The Role of Social Networks in Cultural Assimilation

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Abstract

We develop a model where, in the first stage, minority individuals have to decide whether or not they want to assimilate to the majority culture while, in the second stage, all individuals (both from the majority and the minority group) embedded in a network have to decide how much effort they exert in some activity (say education). We show that the more central minority agents are in the social network, the more they assimilate to the majority culture. We also show that denser networks tend to favor assimilation so that, for example, it is easier to assimilate in a complete network than in a star-shaped network. Finally, we show that the subgame-perfect equilibrium is not optimal because there is not enough activity and assimilation. To restore the first best, we find that it is optimal for the planner to give higher effort subsidies (to all individuals) but lower assimilation subsidies (to minority individuals) to more central agents in the social network.

Keywords: Social networks, assimilation, majority individuals, minority individuals, network centrality, network structure.

JEL Classification: D85, J15, Z13.

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1 Introduction

An intense political and intellectual debate is taking place in Europe around migration issues. Rather than being centered on the economic costs and benefits of such inflows, the debate has instead focused on the perceived costs and benefits of cultural diversity. The attention paid to this issue is relatively novel in Europe and does represent a departure from the long-standing debate which has tended to emphasize racial discrimination as the key explanation of ethnic disadvantage. This is well illustrated by the hot debate in Europe about the veiling among Muslim women. Similarly, the recent votes in Switzerland against the construction of Muslim mosques and against immigrants clearly show how heated and emotional the debates on ethnic and religious identity have recently become. Many European countries are concerned about the cultural integration and the (lack of) assimilation of immigrants, that is whether the basic norms and values of the majority society are adopted by existing minority groups.1

Indeed, the assimilation outcomes of second generation youths have been hotly debated amongst scholars, especially in the United States (Alba et al., 2011; Haller et al., 2011) where the immigrant population’s growth in recent decades has raised questions about whether and how their children, the second generation, will integrate into American society. Current perspectives on second generation integration have evolved and are varied. Some scholars adhere to the segmented assimilation framework in which the second generation will assimilate into different segments of American society based on structural barriers and prejudices (Portes and Rumbaut, 2001; Portes and Zhou, 1993). Other scholars believe that the outcome between immigrant and mainstream culture is less dichotomous, and that immigrants and the American mainstream will eventually coalesce as lifestyles and patterns gradually become similar over time (Alba and Nee, 2003). Different studies have shown different significant influences on the assimilation process for immigrants: the quality of immigrant cohorts (Borjas, 1985), country of origin (e.g. Beenstock et al., 2010; Borjas, 1987, 1992; Chiswick and Miller, 2011), ethnic concentration (e.g. Edin et al., 2003; Lazear, 1999) and personal English skill (e.g. Chiswick and Miller, 1995, 1996; Dustmann and Fabbri, 2003; McManus et al., 1983). An often overlooked structural factor of assimilation of ethnic minorities is the role of immigrant networks in the assimilation process. This is what we study in this paper.

To be more precise, we develop a model where all individuals, native and minorities (or immigrants), are embedded in a network. The network we are interested in is the one that forms between immigrants and natives but also between immigrants in the host country. As a result, we are not modeling the migration process and the role of network in migration (for such a model, see, e.g. Giulietti et al., 2014). Instead, we examine the role of networks in the host country on assimilation.

1 For an overview of these issues in Europe, see Kahanec and Zimmermann (2011).
choices and outcomes. In this network, immigrants and natives possess different positions in the network and have different productivities.

There are two stages. In the first stage, minority individuals (immigrants) have to decide whether or not they want to assimilate to the majority culture while, in the second stage, all individuals (both from the majority and the minority group) embedded in a network\(^2\) have to decide how much effort they exert in some activity (say education or work on a job). Here both minority and majority workers belong to the same network. In this network, links can represent social or working relationships between people. There is a trade off for minority workers. If they choose assimilation, they will be more productive (in terms of education or in terms of skills on the job) because they are more adapted to the social norms of the host country (they know better the language and the habits of the host country). However, they need to pay a fixed cost of assimilation because it is costly, for example, to learn a new language. On the other hand, if they choose not to assimilate, they do not need to pay this cost but end up with a lower productivity, which decreases their outcomes (for example, their education level or their wage). As a result, the incentives for an individual belonging to the minority group to assimilate and adopt the culture of the majority are then directly related to the expected gains and costs that such a strategy implies. We consider a model where efforts are strategic complements. If, for example, we think of education, then if someone that I’m linked to studies hard then I enjoy more utility to study hard myself. Similarly, if we think of productivity on the job, then if I’m linked to someone who works hard, then I enjoy more utility from working hard.

We show that the more central agents in the network tend to have higher productivity than the less central ones and thus, the ethnic minorities who are more central in the network tend to assimilate more than those who are less central because the gains of assimilation are higher. We also show that, when the strength of interactions in the network increases, social interactions become more valuable and, because it is costly not to assimilate (in terms of productivity), more people choose to assimilate. This highlights the fact that endogenous assimilation choices affect the contribution to equilibrium efforts. For example, when the cost of assimilation decreases, more agent choose to assimilate to the majority culture, which, in turn, increases social interactions in the network and thus equilibrium productive efforts and outcomes. It is thus interesting here to see how the assimilation space affects the social space. Furthermore, we find that denser networks tend to favor assimilation so that, for example, it is easier to assimilate in a complete network than in a star-shaped network.

We also show that the subgame-perfect equilibrium is not optimal because there is not enough

\(^{2}\)The economics of networks is a growing field. See Jackson (2008), Ioannides (2012), Jackson et al. (2015) and Jackson and Zenou (2015) for recent overviews.
activity and assimilation. To restore the first best, we find that it is optimal for the planner to give higher effort subsidies (to all individuals) but lower assimilation subsidies (to minority individuals) to more central agents in the social network. In the final part of the paper, we consider different communities that are not linked to each other and show that bridging them is always good for assimilation and also for total welfare.

The rest of the paper unfolds as follows. In the section, we relate our model to the relevant literatures and highlight our contribution. In Section 3, we expose the basic model, derive the two stages and characterize the subgame-perfect Nash equilibrium of this game. In Section 4, we illustrate our results with two specific networks: the star-shaped and the complete network. Section 5 is devoted to the welfare analysis and the subsidy policies aiming at restoring the first best. In Section 6, we consider separated communities and analyze how bridging them affects the outcomes of the individuals. Finally, Section 7 concludes.

2 Related literature

Cultural transmission There is a literature related to the assimilation of ethnic minorities and immigrants that has looked at the cultural transmission, in particular, on the transmission of ethnic identity from parents and peers to children. Indeed, based on some works on anthropology and sociology (see, in particular, Boyd and Richerson, 1985, Cavalli-Sforza and Feldman, 1981; Phinney, 1990), there is a literature in economics initiated by Bisin and Verdier (2000, 2001) arguing that the transmission of a particular trait (religion, ethnicity, identity, etc.) is the outcome of a socialization inside (parents) and outside the family (peers and role models). Parents directly make various socialization choices, e.g., the rules and beliefs the family conforms to and how much time they spend with their children. This clearly affects the assimilation process of immigrants.

Ethnic identity The role of parents in the socialization of their own children is nonetheless limited by the children’s pro-active role in choosing who to imitate and learn from, thereby directly shaping their own cultural identity. This is what Akerlof and Kranton (2010) have studied by developing the concept of identity economics, which aims to incorporate into economic theory the fact that people do not only pursue economic objectives, but also exert effort to gain and/or retain acceptance into a social group with which they identify. Many other papers have investigated the role of identity in the assimilation patterns of ethnic minorities (see, e.g. Bisin et al., 2011a,b; Panebianco, 2014; Verdier and Zenou, 2015). In a series of papers, Zimmermann et al. (2007), Constant and Zimmermann (2008), Constant et al. (2009) have proposed a new measure of the

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3For a review, see Bisin and Verdier (2011).

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ethnic identity of migrants by modeling its determinants and explores its explanatory power for various types of their economic performance. They have proposed the *ethnosizer*, a measure of the intensity of a person’s ethnic identity, which is constructed from information on language, culture, societal interaction, history of migration, and ethnic self-identification.

There is also an important literature that studies the concept of *oppositional cultures* among ethnic minorities. In this literature, ethnic groups may “choose” to adopt what are termed “oppositional” identities, that is, some actively reject the dominant ethnic (e.g., white) behavioral norms (they are oppositional) while others totally assimilate to it (see, in particular, Ainsworth-Darnell and Downey, 1998). Studies in the US and in Europe have found, for example, that African American students in poor areas may be ambivalent about learning standard English and performing well at school because this may be regarded as “acting white” and adopting mainstream identities (Fordham and Ogbu, 1986; Wilson, 1987; Delpit, 1995; Ogbu, 1997; Bisin et al., 2011b; Battu and Zenou, 2010; Fryer and Torelli, 2010; Patacchini and Zenou, 2016). On the theoretical side, Akerlof (1997), Austen-Smith and Fryer (2005), Selod and Zenou (2006), Battu et al. (2007), Bisin et al. (2011a) and De Marti and Zenou (2015) have proposed different models analyzing how oppositional identities affect the outcomes of ethnic minorities.

Compared to these two literatures, our main contribution is to put forward the role of social networks in the choice of assimilation of ethnic minorities and immigrants.

**Networks and assimilation** There is a very small literature that looks at the impact of social networks on the assimilation choices of immigrants and ethnic minorities. Zhou and Bankston (1998) suggest that participating in ethnic religious institutions promotes upward assimilation through instilling an ethnic identity onto youths. Nguyen Le (2010) studies how do peer networks formed at youth groups affect assimilation trajectory for Vietnamese Americans. He shows that participation in Buddhist youth groups instills a Vietnamese-American identity on youths in Seattle and, in turn, this ethnic identity can lead to upward assimilation only if the individual is part of a peer network that promotes normative values. Peer networks formed at youth group shape individual behaviors because the peer networks’ ties are remarkably strong. Members can relate to each other better than they can relate to their family members and school friends, since they share life experiences common amongst second generation Vietnamese Americans. Gang and Zimmermann (2000) show that ethnic network size has a positive effect on educational attainment, and a clear pattern is exhibited between countries-of-origin and education even in the second generation. Using the 2000 U.S. Census, Furtado and Theodoropoulos (2010) examine whether having access to native networks, as measured by marriage to a native, increases the probability of immigrant employment. They show that, indeed, marriage to a native increases immigrant employment
rates. They present several pieces of evidence suggesting that networks obtained through marriage play an important part in explaining the relationship between marriage decisions and employment. Mouw et al. (2014) use a unique binational data on the social network connecting an immigrant sending community in Guanajuato, Mexico, to two destination areas in the United States. They test for the effect of respondents’ positions in cross-border networks on their migration intentions and attitudes towards the United States using data on the opinions of their peers, their participation in cross-border and local communication networks. They find evidence of network clustering consistent with peer effects. Using German data and assuming that assimilation is taking place in the acquisition of education in Australia, Maani et al. (2015) show the ethnic capital network effects on immigrants’ earnings assimilation are significant so that immigrants in Australia benefit from being spatially concentrated.

None of these studies, however, look at the exact network structure and the role of the position of the network in the assimilation process.

3 The model

There are \( q + n \) individuals in the economy, \( q \) individuals belonging to the majority group and \( n \) individuals belonging to the minority group. The network \( g \) is a set of individuals \( Q + N \), where \( Q = \{1, \ldots, q\}, q \geq 2 \) (set of majority individuals) while \( N = \{1, \ldots, n\}, n \geq 2 \) (set of minority individuals), and a set of links or direct connections between them. The adjacency matrix \( G = [g_{ij}] \) keeps track of the direct connections in the network. By definition, agents \( i \) and \( j \) are directly connected if and only if \( g_{ij} = 1 \); otherwise, \( g_{ij} = 0 \). We assume that if \( g_{ij} = 1 \), then \( g_{ji} = 1 \), so the network is undirected. By convention, \( g_{ii} = 0 \). \( ^{4} \)

The timing of the model is as follows. In the first stage, each ethnic minority \( i \) decides whether she wants to keep her culture of origin (\( \alpha = \alpha_0 \)) or assimilate to the culture of the majority group of the host country (\( \alpha = \alpha_1 \)). Individuals from the majority group do not need to make this choice since, by definition, they are assimilated to their own culture (i.e. \( \alpha = \alpha_1 \)). In the second stage, given that the position in the network is given, each individual (ethnic minorities and individuals from the majority group) has to decide how much effort to put in some activity, say education. We assume that more assimilated minorities (those who choose \( \alpha_1 \)) are more productive (in terms of acquiring education or in terms of productivity on the job) than less assimilated minorities (those who choose \( \alpha_0 \)) because the latter lack the exact skills required for jobs in the host country (for example, they do not speak fluently the language of the host country, they have different ways of

\footnote{All our results are not affected if we consider a directed network, i.e. \( g_{ij} \neq g_{ji} \), and a weighted network, i.e. \( g_{ij} \in [0, 1] \).}
To be more precise, assimilated minority individuals have an education productivity of \( \alpha_1 = \alpha \) but have a cost of assimilation equal to \( c \) while oppositional minority individuals have a productivity of \( \alpha_0 = \alpha - t \) but have no cost of assimilation. In other words, \( c \) is the assimilation cost (because of peer pressure from the ethnic group or simply because it is costly to learn the new culture and language of the majority group) while \( t \) is the productivity cost of being non-assimilated (because of poorer language skills that affect the productivity of education or on-the-job skill). We have \( \alpha_i \in \{ \alpha - t, \alpha \} \), \( \forall i = 1, 2, \ldots, n \). As usual, we solve the model backwards.

3.1 Second stage: Choosing education effort

Let us solve the second stage where individuals choose simultaneously their education efforts.

3.1.1 Preferences

The preferences of individual \( i \) in network \( g \) is given by:

\[
U_i(x_i, x_{-i}, g) = \alpha_i x_i - \frac{1}{2} x_i^2 + \theta \sum_{j=1}^n g_{ij} x_i x_j, \tag{1}
\]

where \( x_i \) is the education effort\(^5\) that individual \( i \) exerts, \( x_{-i} \) is the corresponding vector of efforts for the other \( q + n - 1 \) agents in the network. The parameter \( \theta > 0 \) measures the synergy of social interactions between linked individuals. When \( \theta \) is positive, an increase in \( x_j \), the education effort of individual \( j \) who is directly linked to \( i \), increases the marginal utility for individual \( i \) of exerting own effort \( x_i \). For the majority group, by definition, \( \alpha_i = \alpha \), \( \forall i = 1, 2, \ldots, q \), while, for the minority group, \( \alpha_i = \alpha \) for those who assimilate to the majority group while \( \alpha_i = \alpha - t \) for those who do not assimilate (i.e. oppositional). We assume \( \alpha > t \), so that \( \alpha_i > 0, \forall i = 1, 2, \ldots n \). The utility function (1) has two parts. The first one, \( \alpha_i x_i - \frac{1}{2} x_i^2 \), is the utility of exerting \( x_i \) when there is no interaction with other individuals. The second part, \( \theta \sum_{j=1}^n g_{ij} x_i x_j \), is what is obtained by interacting with direct friends in the network.

In the second stage, each agent \( i \) chooses \( x_i \) to maximize (1) taking the structure of the network and the effort choices of other agents as given. In Appendix 1, we define the Katz-Bonacich centrality, which will be useful in the characterization of the Nash equilibrium.

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\(^5\)We interpret our model in terms of education effort but, of course, it can be interpreted in terms of skill or productive effort.
3.1.2 Nash equilibrium in education efforts

For each \( i = 1, \ldots, n \), the first-order condition with respect to \( x_i \) is given by:

\[
x_i^* (\alpha_i, \alpha_{-i}, g) = \alpha_i + \theta \sum_{j=1}^{q+n} g_{ij} x_j^* \quad \forall i = 1, 2, \ldots, n + q.
\]  

The optimal education effort choice of agent \( i \) is a linear function of the education efforts of the agents to whom \( i \) is directly connected in the network. Solving for the \( (\theta + 1) \) vector \( x \) and using (21) gives the Nash equilibrium visit vector \( x^* \):

\[
x^* = (I - \theta G)^{-1} \alpha = M \alpha
\]

where \( M \equiv (I - \theta G)^{-1} \) is a \((q + n \times q + n)\) matrix. Denote by \( \rho(G) \) the spectral radius of the adjacency matrix \( G \). Then, we have the following result:

**Proposition 1 (Equilibrium efforts)** If \( \theta \rho(G) < 1 \), there exists a unique, interior Nash equilibrium in effort choices, which, for each individual \( i \), is given by:

\[
x_i^* (\alpha_i, \alpha_{-i}, g) = b_{\alpha_i}(g, \theta).
\]

The proof of this proposition can be found in Ballester et al. (2006) and Calvó-Armengol et al. (2009). The Nash equilibrium number of efforts \( x_i^* (\alpha_i, \alpha_{-i}, g) \) depends on the position in the social network and the assimilation choices of the minority workers. Proposition 1 implies that an agent who is more central in the social network, as measured by her Katz-Bonacich centrality, will make more (education) effort in equilibrium. Intuitively, agents who are better connected have more to gain from interacting with others and so exert higher effort for any vector of assimilation choices. It is straightforward to verify that \( x_i^* (\alpha_i, \alpha_{-i}, g) \) increases with \( \alpha \), the degree of assimilation, and \( \theta \), and the intensity of social interactions but decreases with the productivity cost \( t \).

3.2 First stage: Choosing the degree of assimilation

As stated above, there is an exogenous cost differential \( c > 0 \) associated with assimilation. We consider any network with \( q + n \) agents but only focus on the choice of the ethnic minority population given by \( N = \{1, \ldots, n\} \). We assume that the \( q \) individuals from the majority group occupy the most central position (in terms of Katz-Bonacich centrality) in the network (for historical reasons and because of old-boy networks). In fact, this assumption is not necessary for the results of this section but will facilitate the characterization of equilibrium.
Agents from the minority group choose the $\alpha_i$ that maximizes their net utility, taking the efforts of all other agents (including the majority individuals) as given. We look at subgame-perfect equilibria. We have seen in the previous section (Proposition 1) that, if $\theta \rho(G) < 1$, there exists a unique effort level for each individual $i$ given by: $x_i^* (\alpha_i, \alpha_{-1}, g) = b_{\alpha_i}(g, \theta)$. Using the best-response function (2) and plugging it into (1), we can write the equilibrium utility level of individual $i$ as:

$$U_i^*(x_i^*, x_{-i}^*, g) = \frac{1}{2} [x_i^* (\alpha_i, \alpha_{-1}, g)]^2 = \frac{1}{2} [b_{\alpha_i}(g, \theta)]^2$$

(5)

where $x_i^*(\alpha_0, x_{-1}, g)$ and $x_i^*(\alpha_1, x_{-1}, g)$ are the equilibrium effort of individual $i$ if she is oppositional or not assimilated ($\alpha_i^* = \alpha_0$) and if she is assimilated ($\alpha_i^* = \alpha_1$), respectively. We need now to solve the first stage of the game, i.e. the assimilation choices.

Define $\mathcal{A}$ as the set of assimilated individuals (i.e. all individuals from the minority group who choose $\alpha_1 = \alpha$ and all individuals from the majority group) and $\mathcal{O}$ as the set of oppositional individuals (i.e. all minority individuals who choose $\alpha_0 = \alpha - t$). If individual $i$ choose to assimilate ($\alpha_1 = \alpha$), her equilibrium utility is equal to:

$$U_i^*(x_i^*(\alpha_1, x_{-1}, g), x_{-1, 1}^*) = \frac{1}{2} \left[ \sum_{j \in \mathcal{A}} \sum_{k=0}^{+\infty} \theta k g_{ij}^{|k|} \alpha + \sum_{j \in \mathcal{O}} \sum_{k=0}^{+\infty} \theta k g_{ij}^{|k|} (\alpha - t) + \sum_{k=0}^{+\infty} \theta k g_{ii}^{|k|} \alpha \right]^2 - c$$

We have here decomposed the Katz-Bonacich centrality $b_{\alpha_i}(g, \theta)$ into self-loops ($m_{ii} = \sum_{k=0}^{+\infty} \theta k g_{ii}^{|k|}$) and non self-loops ($m_{ij} = \sum_{k=0}^{+\infty} \theta k g_{ij}^{|k|}$) and give different weights to these paths depending if agents are assimilated (weight $\alpha$) or oppositional (weight $\alpha - t$). If individual $i$ chooses to be oppositional ($\alpha_0 = \alpha - t$), her equilibrium utility is given by:

$$U_i^*(x_i^*(\alpha_0, x_{-1}, g), x_{-1, 1}^*) = \frac{1}{2} \left[ \sum_{j \in \mathcal{A}} \sum_{k=0}^{+\infty} \theta k g_{ij}^{|k|} \alpha + \sum_{j \in \mathcal{O}} \sum_{k=0}^{+\infty} \theta k g_{ij}^{|k|} (\alpha - t) + \sum_{k=0}^{+\infty} \theta k g_{ii}^{|k|} (\alpha - t) \right]^2$$

Thus, individual $i$ will assimilate if and only if $U_i^*(x_i^*(\alpha_1, x_{-1}, g), x_{-1, 1}^*) > U_i^*(x_i^*(\alpha_0, x_{-1}, g), x_{-1, 1}^*)$.

We would now like to deal with the issues of existence and uniqueness of the subgame-perfect equilibrium assimilation-effort. Since all the $q$ individuals from the majority group are the most central agents (in terms of Katz-Bonacich centrality), we can rank them starting with agent 1, who is the most central agent from the majority group, $m_{ii} = \max_i m_{ii}$, $\forall i \in Q$, then agent 2, who has the next highest centrality, etc. until we reach agent $q$ who has the lowest centrality in the network among the majority individuals, i.e. $m_{q,q} = \min_i m_{ii}$, $\forall i \in Q$. Then, we start with the agent who has the highest centrality among the minority individuals, i.e. $m_{q+1,q+1} = \max_i m_{ii}$, $\forall i \in N$.

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*(See Appendix 1 for the interpretations of the $m_{ii}s$ and $m_{ij}s$.*
Then, we have the second minority agent, who has the next highest centrality among the minority group, etc., until we reach agent $n$ who has the lowest centrality in the network among the majority individuals, i.e. $m_{q+n,q+n} = \min_i m_{ii}, \forall i \in N$. Define each minority individual by her type, where the type of an agent is her Katz-Bonacich centrality (or her $m_{ii}$). Since two minority agents can have the same centrality, there are $\omega \leq n$ types in each network of $q + n$ agents. Observe that $\omega = n$ if there are no minority individuals who have the same position (or centrality) in the network. For example, in a star network with 10 minority individuals (and no majority individuals), $\omega = 2$ and $n = 10$ while, in a complete network with the same number of individuals, $\omega = 1$ and $n = 10$. For the characterization of the subgame-perfect equilibrium, the types of the majority individuals do not matter as they are all assimilated.

To characterize the equilibria, we will use a technique similar to the one developed in Helsley and Zenou (2014). Denote by

$$
\Phi^A(m_{i,i}) \equiv t (2\alpha - t) (m_{i,i})^2 + 2t \left( \sum_{j=1, j\neq i}^{q+n} m_{i,j} \right) m_{i,i}, \ i = 1, \ldots, n
$$

(6)

where all the $m_{ii}$s and $m_{ij}$s are defined by the cells of the $(q + n \times q + n)$ matrix $M = [I - \theta G]^{-1}$. In (6), $\Phi^A(m_{i,i})$ is the incentive function for a given ethnic individual $i \in N$ to choose to assimilate or not when all other minority (and, of course, majority) agents are assimilated. We have a first intuitive result:

**Proposition 2** Assume $\theta \rho(G) < 1$ and consider any network of $q + n$ agents with $\omega \leq n$ types for the minority individuals. In any equilibrium, two minority workers with the same Katz-Bonacich centrality make the same assimilation choice and agents with higher Katz-Bonacich centrality cannot be oppositional if agents with lower Katz-Bonacich centrality are assimilated. Moreover, the number of equilibria is equal to the number of types of minority individuals plus one, i.e. $\omega + 1$.

Let us now derive our existence and uniqueness result:

**Proposition 3 (Existence and uniqueness of equilibrium assimilation behaviors)** Assume $\theta \rho(G) < 1$ and consider any network of $q + n$ agents with $\omega \leq n$ types for the minority individuals. If the number of types of minority individuals is the same as the number of minority individuals, we can characterize the subgame-perfect equilibria as follows:

(i) If $2c < \Phi^A(m_{q+n,q+n})$

there exists a unique Assimilation equilibrium where all agents choose to assimilate, i.e. $A = Q + N$ and $O = \emptyset$.
(ii) If
\[ \Phi^A(m_{q+n,q+n}) < 2c < \Phi^A(m_{q+n-1,q+n-1}) - 2\mu^2 m_{q+n-1,q+n+1} \]
there exists a unique Assimilation-Oppositional equilibrium such that \( A = Q + N - \{n\} \) and \( O = \{n\} \).

(iii) If
\[ \Phi^A(m_{q+n-1,q+n-1}) - 2\mu^2 m_{q+n-1,q+n+1} m_{q+n-1,q+n+1} \]
\[ < 2c < \Phi^A(m_{q+n-2,q+n-2}) - 2\mu^2 (m_{q+n-2,q+n-1} + m_{q+n-2,q+n}) m_{q+n-2,q+n-2} \]
there exists a unique Assimilation-Oppositional equilibrium such that \( A = Q + N - \{n-1,n\} \) and \( O = \{n-1,n\} \).

(iv) If
\[ \Phi^A(m_{q+n-2,q+n-2}) - 2\mu^2 (m_{q+n-2,q+n-1} + m_{q+n-2,q+n}) m_{q+n-2,q+n-2} \]
\[ < 2c < \Phi^A(m_{q+n-3,q+n-3}) - 2\mu^2 \left( \sum_{j \in O} m_{q+n-3,q+j} \right) m_{q+n-3,q+n-3} \]
there exists a unique Assimilation-Oppositional equilibrium such that \( A = Q + N - \{n-2,n-1,n\} \) and \( O = \{n-2,n-1,n\} \).

(v) etc. until we arrive at agent \( 1 \) who has the highest centrality among the minority group. Then,

(vi) If
\[ \Phi^A(m_{q+1,q+1}) - 2\mu^2 \left( \sum_{j \in O-\{1\}} m_{q+1,q+j} \right) m_{q+1,q+1} < 2c \]
there exists a unique Oppositional equilibrium where all minority individuals choose not to assimilate, i.e. \( A = Q \) and \( O = N \).

If the number of types is less than the number of minority individuals, then each step described above has to be made by type and not by individual so that each subscript refers to types and not to individuals.

This proposition characterizes the (subgame-perfect Nash) equilibrium assimilation choices and shows that there always exists a unique equilibrium within each interval. The key object is \( \Phi^A(m_{ii}) \), which is the “incentive function” when there is an Assimilated equilibrium, i.e. when all minority individuals choose to assimilate. Indeed, when, for all \( i \), \( \Phi^A(m_{ii}) > 2c \), all minority individuals
choose to assimilate because the cost of assimilation is low enough compared to the gain from assimilation in terms of productivity and synergies from other individuals (both from the majority and minority groups). In that case, there is a unique Assimilation equilibrium. Then, when we start to increase the cost of assimilation, $c$, and change the decision of minority workers from assimilation to oppositional, the weight in the Katz-Bonacich centrality changes from $\alpha$ (when assimilated) to $\alpha - t$ (when oppositional). This corresponds to the terms of both the right-hand side and left-hand side of each inequality since this is what is needed to be compensated for the agents who are oppositional compared to the Assimilation equilibrium where these agents are all assimilated. Observe that there cannot be multiple equilibria within the same set of parameters.

This result is related to the literature on ethnic identity highlighted in Section 2. In this literature, ethnic groups may “choose” to adopt “oppositional” identities, that is, some actively reject the dominant ethnic (e.g., white) behavioral norms (they are oppositional) while others totally assimilate to it. Proposition 3 shows that these choices, especially the assimilation choice, negatively depends on the cost $c$ of assimilating to the majority culture, positively on $t$, the productivity cost of being non-assimilated, and positively on the position (or centrality) of the individual in the network.

The impact of $c$ on assimilation choices is well-documented. Indeed, studying the assimilation of Muslims in France, Adida et al. (2014) show that if the cost of assimilation is sufficiently high (possibly due to taste-based discrimination by the dominant group), it then becomes rational for members of the minority to eschew assimilation. This is consistent with Proposition 3 but also with other theoretical models (see, in particular, Laitin, 1995; Akerlof and Kranton, 2010; Fang and Loury, 2005; Bisin et al., 2011a). Also, focusing on Mexican immigrants in the United States, Mouw et al. (2014) show that respondents who speak English poorly are more likely to report that they would be happier living in Mexico (a measure of assimilation). Finally, Dustmann (1996) documents that immigrants from more distant cultural backgrounds (e.g., Turks, Greeks) exhibit a lower assimilation tendency into German society. Language proficiency and education are other important determinants of cultural assimilation, as is labor market integration (Dustmann, 1994, 1996). In these studies, discrimination, language skills, education and cultural distance are a measure of $c$, the cost of assimilating to the majority culture.

If we now consider the impact of $t$ on assimilation choices, there is also plenty of evidence. For example, Constant et al. (2009), Zimmerman (2007) and Constant and Zimmerman (2008) show that there are positive and significant impacts of assimilation with the host culture on economic behavior such as work probability, and earnings. For example, Constant and Zimmermann (2009) find that assimilation with German culture significantly increases the probability of work for male immigrants, relative to those identified as separated or marginalized. More recently, Gorinas (2014),
shows that immigrants who share social norms with the majority experience significantly better employment outcomes, particularly first-generation immigrant women. These results are also consistent with Proposition 3 where $t$ can capture workers’ productivity but also the probability of finding a job.

Finally, the most original result in Proposition 3 is the role of social networks, in particular, the role of the centrality of each individual in the assimilation process. For example, if the cost of assimilation $c$ is relatively large (as in case (iv) in the proposition), then the less (more) central are the agents, the more (less) likely they will (not) assimilate to the majority culture. This is because the benefits of assimilation (better outcomes in terms of education or employment) outweigh the costs and the more central are the individuals, the more externalities in terms of productivity they obtain from being assimilated. As stated in Section 2, there are very few papers that study the role of networks on assimilation of immigrants. Some papers, such as the ones by Gang and Zimmermann (2000), Furtado and Theodoropoulos (2010), Mouw et al. (2014) and Maani et al. (2015) have examined the impact of immigrant networks on outcomes and find positive effects. An interesting paper is that of Furtado and Theodoropoulos (2010) who show that having access to native networks increases the probability of immigrant employment. This is what is found in Proposition 3 where some minorities are linked to some majorities workers and this helps them become assimilated and improve their productivity and thus their situation in the labor market.

4 Examples

4.1 Star-shaped network

To illustrate the previous results, consider the star-shaped network $g$ depicted in the following figure:

![Figure 1: A star network with 3 individuals](image)

In this figure, there are three agents (i.e. $q + n = 3$) and agent 1 holds a central position whereas agents 2 and 3 are peripherals. Suppose that individual 1 belongs to the majority group while individuals 2 and 3 belong to the minority group, i.e. $Q = \{1\}$ and $N = \{2, 3\}$. Since there is only one type for the minority workers, i.e. $\omega = 1$, there will be only two possible equilibria, which are given by the following proposition:
Proposition 4 (Assimilation choices in a star-shaped network) Consider the star-shaped network depicted in Figure 1 where individual 1 belongs to the majority group and individuals 2 and 3 are minority workers. Assume \( \theta < 1/\sqrt{2} \).

(i) If
\[
c < \frac{t(1-\theta)(1+\theta)^2 [2\alpha - (1-\theta) t]}{2 (1-2\theta)^2}
\]
there exists a unique Assimilation equilibrium where all agents are assimilated, i.e. \( A = \{1,2,3\} \) and \( O = \emptyset \).

(ii) If
\[
c > \frac{t(1-\theta)(1+\theta)^2 [2\alpha - (1-\theta) t]}{2 (1-2\theta)^2}
\]
there exists a unique Oppositional equilibrium where all minority workers are oppositional, i.e. \( A = \{1\} \) and \( O = \{2,3\} \).

This proposition shows the role of \( c \) and \( t \) in the assimilation choices. For fixed values of \( \alpha, t \) and \( \theta \), when we increase \( c \), we switch from an Assimilation equilibrium to an Oppositional equilibrium. As stated above, this is what was found by Adida et al. (2014) who study the assimilation of Muslims in France. In their paper, they measure the cost of assimilation \( c \) by the taste-based discrimination of Muslims by the majority group. Similar results are obtained when \( t \) decreases.

In our model, \( t \) is the productivity cost of being non-assimilated because, for example, of poorer language skills that affect the productivity of education or on-the-job skill. Interestingly, for fixed values of \( \alpha, t \) and \( c \), when we decrease \( \theta \), the degree of social interactions, we obtain the same types of result because an increase in \( \theta \) means that social interactions are more valuable in terms of outcomes and thus tend to induce people to assimilate more. Let us give some parameter values for which each condition is satisfied given that \( \theta < 1/\sqrt{2} = 0.707 \). For example, for Proposition 4, if we set \( \alpha = 6, t = 1 \) and \( \theta = 0.2 \), then: (i) if \( c < 7.62 \), there exists a unique Assimilation equilibrium where \( A = \{1,2,3\} \) and \( O = \emptyset \); (ii) if \( c > 7.62 \), there exists a unique Oppositional equilibrium where \( A = \{1\} \) and \( O = \{2,3\} \).

4.2 Complete networks

Let us now consider a complete network where, as in the previous example, we set \( q + n = 3 \). Since there is necessary only one type of minority worker, there will always only be the two following equilibria: an Assimilation equilibrium and an Oppositional equilibrium. Assume that agent 1 belongs to the majority group and individuals 2 and 3 to the minority group.
Proposition 5 (Locational equilibrium for a complete network) Consider the complete network with 3 agents where individual 1 belongs to the majority group and 2 and 3 are minority workers. Assume that $\alpha < 1$. If

$$c < \frac{t(1-\theta)^2(2\alpha + 4\alpha t - t)}{2(1-\theta - 2\theta^2)^2}$$

there exists a unique Assimilation equilibrium where all minority agents assimilate, i.e. $A = \{1, 2, 3\}$ and $O = \emptyset$.

(ii) If

$$c > \frac{t(1-\theta)^2(2\alpha + 4\alpha t - t)}{2(1-\theta - 2\theta^2)^2}$$

there exists a unique Oppositional equilibrium where all minority agents are oppositional, i.e. $A = \{1\}$ and $O = \{2, 3\}$.

This proposition completely characterizes the equilibrium configuration for a complete network. We can give parameter values for which each condition is satisfied given that $\theta < 1/2$. For example, if take exactly the same parameters as for the star network, i.e. $\alpha = 6$, $t = 1$ and $\theta = 0.2$, then: (i) if $c < 21.61$, there exists a unique Assimilation equilibrium where $A = \{1, 2, 3\}$ and $O = \emptyset$; (ii) if $c > 21.61$, there exists a unique Oppositional equilibrium where $A = \{1\}$ and $O = \{2, 3\}$.

When we compare the star network and the complete network with 3 agents, where individual 1 belongs to the majority group and 2 and 3 are minority workers, we see that there is much more assimilation among minority workers in the complete network than in the star-shaped network. Indeed, if we again consider the parameters $\alpha = 6$, $t = 1$ and $\theta = 0.2$, then when $7.62 < c < 21.61$, all minority workers are assimilated in the complete network while they all are oppositional in the star network. This is because there are much more interactions in the complete than in the star network because, in the former, everybody interact directly with everybody while, in the latter, agents 1 and 2 interact directly with the star (agent 1) but only indirectly with each other. This is in fact a general result, which is straightforward to prove, which says that denser networks will have more assimilated individuals than less dense networks. To summarize,

Proposition 6 (Aggregate interactions) Assume $\theta \rho(G) < 1$. There is more assimilation in denser networks than in less dense networks.

This result is due to the fact that, because of (local) complementarities in effort in the utility function, aggregate interactions as well as the entire vector of individual interactions increase with
the density of the network. As a result, minority individuals find it more beneficial to assimilate in denser networks. As White et al. (1976) remind us, social networks, by themselves, do not produce a uniformly simple effect, nor are they simply the conduit of contextual influence. Rather, they can facilitate or inhibit assimilation by structuring interactions between initiates and members and by tying both into the larger social structure. At each stage, then, networks constrain or facilitate contact with members, condition members’ reactions to initiates, and influence initiates’ attitudes, values and beliefs. Proposition 6 formalizes this intuition in terms of network density.

5 Welfare analysis and subsidy policies

Let us first look at the second stage of the game where all agents decide their optimal effort level. Let us therefore study welfare policies for a given assimilation equilibrium.

5.1 First-best analysis when assimilation is given

5.1.1 First-best analysis

We would like to see if the equilibrium outcomes are efficient in terms of productive interactions. For that, the planner chooses $x_1, ..., x_n$ to maximize total welfare, that is:

$$
\max_{x_1, ..., x_n} \mathcal{W} = \max_{x_1, ..., x_n} \sum_{i=1}^{i=q+n} U_i(x_i, x_{-i}, g)
$$

$$
= \max_{x_1, ..., x_n} \left\{ \sum_{i=1}^{i=q+n} \left[ \alpha_i x_i - \frac{1}{2} x_i^2 \right] + \theta \sum_{i=1}^{i=q+n} \sum_{j=1}^{q+n} g_{ij} x_i x_j \right\}
$$

First-order condition gives for each $i = 1, ..., q + n$:

$$
\alpha_i - x_i + \theta \sum_j g_{ij} x_j + \theta \sum_j g_{ji} x_j = 0
$$

which implies that (since $g_{ij} = g_{ji}$):

$$
x_i^O = \alpha_i + 2\theta \sum_j g_{ij} x_j
$$

---

7 We define network density as follows. Consider an alternative social network $g'$, $g' \neq g$ such that for all $i, j$, $g'_{ij} = 1$ if $g_{ij} = 1$. It is conventional to refer to $g$ and $g'$ as nested networks, and to denote their relationship as $g \subset g'$. As discussed in Ballester et al. (2006), the network $g'$ has a denser structure of network links: some agents who are not directly connected in $g$ are directly connected in $g'$.

8 It is easily checked that there is a unique maximum for each $x_i$.

9 The superscript $O$ refers to the “social optimum” outcome while a star refers to the “Nash equilibrium” outcome.
Using (2), we easily see that:

\[ x_i^O = v_i^* + \theta \sum_j g_{ij} x_j \]  

(10)

where \( x_i^* \) is the Nash equilibrium efforts given in (2). This means that individuals are exerting too little effort at the Nash equilibrium as compared to the social optimum outcome. Equilibrium interaction effort is too low because each agent ignores the positive impact of her effort on the efforts of others, that is, each agent ignores the positive externality arising from complementarity in effort choices. As a result, the market equilibrium is not efficient and the planner would like to subsidize the (productivity) effort of each agent.

### 5.1.2 Subsidizing social interactions

Letting \( S_i^O \) denote the optimal subsidy to per effort, comparison of (9) and (10) implies:

\[ S_i^O = \theta \sum_j g_{ij} x_j \]  

(11)

or in matrix form

\[ S^O = \theta Gx \]

If we add one stage before the visit game is played, the planner will announce the optimal subsidy \( S_i^O \) to each agent \( i \) such that:

\[ U_i = (\alpha_i + S_i^O) x_i - \frac{1}{2} x_i^2 + \theta \sum_j g_{ij} x_i x_j = \alpha_i x_i - \frac{1}{2} x_i^2 + 2\theta \sum_j g_{ij} x_i x_j \]

By doing so, the planner will restore the first best. Observe that the optimal subsidy is such that

\[ x^O = (I - \theta G)^{-1} (\alpha_i + S_i^O) \quad 1 = (I - 2\theta G)^{-1} \alpha \]

where \( \alpha = (\alpha_1, ..., \alpha_n)^T \), which means that

\[ x_i^O = \sum_{j=1}^{q+n} \sum_{k=0}^{+\infty} \theta^k g_{ij}^{[k]} (\alpha_j + S_j^O) = \sum_{j=1}^{q+n} \sum_{k=0}^{+\infty} 2^k \theta^k g_{ij}^{[k]} \alpha_j \]

and thus

\[ U_i^O = \frac{1}{2} [b_{\alpha + S^O}(\theta, g)]^2 = \frac{1}{2} [b_{\alpha}(2\theta, g)]^2 \]

In particular, the optimal subsidy is given by:

\[ S_i^O = \theta \sum_j g_{ij} x_j^O = \theta \sum_{j=1}^{q+n} \sum_{k=0}^{+\infty} 2^k \theta^k g_{ij}^{[k]} \alpha_i \]  

(12)
What is interesting here is that the planner will give a larger subsidy to more central agents (independently of their ethnicity) in the network. Since we assumed that the individuals from the majority group are more central than the individuals from the minority group, then the planner will subsidize more the activities (in terms of education or work) of the majority group.

**Proposition 7 (Optimal level of social interactions)** *The Nash equilibrium outcome in terms of productive efforts is not efficient since there are too few of them. If the planner proposes a subsidy \( S_i^O = \theta \sum_j g_{ij} x_j \) to each individual \( i \), then the first-best outcome can be restored. In that case, it is optimal for the planner to give higher subsidies to more central agents in the network.*

### 5.2 Endogenous choice of assimilation

#### 5.2.1 Effort subsidies

Assume that the planner cannot control assimilation choices but only effort decisions. In that case, in the second stage, she will choose a higher level of effort given by

\[
x_i^O = \sum_{j=1}^{g+n} \sum_{k=0}^{+\infty} 2^k \theta^k g_{ij}^{[k]} \alpha_j
\]

and then let the minority agents choose whether they want to assimilate or not. In other words, the timing is as follows: First, minority agents choose whether to assimilate or not and then the government chooses the optimal effort for all agents. The second stage is solved as above and the optimal effort is given by (13). We can characterize the equilibrium assimilation and it is clear that more minority agents will assimilate compared to the case when there are no subsidies.

Furthermore, if we investigate a constrained efficient allocation in which the planner can subsidize interactions (i.e., provide a subsidy of \( S_i^O \) per effort by agent \( i \)) but cannot directly control assimilation choices, then more agents will assimilate compared to the case without subsidies. Indeed, since all agents devote more effort to interacting with others under the subsidy program (11), the incentives for assimilation must be stronger under that allocation than in the Subgame-Perfect Nash equilibrium.

**Proposition 8 (Equilibrium versus optimal assimilation choices)** *If the planner proposes a per effort subsidy \( S_i^O \) to each individual \( i \), then, compared to the Subgame-Perfect Nash equilibrium described in Proposition 3, more minority agents will assimilate.*

#### 5.2.2 Assimilation subsidies

Let us now consider the case where the planner subsidizes assimilation but not effort. Since there are more interactions when agents assimilate and since interactions increase utility, then the plan-
ner could subsidize the assimilation cost \( c \). For example, the government can make it easier for immigrants to assimilate by helping them moving to more mixed areas\(^\text{10}\) or by having language courses for new immigrants (for example, the Swedish For Immigrants course in Sweden for new immigrants). The law on veiling, by preventing young Muslim women to wear the veil in public areas, could also be seen as a way to help young ethnic minorities to assimilate to the majority culture.\(^\text{11}\)

In this policy, the government gives a per-cost subsidy \( \sigma \) so that the cost of assimilating would be \((1 - \sigma) c\) instead of \( c \). The timing is now as follows. In the first stage, the planner announces the subsidy to ethnic minorities who decide to assimilate. In the second stage, agents decide whether they will assimilate or not and, in the last stage, their decide their effort level. As for the subsidy effort, this will clearly generate more assimilation but the mechanism is different since, in the latter, the effect is direct while, in the former, it is indirect. In that case, equilibrium efforts will still be determined by (4) while assimilation decisions will be characterized by Proposition 3 where \( c \) has to be replaced by \((1 - \sigma) c\).

In this model, it is clear that, if the planner wants to reach the first best in terms of assimilation, she will subsidize \( c \) so that all agents will be assimilated. This maximizes aggregate interactions and thus total welfare. For example, in the case of the star network described in Figure 1, where individual 1 is from the majority group and individuals 2 and 3 are from the minority group, we have shown that if \( \alpha = 6, t = 1 \) and \( \theta = 0.2 \), then: (i) if \( c < 7.62 \), there exists a unique Assimilation equilibrium; (ii) if \( c > 7.62 \), there exists a unique Oppositional equilibrium. As a result, if, for all agents, \((1 - \sigma) c \leq 7.62\), which is equivalent to \( \sigma \geq 1 - (7.62/c) \), then the first best is reached and all minority workers assimilate. For example, if \( c = 20 \), then planner needs to subsidize 61.9 percent of the cost of assimilation of all agents. Interestingly, this result depends on the network structure. For the complete network with 3 agents, we have seen that, with exactly the same parameters, \( \alpha = 6, t = 1 \) and \( \theta = 0.2 \), then: (i) if \( c < 21.61 \), there exists a unique Assimilation equilibrium; (ii) if \( c > 21.61 \), there exists a unique Oppositional equilibrium. In that case, we need to subsidize \( \sigma \geq 1 - (21.61) / c \) percent of \( c \) for all agents to reach the first best. Thus, for the complete network, if \( c = 20 \), the planner does not need to subsidy any worker to reach the first best in efforts since \( 20 < 21.61 \). Using this reasoning and looking at Proposition 3, the optimal subsidy for any network

\(^{10}\)For example, the Moving to Opportunity (MTO) programs do that by giving housing vouchers to poor family, usually blacks or Hispanics, to help them move to richer areas. See Ludwig et al. (2001) and Kling et al. (2005).

\(^{11}\)This is a controversial law and its effects on assimilation are not clear. For example, Carvalho (2013) models the veiling among Muslim women as a form of cultural resistance, which inhibits the transmission of secular values. His theory predicts that veiling is highest when individuals from highly religious communities interact in highly secular environments.
with $n$ minority individuals is given by:

$$\sigma^O > 1 - \frac{\Phi^A(m_{q+n,q+n})}{2c} \quad (14)$$

where, from (6), we have:

$$\Phi^A(m_{q+n,q+n}) \equiv t (2\alpha - t) (m_{q+n,q+n})^2 + 2t \left( \sum_{j=1}^{q+n-1} m_{q+n,j+n} \right) m_{q+n,q+n} \quad (15)$$

Observe that equation (14) gives the subsidy for the minority individual $q+n$ who has the lowest centrality in the network. Indeed, if the planner gives a $c-$subsidy of $1 - [\Phi^A(m_{q+n,q+n})/2c]$ to all agents, the first best will be reached since all individuals will be induced to assimilate. This is clearly a sufficient condition. The planner could also discriminate between agents and gives a different subsidy to each agent so that the higher is the centrality of an agent in a network, the lower is the subsidy. In that case, the subsidy to be given to each agent $i$ will be equal to:

$$\sigma^O_i > 1 - \frac{\Phi^A(m_{q+i,q+i})}{2c} \quad (16)$$

for all $i = 1, \ldots, n$, where $\Phi^A(m_{q+i,q+i})$ is defined by (6). Of course, this policy is much more complicated to implement (and more costly) because it requires that the planner knows the position of all agents in the network. On the contrary, the previous policy (14) only requires to know the minority worker who has the less central position in the network.

Observe also that if $\Phi^A(m_{q+n,q+n}) > 2c$, meaning that $1 - \frac{\Phi^A(m_{q+n,q+n})}{2c} < 0$, the condition (14) is always satisfied. This is because, in this case, we do not need to subsidize any worker to obtain an Assimilation equilibrium because $\Phi^A(m_{q+n,q+n}) > 2c$ is precisely the condition for which an Assimilation equilibrium exists and is unique (see Proposition 3(i)). Assuming that, when a worker is indifferent between assimilating and not assimilating, she always chooses to assimilate, then the subsidy (16) can be written as:

$$\sigma^O_i = \max \left\{ 0, 1 - \frac{\Phi^A(m_{q+i,q+i})}{2c} \right\} \quad (17)$$

### 5.2.3 Effort versus assimilation subsidy

Let us now study both the effort and assimilation subsidies. We have seen that if the planner only subsidizes effort, then the optimal subsidy is given by (12), that is

$$S^O_i = \theta \sum_{j=1}^{q+n} \sum_{l=1}^{q+n} \sum_{k=0}^{\infty} 2^k g^j g^l \alpha_l$$
This optimal subsidy clearly depends on agents’ assimilation behaviors. As a result, the first best when both assimilations and efforts are taken into account should be when the effort subsidy is $S_i^O$ and all agents assimilate. The timing is now as follows. First, the planner announces the assimilation and effort subsidies. Second, agents choose to assimilate or not. Third, agents choose efforts.

Denote $M^O = (I - 2\theta G)^{-1}$ so that the element of the $i$th and $j$th of $M^O$ is $m^O_{ij}$. Then, using the same reasoning as above, the assimilation subsidy and the effort subsidy that guarantee that the first best is achieved is determined in the following proposition:

**Proposition 9 (First best with effort and assimilation subsidies)** Assume $2\theta \rho(G) < 1$ and consider any network of $q + n$ agents. If the assimilation subsidy for minority individuals and the effort subsidy for all individuals are such that

$$
\left\{ \begin{array}{l}
\sigma^O_i = \max \left\{ 0, 1 - \frac{2\Phi^C(m^O_i)}{\epsilon} \right\}, \text{ for } i \in N \\
S^O_i = \theta \alpha \sum_{j=1}^{q^n} \sum_{k=0}^{q^n} \sum_{k=0}^{+\infty} \frac{\phi^{k\theta^k} g_{ij}^k g_{jl}^k}{(1+\theta^2)^k}, \text{ for } i \in Q + N
\end{array} \right.
$$

then all agents choose to assimilate and provide optimal interaction efforts and therefore the first best is achieved.

This proposition implies that, to reach the first best, it is optimal for the planner to give higher effort subsidies but lower assimilation subsidies to more central agents in the social network.

If we consider the star network of Figure 1, it is readily verified that, if $\theta < 1/(2\sqrt{2}) = 0.35$, then, if all agents assimilate (i.e. $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$), we have:

$$
M^O \alpha = (I - 2\theta G)^{-1} \alpha = \frac{\alpha}{(1 - 8\theta^2)} \begin{pmatrix}
1 + 4\theta \\
1 + 2\theta \\
1 + 2\theta
\end{pmatrix}
$$

so that

$$
\begin{pmatrix}
x_1^O \\
x_2^O \\
x_3^O
\end{pmatrix} = \frac{\alpha}{(1 - 8\theta^2)} \begin{pmatrix}
1 + 4\theta \\
1 + 2\theta \\
1 + 2\theta
\end{pmatrix}
$$

Since the optimal effort subsidy for each agent $i$ is given by:

$$
S_i^O = \theta \sum_j g_{ij} x_j^O
$$

or in matrix form

$$
S^O = \theta G x^O
$$
We have
\[ S_1^O = \theta (x_2^O + x_3^O) = 2\alpha \theta \left( \frac{1 + 2\theta}{1 - 8\theta^2} \right) \]
\[ S_2^O = S_3^O = \theta x_1^O = \alpha \theta \left( \frac{1 + 4\theta}{1 - 8\theta^2} \right) \]

Not surprisingly, the planner gives more effort subsidy to more central agents since \( S_1^O > S_2^O \).

Now, let us calculate the assimilation subsidy. The matrix \( M^O \) is given by
\[
M^O = (I - 2\theta G)^{-1} = \frac{1}{(1 - 8\theta^2)^2} \begin{pmatrix} 1 & 2\theta & 2\theta \\ 2\theta & 1 - 4\theta^2 & 4\theta^2 \\ 2\theta & 4\theta^2 & 1 - 4\theta^2 \end{pmatrix}
\]

As a result,
\[
\Phi^A(m_{q+1,q+1}^O) = t (2\alpha - t) (m_{q+1,q+1}^O)^2 + 2t\alpha (m_{q+1,q+2}^O + m_{q+1,q+3}^O) m_{q+1,q+1}^O
= \frac{t [2\alpha (1 + 4\theta) - t]}{(1 - 8\theta^2)^2}
\]

Thus, the optimal subsidy given to agent 1 is equal to:
\[
\sigma_1^O = \max \left\{ 0, 1 - \frac{t [2\alpha (1 + 4\theta) - t]}{2c (1 - 8\theta^2)^2} \right\}
\]

Similarly, we have:
\[
\Phi^A(m_{q+2,q+2}^O) = \Phi^C(m_{q+3,q+3}^O) = t (2\alpha - t) (m_{q+2,q+2}^O)^2 + 2t\alpha (m_{q+2,q+3}^O + m_{q+2,q+3}^O) m_{q+2,q+2}^O
= \frac{t \left( 1 - 4\theta^2 \right) (1 + 2\theta) [2\alpha - t (1 - 2\theta)]}{(1 - 8\theta^2)^2}
\]

Thus, the subsidy \( \sigma_2^O = \sigma_3^O \) to give to agents 2 and 3 is:
\[
\sigma_2^O = \max \left\{ 0, 1 - \frac{t \left( 1 - 4\theta^2 \right) (1 + 2\theta) [2\alpha - t (1 - 2\theta)]}{2c (1 - 8\theta^2)^2} \right\}
\]

As expected, it is easily verified that \( \sigma_1^O < \sigma_2^O \), i.e. the planner gives less assimilation subsidy to more central agents.

To summarize:

**Proposition 10 (Effort versus assimilation subsidies)** Assume \( 2\theta \rho(G) < 1 \) and consider any network of \( q + n \) agents. To restore the first best, it is optimal for the planner to give higher effort subsidies (to all individuals) but lower assimilation subsidies (to minority individuals) to more central agents in the social network.

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Let us now show, with an example, the importance of social network for this last result. Take again \( \alpha = 6, \tau = 1 \) and \( \theta = 0.2 \), then the planner needs to give the following subsidies:

\[
S_1^O = 4.94 \text{ and } S_2^O = S_3^O = 3.18
\]

and

\[
\sigma_1^O = \max \left\{ 0, 1 - \frac{22.28}{c} \right\} \text{ and } \sigma_2^O = \sigma_3^O = \max \left\{ 0, 1 - \frac{14.50}{c} \right\}
\]

to reach the first best. If, for example, \( c = 20 \), then the planner does not need to subsidize agent 1 but needs to subsidy 27.5 percent of the assimilation cost \( c \) for agents 2 and 3 in order for them to assimilate.

Consider now the complete network with the three agents who assimilate. It is easily verified that, if \( \theta < 0.25 \), then

\[
S_1^O = S_2^O = S_3^O = S^O = \frac{2\alpha\theta (1 + 2\theta)}{(1 - 2\theta - 8\theta^2)}
\]

\[
\sigma_1^O = \sigma_2^O = \sigma_3^O = \sigma^O = \max \left\{ 0, 1 - \frac{t(1 - 2\theta) [2\alpha (1 + 2\theta) - t (1 - 2\theta)]}{2c(1 - 2\theta - 8\theta^2)^2} \right\}
\]

If we take the same parameter values, \( \alpha = 6, t = 1 \) and \( \theta = 0.2 \), then \( S^O = 12 \) and

\[
\sigma^O = \max \left\{ 0, 1 - \frac{17.36}{c} \right\}
\]

If we compare the two networks, it easily verified that the planner needs to subsidize much more the social effort of all agents in the complete network (there are more network externalities in the complete network compared to the star network) while, for a given \( c \) and for given assimilation subsidies, she needs to subsidize less agent 1 but more agents 2 and 3 in the star network. In terms of network design, this means that the planner would not always like to choose the complete network, even though it is the network that generates most interactions among all possible networks. The optimal network will clearly depend on parameters values and will be, in general, difficult to determine.

6 Integrated versus segregated communities

So far, we have assumed that the majority group and the minority group always interact with each other, i.e. they belong to the same social network. We know that this not always true in the real world because, for example, minority and majority individuals do not go to the same school, the same college or do not work in the same workplace or do not live in the same area (segregation). Let us consider the case when this is not the case and analyze its consequences on assimilation.
behavior. To illustrate the result, let us have the following two complete networks where the two communities are totally separated.

\[ \begin{array}{ccc}
\text{Minority group} & \text{Majority group} \\
1 & 3 \\
2 & 4 \\
\end{array} \]

Figure 2: Two separated communities

In that case, since minority workers 1 and 2 are of the same type (i.e. same centrality), there can only be two equilibria: an Assimilation Equilibrium for which \( A = \{1, 2\} \) and \( O = \emptyset \), and an Oppositional Equilibrium for which \( A = \emptyset \) and \( O = \{1, 2\} \). Thus, if \( \theta < 1 \), we have:

\[
M = \frac{1}{(1 - \theta^2)} \begin{pmatrix} 1 & \theta \\ \theta & 1 \end{pmatrix}
\]

and

\[
\Phi^A(m_{11}) = \Phi^A(m_{22}) = t (2\alpha - t) (m_{11})^2 + 2t \alpha m_{12}m_{11}
\]

\[
= t (2\alpha - t) (m_{22})^2 + 2t \alpha m_{21}m_{22} = \frac{t [2\alpha (1 + \theta) - t]}{(1 - \theta^2)^2}
\] (18)

As a result, (i) if \( c < \frac{t [2\alpha (1 + \theta) - t]}{2(1 - \theta^2)^2} \), there is a unique Assimilation Equilibrium while if \( c > \frac{t [2\alpha (1 + \theta) - t]}{2(1 - \theta^2)^2} \), there is a unique Oppositional Equilibrium. The planner would certainly want that both groups interact more since this will increase peer effects and thus total productivity in the economy.

Imagine that the planner “creates” a bridge between the two communities so that the two networks are now as follows:
The planner can create such a bridge by moving minority individuals to more mixed areas (such as Moving to Opportunity programs mentioned above) or by integrating minority individuals to schools or colleges (such as the Affirmative Action or busing policies) or by helping them find a job in “majority” types of jobs. Let us analyze the consequence of this bridge on the assimilation process. Observe first that since the minorities are now of two types (in terms of centralities), there will be three equilibria. If \( \theta < \frac{2}{1+\sqrt{5}} = 0.618 \), we have:\(^{12}\)

\[
M_B = \frac{1}{(1 - 3\theta^2 + \theta^4)} \begin{pmatrix} 1 - \theta^2 & \theta (1 - \theta^2) & \theta & \theta^2 \\ \theta (1 - \theta^2) & 1 - 2\theta^2 & \theta^2 & \theta^3 \\ \theta & \theta^2 & 1 - \theta^2 & \theta (1 - \theta^2) \\ \theta^2 & \theta^3 & \theta (1 - \theta^2) & 1 - 2\theta^2 \end{pmatrix}
\]

and

\[
\Phi_B^A(m_{22}) = t \frac{(2\alpha - t) (m_{22})^2 + 2\alpha (m_{21} + m_{23} + m_{24}) m_{22}}{(1 - 2\theta^2) \left[ (2\alpha - t) (1 - 2\theta^2) + 2\alpha \theta (1 + \theta) \right]} = \frac{(2\alpha - t) (m_{22})^2 + 2\alpha (m_{21} + m_{23} + m_{24}) m_{22}}{(1 - 3\theta^2 + \theta^4)^2}
\]

As a result, if

\[
c < t \frac{(2\alpha - t) (1 - 2\theta^2) + 2\alpha \theta (1 + \theta)}{2 (1 - 3\theta^2 + \theta^4)^2}
\]

\(^{12}\)The subscript \( B \) indicates that there is a bridge between the two communities.
there is a unique Assimilation Equilibrium for which $\mathcal{A} = \{1, 2\}$ and $\mathcal{O} = \emptyset$. If

$$
t \frac{(1 - 2\theta^2) [(2\alpha - t)(1 - 2\theta^2) + 2\alpha t (1 - \theta^2)]}{2 \left(1 - 3\theta^2 + \theta^4\right)^2} < c < \frac{t \left(1 - \theta^2\right) [(1 - \theta^2) (2\alpha - t - 2t\theta) + 2\alpha \theta (2 + \theta - \theta^2)]}{2 \left(1 - 3\theta^2 + \theta^4\right)^2}
$$

there exists a unique Assimilation-Oppositional equilibrium for which $\mathcal{A} = \{1\}$ and $\mathcal{O} = \{2\}$. Finally, if

$$
c > \frac{t \left(1 - \theta^2\right) [(1 - \theta^2) (2\alpha - t - 2t\theta) + 2\alpha \theta (2 + \theta - \theta^2)]}{2 \left(1 - 3\theta^2 + \theta^4\right)^2}
$$

there exists a unique Oppositional equilibrium for which $\mathcal{A} = \emptyset$ and $\mathcal{O} = \{1, 2\}$.

It is clear that there is much more assimilation when there is a bridge between the two communities (Figure 3) than when there is not (Figure 2). To see that, take again the same parameter values as above: $\alpha = 6$, $t = 1$ and $\theta = 0.2$. If there is no bridge, we have that, if $c < 7.27$, there is a unique Assimilation equilibrium while if $c > 7.27$, there is a unique Oppositional equilibrium. When there is a bridge, we have that: (i) if $c < 7.69$, there is a unique Assimilation equilibrium; (ii) if $7.69 < c < 9.49$, there is a unique Assimilation-Oppositional equilibrium; (iii) if $c > 9.49$, there is a unique Oppositional equilibrium. So, for example, if $7.27 < c < 7.69$, then in the network with separated communities (Figure 2), all minority workers are oppositional while in the network with the bridge between the communities (Figure 3), individual 1 is assimilated while individual 2 is oppositional. This is a general result that is true for any network. It is due to the fact that, by creating a bridge between the two communities, positive externalities in terms of effort productivity increase. Indeed, if we look at individual 2, we can check that what she receives from individual 1 (i.e. $m_{21}$) increases from $\frac{\theta}{1 - \theta^2}$ (before the bridge) to $\frac{\theta(1 - \theta^2)}{1 - 3\theta^2 + \theta^4}$ (after the bridge is created). Similarly, her own centrality $m_{22}$ (i.e. self-loops) increases from $\frac{1}{(1 - \theta^2)}$ to $\frac{1 - 2\theta^2}{(1 - 3\theta^2 + \theta^4)}$. More generally, when creating a bridge between the two communities, the function $\Phi^A(m_{ii})$ that gives the incentive to assimilate always increases for both individuals because the centralities of the individuals from the majority group, i.e. $m_{13}$ and $m_{14}$ for individual 1 and $m_{23}$ and $m_{24}$ for individual 2, are added to this function (compare (18) and (19)). We have therefore the following general result:

**Proposition 11 (Bridge and assimilation)** Assume $\theta \rho(G) < 1$ and consider any network of $q + n$ agents. Creating bridge(s) between the majority and the minority group always (weakly) increases assimilation.

This proposition demonstrates that assimilation is easier when ethnic minorities interact with individuals from the majority group. This is observed empirically. Using data from the German Socio-Economic Panel for the period 1996-2011, Facchini et al. (2015) document that migrants
with a German friend are more similar to natives than those without a local companion along several important dimensions, including engagement in social activities, concerns about the economy, interest in politics and broad policy issues like the environment, crime and xenophobia. Using data from the European Community Household Panel, De Palo et al. (2006) study the assimilation of immigrants in the host country society by analyzing a question on the number of interactions between the respondent and his/her neighbor, as related to the number of interactions with friends or relatives not living with them. Their results suggest that non-EU migrants tend to socialize less with natives, even after controlling for individual characteristics. They also find that migrants’ behavior tends to slowly assimilate to that of native migrants (ethnic minorities). They suggest that education has a significant impact on the type of social activities undertaken by the individuals. More-educated people tend to relate somewhat less with close neighbors, but socialize more intensively with the broader community.

More generally, Proposition 11 shows that, by creating bridges between communities all workers are better off in terms of productivities and thus utilities. This means that, for example, if a planner provides a job to a minority worker that is usually reserved to majority workers (thus creating a bridge between the communities), it does not just increase the productivity and utility of this worker but it also increases the productivity of other minority workers from the same community. In other words, helping one minority individual benefits the whole community she belongs to because of the spillovers it generates. Since network structure plays an important role here, it would always be more efficient to create a bridge link between the most central individual from the minority group and the most central individual from the majority group. This is because we have positive complementarities in efforts so that more central agents provide more positive externalities than less central agents. This is, in turn, positively affects the assimilation behavior of minorities because of the effect on their productivity. The potential gains from bridging different parts of a network were important in the early work of Granovetter (1974) and are central to the notion of structural holes developed by Burt (1994). The latter postulates that social capital is created by a network in which people can broker connections between otherwise disconnected segments. In recent years, a number of empirical studies have shown that individuals or organizations who bridge ‘structural holes’ in networks gain significant payoff advantages.13

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13See Burt (1994) and Mehra et al. (2003) for influence of structural positions on promotions and performance evaluation, Podolny and Baron (1997) for work on network positions and mobility, and Ahuja (2000) for the influence of a firm’s position in inter-organizational networks on its innovativeness and overall performance. See also Burt (2004) who explores the influence of individual position in social networks in shaping the generation of creative ideas.
7 Concluding remarks

In this paper, we have proposed a simple two stage-model where ethnic minorities choose whether to assimilate or not to the majority culture in the first stage while, in the second stage, both ethnic minorities and majority workers decide how much effort to put in some activity. The network is taken as given. We show that the more central agents in the network tend to assimilate more than the less central agents. We also show that there is more assimilation in denser networks and the Nash equilibrium is not efficient because of positive externalities created by effort choices. We then propose different policies that can restore the first best, looking at both effort and assimilation subsidies. We also show that bridging networks between the majority and the majority group is always beneficial for assimilation behaviors. As a result, our results suggest that ethnic enclaves may not be good for assimilation while providing host country language proficiency could be a more effective policy. They also suggest that targeting the most central individuals among the minority group (such as community leaders) could be an efficient policies if they are themselves assimilated because it will generate the most positive externalities on other minority individuals in the network.

References


14 Interestingly, this is also the conclusion of Angelini et al. (2015) who investigate the association between assimilation with a host culture and immigrants’ subjective well-being in Germany.


Appendix 1: The Katz-Bonacich network centrality measure

The Katz-Bonacich centrality measure (due to Katz, 1953, and Bonacich, 1987) has proven to be extremely useful in game theoretical applications (Ballester et al., 2006; Dequiedt and Zenou, 2014). Let $G^k$ be the $k$th power of $G$, with elements $g^{[k]}_{ij}$, where $k$ is an integer. The matrix $G^k$ keeps track of the indirect connections in the network: $g^{[k]}_{ij} \geq 0$ gives the number of walks or paths of length $k \geq 1$ from $i$ to $j$ in the network $G$. In particular, $G^0 = I$. Consider the matrix $M = \sum_{k=0}^{+\infty} \theta^k G^k$. The elements of this matrix, $m_{ij}$, count the number of walks of all lengths from $i$ to $j$ in the network $G$, where walks of length $k$ are weighted by $\theta^k$. The Katz-Bonacich centrality of agent $i$, denoted, $b_i(g, \theta)$ is equal to the sum of the elements of the $i$th row of $M$:

$$b_i(g, \theta) = \sum_{j=1}^{q+n} m_{ij} = \sum_{j=1}^{q+n} \sum_{k=0}^{+\infty} \theta^k g^{[k]}_{ij}.$$  \hspace{1cm} (20)

Note that the Katz-Bonacich centrality is well-defined if $\theta$ is low enough. In matrix form, the vector of Katz-Bonacich centralities can be written as:

$$b(g, \theta) = M\mathbf{1} = [I - \theta G]^{-1} \mathbf{1},$$ \hspace{1cm} (21)

where $\mathbf{1}$ is a vector of ones. We can also define the *weighted* Katz-Bonacich centrality of agent $i$ as:

$$b_{\alpha_i}(g, \theta) = \sum_{j=1}^{q+n} \sum_{k=0}^{+\infty} \theta^k g^{[k]}_{ij} \alpha_j,$$  \hspace{1cm} (22)

where the weight attached to the walks from $i$ to $j$ is $\alpha_j$. For any $n$–dimensional vector $\alpha$, the matrix equivalent of (22) is given by:

$$b_{\alpha}(g, \theta) = M\alpha = [I - \theta G]^{-1} \alpha.$$