework to allow individuals to freely move between two cities. We totally characterize the three different stable equilibria as a function of commuting costs. When the commuting costs are high enough, the population of workers and criminals is evenly distributed between the two cities. When the commuting costs take intermediate values, there is a large and a small city where in the former there are more criminals and workers than in the latter. When commuting costs are low enough, there is a single city. We also show that criminal activity increases in the economy when commuting costs decline only if the size of the agglomeration is high enough. Finally, while we have shown in the short run (only one city) that better accessibility to jobs decreases crime, we show that, in the long run (free mobility between the two cities), this policy may, on the contrary, increase crime. This means that policy makers should be careful in implementing a transportation policy because it may backfire on crime. Our paper provides an explanation of why this might be the case.

References


APPENDIX

Appendix 1. Proof of Lemma 2

The crime rate is given by (25) where

$$\theta^*(0) = \frac{(t/2 + \xi - \varphi)\lambda}{1 + t\lambda/2} > 0$$

By differentiating (25), we easily obtain:

$$\frac{\partial \theta^*}{\partial \tau} = \frac{-\xi\lambda f'(T) (1 - \theta) + 1}{1 + t\lambda/2 - \xi\lambda f'(T)\tau} = 0$$

which is (26). Denote by

$$N \equiv -\xi\lambda f'(T) (1 - \theta) + 1$$

and by

$$D \equiv 1 + t\lambda/2 - \xi\lambda f'(T)\tau$$

Observe that

$$\frac{\partial \theta^*}{\partial \tau} \bigg|_{\tau=0} < 0$$

$$\frac{\partial^2 \theta^*}{\partial \tau^2} = \frac{[-\xi\lambda^3 f''(T) (1 - \theta)^2 + \xi\lambda^2 f'(T) \frac{\partial \theta^*}{\partial \tau}] D + \xi\lambda^2 [f''(T) (1 - \theta) \lambda\tau + f'(T)] N}{D^2}$$

We look for a minimum and thus $\frac{\partial \theta^*}{\partial \tau} = 0$, which implies that $N = 0$ and $\frac{\partial \theta^*}{\partial \tau} = 0$. Thus

$$\frac{\partial^2 \theta^*}{\partial \tau^2} = \frac{-\xi\lambda^3 f''(T) (1 - \theta)^2}{D} > 0$$

and thus there is a unique minimum that we denote by $\tau = \tau^c$.

Observe that $\tau^c$ is defined by:

$$-\xi\lambda f'(\tau^c L) (1 - \theta) + 1 = 0$$

which is equivalent to

$$\tau^c = \frac{f^{t-1}}{L} \frac{1}{\xi\lambda (1 - \theta)} > 0$$

which proves that $\tau^c > 0$. ■
Appendix 2. Existence and stability of equilibrium spatial configurations
with a system of two cities

We investigate in more detail all the possible equilibria. First, full dispersion ($\lambda^* = 1/2$) is always an equilibrium whatever the value of $t$ since $\Delta \text{EV}(1/2) = 0$. Second, there is an equilibrium with full agglomeration ($\lambda^* = 0$ or $\lambda^* = 1$) if and only if $\Delta \text{EV}(1) > 0$ and $\Delta \text{EV}(0) < 0$. Using (28), it is easily checked that these two conditions are satisfied if and only if $t < \bar{\xi}(\xi)$, where

$$\bar{\xi}(\xi) \equiv 2 \left[ -\xi^2 + \xi \varphi + \varphi + (1 + \varphi - \xi) \sqrt{1 + (1 + \xi)^2} \right]$$

which is (29). In Figure 4, we have depicted $\bar{\xi}(\xi)$, which is a non-linear curve that increases and then decreases up to $\bar{\xi}(\xi) = 0$ for which $\xi = \bar{\xi} \equiv \sqrt{3 + 4\varphi + \varphi^2} - 1$, with $\bar{\xi} \in (\varphi, \varphi + 1)$. Notice that $\bar{\xi}(\xi) > 2(\varphi - \xi)$ for all $t, \xi > 0$, which guarantees that $\theta^*_r > 0$ where there is no full agglomeration. Let us now study partial agglomeration ($\lambda^* \in (1/2, 1)$), which occurs when $\Gamma(\lambda^*) = 0$. This is the case when $t > \bar{\xi}(\xi)$ so that we have $\Delta \text{EV}(1) < 0 < \Delta \text{EV}(0)$.

Let us now look at the stability of the interior equilibria since full-agglomeration equilibrium is always stable when it exists (see Definition 4). An interior solution $\lambda = \lambda^*$ is stable if and only if

$$\left. \frac{d\Delta \text{EV}}{d\lambda} \right|_{\lambda=\lambda^*} = \Gamma(\lambda^*, t) + \left( \lambda^* - \frac{1}{2} \right) \frac{d\Gamma}{d\lambda} < 0$$

We have two types of interior solutions: full dispersion with $\lambda^* = 1/2$ and partial agglomeration with $\lambda^* \in (1/2, 1)$.

**Stability for a full dispersion equilibrium.** We determine the conditions under which a full dispersion equilibrium ($\lambda^* = 1/2$) is a stable equilibrium (or equivalently $\Gamma(1/2, t) < 0$). Let $\Gamma(1/2, t) \equiv \left. \frac{d\Delta \text{EV}}{d\lambda} \right|_{\lambda=1/2}$ be the slope of $\Delta \text{EV}$ at $\lambda = 1/2$ where

$$\Gamma(1/2, t) = \frac{(1 + \xi)(\varphi - \xi)t^2 + 4[3\xi(\varphi - \xi) - (2 - \varphi - \xi)]t - 16(\xi - \bar{\xi})(\xi + 2 + \bar{\xi})}{4(1 + t)^3}$$

with

$$\bar{\xi} = \sqrt{3 + 4\varphi + \varphi^2} - 1 \in (\varphi, \varphi + 1)$$

where $\Gamma(1/2, 0) = 4(\xi - \bar{\xi})(\xi + 2 + \bar{\xi})$ and

$$\left. \frac{d\Gamma(1/2, t)}{dt} \right|_{t=0} = (7 + 3\varphi) \left[ \xi - \frac{3\varphi + 8}{3\varphi + 7}(\varphi + 1) \right] < 0 \quad (31)$$
which is negative as long as $\xi < \varphi + 1$. In addition, $\Gamma(1/2, t) = 0$ when $t = \bar{\tau}_1$ or $t = \bar{\tau}_2$ with

$$
\bar{\tau}_1 \equiv 2 \frac{(2 - \varphi - \xi) - 3\xi(\varphi - \xi) + (2 + \varphi - \xi)\sqrt{(1 + \xi)^2 + 4(\xi^2 - \xi\varphi - \varphi)}}{(1 + \xi)(\varphi - \xi)}
$$

$$
\bar{\tau}_2 \equiv 2 \frac{(2 - \varphi - \xi) - 3\xi(\varphi - \xi) - (2 + \varphi - \xi)\sqrt{(1 + \xi)^2 + 4(\xi^2 - \xi\varphi - \varphi)}}{(1 + \xi)(\varphi - \xi)}
$$

where $2 + \varphi - \xi > 0$ when $\xi < 1 + \varphi$, $\bar{\tau}_2 = 0$ when $\xi = \bar{\xi}$ and $\bar{\tau}_2 > 0 > \bar{\tau}_1$ when $\xi > \varphi$.

As a result,

(i) When $\xi \geq \bar{\xi}$, the function $\Gamma(1/2, t)$ is concave with $\Gamma(1/2, 0) \leq 0$ and $\Gamma(1/2, t) < 0$ as long as $t > 0$. Indeed, if $t = 0$ then $\Gamma(1/2, t) \leq 0$ (when $\xi \geq \bar{\xi}$) and $d\Gamma(1/2, t)/dt < 0$ when $t = 0$. By implication, $\lambda^* = 1/2$ is a stable spatial configuration regardless of $t \geq 0$ when $\xi \geq \bar{\xi}$.

(ii) When $\bar{\xi} > \xi > \varphi$, the function $\Gamma(1/2, t)$ is still concave where we have now $\Gamma(1/2, 0) > 0$ and $\bar{\tau}_2 > 0 > \bar{\tau}_1$. Thus, there exists a single positive value of $t$ ($\bar{\tau}_2$) such that $\Gamma(1/2, \bar{\tau}_2) = 0$ and $\Gamma(1/2, t) < 0$ if and only if $t > \bar{\tau}_2$. Hence, when $\bar{\xi} > \xi > \varphi$, $\lambda^* = 1/2$ is a stable spatial configuration if and only if $t > \bar{\tau}_2$.

(iii) When $\xi = \varphi$, there exists a single positive value of $t$ ($4(1 + \varphi)/(1 - \varphi)$) such that $\Gamma(1/2, t) = 0$ and $\Gamma(1/2, t) < 0$ if and only if $t > 4(1 + \varphi)/(1 - \varphi)$. Hence, when $\xi = \varphi$, $\lambda^* = 1/2$ is a stable spatial configuration if and only if $t > \bar{\tau}_3$ with

$$
\bar{\tau}_3 \equiv \frac{4(1 + \varphi)}{1 - \varphi}
$$

(iv) When $\varphi > \xi$, we have $\bar{\tau}_1 > \bar{\tau}_2$ and the function $\Gamma(1/2, t)$ becomes convex. Hence, $\Gamma(1/2, t) < 0$ if and only if $\bar{\tau}_1 > t > \bar{\tau}_2$. Note that $\bar{\tau}_2 > 0$ if and only if $\xi > \max\{\xi, 0\}$ with

$$
\xi \equiv \frac{2\varphi}{5} - \frac{1}{5} + \frac{2}{5} \sqrt{\varphi^2 + 4\varphi - 1}
$$

where $\underline{\xi} = \varphi$ when $\varphi = 1$ and $\underline{\xi} < \varphi$ if and only if $\varphi < 1$. Hence, $\lambda^* = 1/2$ is a stable spatial configuration if and only if $\bar{\tau}_1 > t > \bar{\tau}_2$ and $\varphi > \xi > \max\{\xi, 0\}$. Note that that $\varphi > \max\{\underline{\xi}, 0\}$ if and only if $\varphi < 1$. Hence, $\lambda^* = 1/2$ is not a spatial equilibrium when $\varphi > 1$ or when $\xi < \max\{\xi, 0\}$.

To sum up,

**Lemma 7** Full dispersion ($\lambda = 1/2$) is a stable spatial configuration

(i) when $\xi \geq \bar{\xi}$ regardless of commuting costs $t$;
(ii) when $\bar{\xi} > \xi > \varphi$ if and only if $t > \bar{t}_2$;

(iii) when $\xi = \varphi$ if and only if $t > \bar{t}_3$;

(iv) when $\xi < \varphi$ if and only if $\bar{t}_1 > t > \bar{t}_2$ and $\varphi > \xi > \max\{\xi, 0\}$.

**Stability for a partial agglomeration equilibrium.** In a partial agglomeration equilibrium $\lambda^* \in (1/2, 1)$ such that $\Gamma(\lambda^*, t) = 0$ is stable if and only if $d\Gamma/d\lambda < 0$ when $\lambda = \lambda^*$. Note that $\Gamma(\lambda^*, t) = 0$ has at most one solution when $\lambda \in (1/2, 1)$. Let $\lambda^*$ be the implicit solution of $\Gamma(\lambda, t) = 0$. If $\Gamma(1/2, t) < 0$ (where $\Gamma(1/2, t) = d\Delta EV/d\lambda$ when $\lambda^* = 1/2$) and $\Delta EV(1) > 0$, then $\lambda^*$ exists but is unstable. By contrast, if $\Gamma(1/2, t) > 0$ and $\Delta EV(1) < 0$, then $\lambda^*$ exists and is stable. In other words, an asymmetric spatial configuration emerges when commuting costs take intermediate values. In addition, under this spatial configuration, we have

$$L^*_A - L^*_B = \left(\lambda^* - \frac{1}{2}\right) \Lambda(\lambda^*)$$

where

$$\Lambda \equiv \frac{2(1 - \xi + \varphi) + t(\varphi - \xi)(1 - \lambda^*)\lambda^*}{(1 + t\lambda^*/2)[1 + t(1 - \lambda^*)/2]}$$

and where $L^*_A = L^*_B$ when $\lambda^* = 1/2$ and $L^*_A > L^*_B$ when $\lambda^* > 1/2$ because $\partial \Lambda(\lambda)/\partial \lambda > 0$.  

[34]
Figure 1: Equilibrium land use
Figure 2: Tax Policy and Criminal activity
Figure 3: Equilibrium tax rate and commuting costs
Figure 4: Spatial equilibria and crime in \((t, \xi)\)-space.

(a) high \(\varphi\) \((\varphi>1)\)

\[
C^* = \sum_{r} \frac{2(t/2+\xi-\varphi)(\lambda^*_r)^2}{2+t\lambda^*_r}
\]

\[
\lambda^*_1 = \lambda^*_2 = \frac{1}{2}
\]

\[
C^* = \frac{t/2+\xi-\varphi}{1+t/2}
\]

(b) low \(\varphi\) \((\varphi<1)\)

\[
C^* = \sum_{r} \frac{2(t/2+\xi-\varphi)(\lambda^*_r)^2}{2+t\lambda^*_r}
\]

\[
\lambda^*_1 = \lambda^*_2 = \frac{1}{2}
\]

\[
C^* = \frac{t/2+\xi-\varphi}{2+t}
\]
Figure 5: Relationship between total crime and agglomeration
Figure 6: The impact of commuting costs on criminal activities

(a) high $\varphi (\varphi > \hat{\varphi})$

(b) low $\varphi (\varphi < \hat{\varphi})$