## Macro II, Spring 2015

John Hassler and Karl Walentin

**Instructions.** The exam consists of two parts, part one covering John's part of the course and part two covering Karl's.

**Exam score.** Part one contains three exam questions with a total maximum of 100 points. Part two contains two exam questions with a total maximum of 100 points. The exam score is the sum of the scores on part one and two divided by two.

**Course score** The course score is a weighted average of the exam score and the score on the problem sets. Weights are 4/5 on the exam and 1/5 on the problem sets. The threshold for pass is 50 and for pass with distinction 75.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

Part I

1. The RBC model (35 points) Consider the planning problem

$$\max_{\{C_{t+s}, K_{t+1+s}, L_{t+s}\}_{s \ge 0}} \sum_{s=0} \beta^{s} U(C_{t+s}, L_{t+s})$$
  
s.t.  $C_{t+s} + K_{t+1+s} = Y_{t+s} + (1-\delta) K_{t+s} \forall s \ge 0$   
 $Y_{t+s} = Z_{t+s} F(K_{t+s}, (1-L_{t+s}))$   
 $K_{t}$  given

and where  $Z_t$  is a stationary Markov process,  $C_t$  consumption,  $L_t$  leisure and  $K_t$  capital.  $\delta$  denotes depreciation and U and F satisfy standard concavity conditions.

- (a) (10) Derive the optimality conditions for the planner by using the first order conditions for  $C_{t+s}, K_{t+1+s}$  and  $L_{t+s}$ . Express your results as
  - 1. an Euler condition expressing the intertemporal optimality condition involving a trade-off between  $C_t$  and  $C_{t+1}$  and
  - 2. an intratemporal optimality condition involving a trade-off between labor and leisure. Show the steps in your derivations!
- (b) (10) Suppose that

$$U(C_t, L_t) = \ln(C_t) + \frac{\nu}{\nu - 1} \phi L_t^{\frac{\nu - 1}{\nu}}$$

and that output is given by labor only

$$Y_t = Z_t (1 - L_t)^{\alpha}, \alpha \in [0, 1].$$

Also assume that individuals cannot save or borrow so that the budget constraint is simply

$$Y_t = C_t$$
.

- 1. Derive the intratemporal optimality condition for this special case.
- 2. How does labor supply respond to productivity shocks in this case?
- 3. Now focus on the case when  $\nu = 1$ . We can then write utility as  $U(C_t, L_t) = \ln(C_t) + \phi \ln L_t$ . Solve for labor supply.

(c) (15) Now suppose there is a government that spends resources on government consumption,  $G_t$ and finance it with lump sum taxation,  $T_t$  ( $T_t = G_t$ ). Individuals derive utility  $U(C_t, L_t, G_t) = \ln(C_t) + \phi \ln L_t + \gamma \ln G_t$ . The budget for the individual is

$$C_t = w_t \left( 1 - L_t \right) - T_t$$

where  $w_t$  is the wage given by marginal labor productivity

$$\alpha Z_t \left(1 - L_t\right)^{\alpha - 1}$$

- 1. Suppose workers take  $w_t, T_t$  and  $G_t$  as given. Derive the intra-temporal first-order condition for an individual who maximizes her utility given  $w_t, T_t$  and  $G_t$ .
- 2. Solve for labor supply  $(1 L_t)$  and show how it responds to changes in  $T_t$ . Explain your results in words.
- 3. Compare how labor supply responds to wage changes in the two cases  $T_t = 0$  and  $T_t > 0$ . Explain your results in words.
- 2. Monopolistic competition (40p). A key concept in the New Keynesian model is monopolistic competition.
  - (a) (5) Explain briefly why we use monopolistic rather than perfect competition in the New Keynesian model.
  - (b) (10) In the model we discussed in class, we had a large number of different consumption goods. Now instead assume that there is only one final good Y. However, this good is produced with a number of intermediate goods  $Y_i$ ,  $i \in \{1, ..., N\}$  according to the CRS function

$$Y = \left(\frac{1}{N}\sum_{i=1}^{N}Y_{i}^{q}\right)^{\frac{1}{q}}, q \in (0,1)$$

Suppose there is a large number of final good producers acting on a competitive market but that each intermediate good is produced by a monopolist. Let P be the price of the final good and let  $P_i$  be the price of intermediate good i.

Setup the profit function of a representative final good producing firm and take the first-order condition for the purchase of good i. Use this to derive aggregate demand function for good i. Express this as the price of good i as a function of aggregate output Y, the price P and  $Y_i$ .(Hint:

Use the first order condition for  $Y_i$  and note that  $\left(\frac{1}{N}\sum_{i=1}^N Y_i^q\right)^{\frac{1}{q}-1} = Y^{1-q}$ .

(c) (10) Suppose that the production function of a representative intermediate goods producing firm i is

$$Y_i = \phi \left( L_i - c \right)$$

where  $\phi$  is a parameter determining marginal labor productivity, c is a fixed cost expressed in labor requirements and  $L_i$  is labor input and .

- 1. Write the cost function of the representative intermediate firm, i.e., nominal costs as a function of its output and the nominal wage W.
- 2. Write the nominal profit function of the representative intermediate firm using the cost function you just derived and the demand function from b.

- (d) (10) Use the first-order condition for the representative intermediate firm to derive an expression for the real wage. Assume N is large enough for the intermediate firm to take Y and P as given. (Hint: in equilibrium,  $Y_i = Y$ ).
- (e) (5) Is the economy efficient? If not, is there a tax or subsidy that could make it efficient?

## 3. Short Questions (25 points).

- (a) (8) In the new Keynesian models price stickiness is assumed. Describe the different standard varieties of price stickiness!
- (b) (8) Under the assumption of price stickyness, a New Keynesian Phillips curve of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

can be derived. Explain in words why expected inflation affects current inflation positively.

(c) (9) Discuss very briefly why we cannot expect the fiscal policy multiplier to be a constant and some results regarding this inconstancy.

## Part II

- 4. Short questions. (50 points) Please answer briefly.
  - (a) (5 points) Describe the main mechanism in one paper on the syllabus where heterogeneity matters for aggregate prices, quantities or welfare.
  - (b) (5 points) What's the relationship between income and wealth inequality (say, e.g., in terms of cross-sectional standard deviation)? Does the Aiyagari (1994) model generate too much or too little wealth inequality for a standard earnings calibration? Mention a mechanism or changed assumption that could reduce this discrepancy between model and data.
  - (c) (5 points) For the US in the last 100 years, what's the key time series pattern for income inequality that Piketty reports? Is the change since 1970 mainly driven by capital income or labor income? How is Europe different in this respect?
  - (d) (5 points) Which assumption(s) or condition(s) are required to justify representative agent modeling of aggregate dynamics?
  - (e) (6 points) Precautionary savings: Assume that marginal utility is convex, U''' > 0, and that  $\beta(1+r_t) = 1$ . The budget constraint is  $c_t + a_{t+1} = (1+r_t)a_t + y_t$  where  $y_t$  denote earnings. Derive the partial equilibrium result that savings are higher in a world with idiosyncratic earnings uncertainty compared to a world without such uncertainty.
  - (f) (8 points) Some people think that the equilibrium real interest rate has decreased after the recent financial crisis. Make an argument in that direction using the Aiyagari model (without assuming a change in the degree of earnings uncertainty). To get full points, please draw the demand and supply of capital curves (capital K on the x-axis, interest rate r on the y-axis) as well as the borrowing limit and the interest rate that would obtain if markets where complete.
  - (g) (8 points) In non-search models of unemployment the wage is necessarily such that labor supply exceeds labor demand. "Efficiency wages" is one theory of why wages don't clear the market. Describe the key parts of this theory.
  - (h) (8 points) In class we discussed three different models of endogenous labor force participation over the business cycle. Please describe the basics of the Christiano, Trabandt and Walentin (2012) model, including the fundamental reason for unemployment in that model. In addition, state why individual agents' unemployment risk can not be fully insured against.

## 5. The Diamond-Mortensen-Pissarides model. (50 points)

Consider a setup similar to the standard DMP model discussed in class (although here notation will be slightly different, intentionally). In particular, we have: risk-neutrality, homogenous workers, homogenous firms, a worker-firm match produces output  $y_t$ , wages,  $w_t$ , are determined by Nash bargaining, the cost of creating a vacancy is a, free entry of firms, unemployment benefits, i, exogenous break-up rate, s, constant labor force participation normalized to unity, no intensive margin (choice of hours/worker). The discount factor is  $\beta$ . Denote the job finding rate with  $f_t$  and the vacancy filling rate with  $q_t$ . Please define clearly any notation that you add.

- (a) (8 points) Please write down and explain the value, W, to a worker of being employed, compared to being unemployed. Assume that a worker that loses his job doesn't have the ability to look for a new job until the next period.
- (b) (7 points) Write down and explain the value for a firm of being matched with a worker.
- (c) (7 points) State the job creation condition implied by free entry, assuming that a vacancy that is filled today becomes a productive match next period.
- (d) (7 points) Assume a matching function  $M_t = \gamma V_t^{1-\lambda} U_t^{\lambda}$  where V denotes vacancies and U unemployment. Derive expressions for the job finding rate and the vacancy filling rate in terms of market tightness,  $\theta_t \equiv V_t/U_t$ . Describe a mechanism related to the matching function and the job creation condition that dampens fluctuations in  $\theta_t$ .
- (e) (7 points) Using equations for flows into and out of unemployment in steady state, derive a steady state expression for the job finding rate f as a function of the job separation rate and unemployment.
- (f) (7 points) Without necessarily providing detailed derivation, please describe how an increase in the bargaining power of workers would affect equilibrium unemployment in this model.
- (g) (7 points) What is the Shimer (2005) puzzle? Name two suggestions in the literature on how to solve or reduce this puzzle.