A Dynamic Model of Weak and Strong Ties in the Labor Market

Yves Zenou, Stockholm University, IFN, and GAINS

The study develops a simple model where workers can obtain a job through either their strong or weak ties. It shows that increasing the time spent with weak ties raises the employment rate of workers. It also shows that when the job-destruction rate or the job-information rate increases, workers choose to rely more on their weak ties to find a job. The model is extended so unemployed workers can also learn of a vacancy directly from an employer. Results show that equilibrium employment and time spent with weak ties are sometimes, but not in all cases, positively related.

I. Introduction

A number of studies for a range of countries have emphasized the popularity of using friends and family as a job search mechanism and indicate that this is an effective mechanism for obtaining job offers (Rees 1966; Granovetter 1974, 1979; Blau and Robins 1990; Topa 2001; Wahba and Zenou 2005; Bayer, Ross, and Topa 2008; Bentolila, Michelacci, and Suarez 2010; Pellizzari 2010). The empirical evidence reveals that around 50% of individuals obtain or hear about jobs through friends and family (Holzer 1987, 1988; Montgomery 1991; Addison and Portugal 2002). This method has the advantage that it is relatively less costly and may provide more reliable information about jobs compared to other methods. Little is known, however, about the way people search and what kinds of mechanisms lead to finding a job.

I would like to thank the editor, Jonathan Guryan, for very helpful comments. Contact the author at yves.zenou@ne.su.se.
Building on Granovetter (1973, 1974, 1983)’s idea that weak ties are superior to strong ties for providing support in getting a job,¹ I develop a dynamic model of the labor market in which dyad members do not change over time so that two individuals belonging to the same dyad hold a strong tie with each other. However, each dyad partner can meet other individuals outside the dyad partnership, referred to as weak ties, or random encounters. By definition, weak ties are transitory and only last for one period. The process through which individuals learn about jobs results from a combination of a socialization process that takes place inside the family (in the case of strong ties) and a socialization process that takes place outside the family (in the case of weak ties).² Thus, information about jobs is essentially obtained through strong and weak ties and thus through word-of-mouth communication. In particular, I assume that an unemployed (employed) individual meets either a strong tie or a weak tie during a given period, potentially gaining (providing) information on a job vacancy from the tie she meets only if that tie is employed (unemployed).³

I first characterize the steady-state equilibrium employment rate and determine the time spent in a $d_0$-dyad (both workers are unemployed), in a $d_1$-dyad (one worker is employed and the other one is unemployed), and in a $d_2$-dyad (both workers are employed). I then show that the higher is the job-destruction rate or the lower is the job-information rate, the lower is the employment rate and more workers spend time in a $d_0$-dyad. I also

¹ Granovetter (1973, 1974, 1983) defines weak ties in terms of lack of overlap in personal networks between any two agents, i.e., weak ties refer to a network of acquaintances who are less likely to be socially involved with one another. Formally, two agents A and B have a weak tie if there is little or no overlap between their respective personal networks. Vice versa, the tie is strong if most of A’s contacts also appear in B’s network. In other words, for Granovetter, weak ties are useful because they essentially increase the information set relative to strong ties by providing access to different information on job openings than can be provided by strong ties.

² This idea was first put forward by Bisin and Verdier (2000, 2001) in the context of the transmission of a trait like, for example, religion or identity.

³ More recent empirical evidence lends some support to the strength of weak ties in finding a job. Yakubovich (2005) uses a large-scale survey of hires made in 1998 in a major Russian metropolitan area and finds that a worker is more likely to find a job through weak ties than through strong ones. These results come from a within-agent fixed effect analysis, so they are independent of workers’ individual characteristics. Using data from a survey of male workers from the Albany, New York, area in 1975, Lin, Ensel, and Vaughn (1981) find similar results. Marsden and Hurlbert (1988) and Lai, Lin, and Leung (1998) also find that weak ties facilitate the reach to a contact person with higher occupational status, which in turn leads to better jobs, on average. Finally, Giulietti, Wahba, and Zenou (2014) show that weak ties are important for rural-urban migration in China because they help migrants obtain an urban job. See also Patacchini and Zenou (2008), who find evidence of the strength of weak ties in crime.
show that by increasing the probability $q$ of meeting new workers (i.e., weak ties), the steady-state employment rate increases, formally demonstrating Granovetter’s informal idea of the strength of weak ties in finding a job. This is not a trivial result, since, by increasing $q$, we have different and opposite effects on the job formation/destruction process. On the one hand, we increase the probability of getting out of unemployed dyads, while on the other hand, we potentially give up the information of an employed partner in favor of a link with an unemployed one. I obtain this result because, in my model, it is indeed better to meet weak ties because a strong tie does you no good in state $d_0$ since your best friends are all unemployed, but a weak tie can do you good in any state because that person might be employed. So there is an asymmetry that is key to the model and that can explain why some workers may be stuck in unemployment traps (i.e., $d_0$-dyads), having little contact with weak ties that can help them leaving the $d_0$-dyad.

Observe that my definition of weak ties is slightly different than Granovetter’s. As underscored by Granovetter, in a close network where everyone knows each other, information is shared and so potential sources of information are quickly shaken down so that the network quickly becomes redundant in terms of access to new information. In contrast to the current study, Granovetter stresses the strength of weak ties involving a secondary ring of acquaintances who have contacts with networks outside ego’s network and therefore offer new sources of information on job opportunities. In his 1983 paper, Granovetter reviews some of the empirical literature up to that time, including his own work, where the definition of strong and weak ties varies depending on the paper (and on data availability). As a result, what constitutes a strong tie and a weak tie is somewhat unclear in this literature. Here I adopt definitions of weak and strong ties based on the time spent together so that weak ties are more transient and changing than strong ties. This enables me to show that weak ties provide better job access in many cases because they provide an unemployed worker essentially with more draws at the underlying probability of becoming employed.

I then extend this framework by endogeneizing social interactions. When workers decide how much time they spend with their weak and strong ties, I show that, when the job-destruction rate or the job-information rate increases, workers spend more time with their weak ties. This means that, in downturn periods where jobs are destroyed at a faster rate, workers tend to spend more time with their weak ties because they know they will help them exit unemployment. In an economy where the flow of job information is faster, the same results occur.

Finally, I allow for the possibility that the unemployed workers can learn of a vacancy directly from an employer rather than via a social interaction. In the new model, employed and unemployed workers can hear
directly from a job and the unemployed workers can, as before, find a job through their social network. I show that the equilibrium employment and time spent with weak ties are sometimes, but not in all cases, positively related.

There is a growing interest in theoretical models of peer effects and social networks (see, e.g., Glaeser, Sacerdote, and Scheinkman 1996; Akerlof 1997; Ballester, Calvó-Armengol, and Zenou 2006; Goyal 2007; Jackson 2008, 2014; Calvó-Armengol, Patacchini, and Zenou 2009; Ioannides 2012; Jackson, Rogers, and Zenou 2015; and Jackson and Zenou 2015), especially in the labor market. However, few models of social networks in the labor market are dynamic. Montgomery (1994) and Calvó-Armengol, Verdier, and Zenou (2007) propose a dynamic model of weak and strong ties, but the former focuses on inequality, while the latter focuses on the interaction between crime and labor markets. Calvó-Armengol and Jackson (2004, 2007) have a richer network analysis (since they can encompass any network structure) by modeling strong ties in a more general way so that workers who hear about a job vacancy do not always pass along this information to their strong ties. This will depend on the number of their friends. On the other hand, I model weak ties in a richer way since, in my framework, weak ties can directly provide information about jobs while they cannot in the Calvó-Armengol and Jackson (2004) model, and workers have to decide how much time they spend with their weak and strong ties.4


My model is simpler than these models in the sense that I do not model the whole network structure but only focus on dyads. However, my model is richer since it explicitly analyzes the relationship between weak and strong ties and their impact on the labor market outcomes of workers. Because of its simplicity, my model is tractable and all my equilibrium variables can be written in closed-form solutions. This enables me to extend my model in different directions. In particular, I allow workers to choose the time spent with weak and strong ties. I also look at a model where jobs can be found either directly or through strong and weak ties.

The paper unfolds as follows. In the next section, I present the basic model of weak and strong ties. In Section III, I characterize the steady-state equilibrium, derive some important comparative statics results, calculate

4 See Sec. V, where we have a more thorough discussion on the comparison of our model and that of Calvó-Armengol and Jackson (2004).
the correlation in employment status between workers in the same dyad, and endogeneize the choice of social interactions. In Section IV, I allow unemployed workers to directly hear from a job. In Section V, I compare my model to that of Calvó-Armengol and Jackson (2004). Finally, Section VI concludes. All proofs can be found in the appendix.

II. The Model

Consider a population of individuals of size one.

A. Dyads

As in Montgomery (1994), I assume that individuals belong to mutually exclusive two-person groups, referred to as dyads. Two individuals belonging to the same dyad hold a strong tie to each other. I assume that dyad members do not change over time. A strong tie is created once and forever and can never be broken. Thus, we can think of strong ties as links between members of the same family or between very close friends.

Individuals can be in either of two different states: employed or unemployed. Dyads, which consist of paired individuals, can thus be in three different states, as follows: (i) both members are employed—we denote the number of such dyads by \( d_2 \); (ii) one member is employed and the other is unemployed—we denote the number of such dyads by \( d_1 \); (iii) both members are unemployed—we denote the number of such dyads by \( d_0 \).

B. Aggregate State

By denoting the employment rate and the unemployment rate at time \( t \) by \( e(t) \) and \( u(t) \), where \( e(t), u(t) \in [0, 1] \), we have

\[
\begin{align*}
\begin{cases}
e(t) = 2d_2(t) + d_1(t), \\
u(t) = 2d_0(t) + d_1(t).
\end{cases}
\end{align*}
\]

The population normalization condition can then be written as

\[
e(t) + u(t) = 1,
\]

or, alternatively, as

\[
d_2(t) + d_1(t) + d_0(t) = \frac{1}{2}.
\]

5 The inner ordering of dyad members does not matter.
C. Social Interactions

Time is continuous, and individuals live forever. I assume repeated random pairwise meetings over time. Matching can take place between dyad partners or not. At time $t$, each individual can meet a weak tie with probability $\omega(t)$; thus $1 - \omega(t)$ is the probability of the individual meeting her strong-tie partner at time $t$. In Sections II and IV, I assume these probabilities to be constant and exogenous, not to vary over time, and thus, they can be written as $\omega$ and $1 - \omega$. I endogeneize $\omega$ in Section III.E.

I will refer to matchings inside the dyad partnership as strong ties and to matchings outside the dyad partnership as weak ties, or random encounters. Within each matched pair, information is exchanged, as explained below. Observe that I assume symmetry within each dyad, that is, if I meet a strong (or a weak) tie, then my strong (or weak) tie has to meet me. In the language of graph theory, this means that the network of relationships is undirected (Jackson 2008).

D. Information Transmission

Each job offer is taken to arrive only to employed workers, who can then direct it to one of their contacts (through either strong or weak ties). This is a convenient modeling assumption, which stresses the importance of on-the-job information. To be more precise, employed workers hear of job vacancies at the exogenous rate $\lambda$, while they lose their job at the exogenous rate $\delta$. All jobs and all workers are identical, so all employed workers obtain the same wage. Therefore, employed workers who hear about a job pass this information on to their current matched partner, who can be a strong tie or a weak tie. Thus, information about jobs is essentially obtained through social networks.

This information transmission protocol defines a Markov process. The state variable is the relative size of each type of dyad. Transitions depend on labor market turnover and the nature of social interactions, as captured by $\omega$. Because of the continuous-time Markov process, the probability of a two-state change is zero (small order) during a small interval of time $t$ and $t + dt$. This means, in particular, that both members of a dyad cannot change their status at the same time.

---

6 If each individual has one unit of time to spend with her friends, then this can also be interpreted as the percentage of time spent with weak ties.

7 There is strong evidence that firms rely on referral recruitment (Bartram et al. 1995; Mencken and Winfield 1998; Barber et al. 1999; Pellizzari 2010), and it is even common and encouraged strategy for firms to pay bonuses to employees who refer candidates who are successfully recruited to the firm (Berthiaume and Parsons 2006).

8 We allow unemployed workers to hear directly about jobs in Sec. IV.
E. Flows of Dyads between States

It is readily checked that the net flow of dyads from each state between \( t \) and \( t + dt \) is given by

\[
\begin{cases}
\dot{d}_2(t) = (1 - \omega + \omega \epsilon(t))\lambda d_1(t) - 2\delta d_2(t), \\
\dot{d}_1(t) = 2\omega \epsilon(t)\lambda d_2(t) - (\delta + [1 - \omega + \omega \epsilon(t)]\lambda) d_1(t) + 2\delta d_2(t), \\
\dot{d}_0(t) = \delta d_1(t) - 2\omega \epsilon(t)\lambda d_0(t).
\end{cases}
\]

(4)

Let us explain in detail these equations of (4). Take the first one. The variation of dyads composed of two employed workers (\( \dot{d}_2(t) \)) is equal to the number of \( d_1 \)-dyads in which the unemployed worker has found a job (through either her strong tie with probability \((1 - \omega)\lambda\) or her weak tie with probability \(\omega \epsilon(t)\lambda\)) minus the number of \( d_2 \)-dyads in which one of the two employed workers has lost her job. In the second equation, the variation of dyads composed of one employed and one unemployed worker (\( \dot{d}_1(t) \)) is equal to the number of \( d_0 \)-dyads in which one of the unemployed workers has found a job (only through her weak tie with probability \(\omega \epsilon(t)\lambda\) since her strong tie is unemployed and cannot therefore transmit any job information) minus the number of \( d_1 \)-dyads in which either the employed worker has lost her job (with probability \(\delta\)) or the unemployed worker has found a job with the help of her strong or weak tie (with probability \([1 - \omega + \omega \epsilon(t)]\lambda\)) plus the number of \( d_2 \)-dyads in which one of the two employed workers has lost her job. Finally, in the last equation of (4), the variation of dyads composed of two unemployed workers (\( \dot{d}_0(t) \)) is equal to the number of \( d_1 \)-dyads in which the employed worker has lost her job minus the number of \( d_0 \)-dyads in which one of the unemployed workers has found a job (only through her weak tie, with probability \(\omega \epsilon(t)\lambda\)). These dynamic equations reflect the flows across dyads. See figure 1 for the graphical representation.

Observe that the assumption stated above that both members of a dyad cannot lose their status at the same time is reflected in the flows described by (4). What is crucial in my analysis is that members of the same dyad (strong ties) always remain together throughout their life. So,

![Diagram of flows in the labor market](image)
for example, if a $d_2$-dyad becomes a $d_0$-dyad, the members of this dyad are exactly the same; they have just changed their employment status.

III. Steady-State Equilibrium and Comparative Statics Analysis

A. Steady-State Equilibrium

In a steady-state $(d^*_2, d^*_1, d^*_0)$, each of the net flows in (4) is equal to zero. Setting these net flows equal to zero leads to the following relationships:

$$d^*_2 = \frac{(1 - \omega + \omega e^*)\lambda}{2\delta} d^*_1, \quad (5)$$

$$d^*_1 = \frac{2\omega e^*\lambda}{\delta} d^*_c, \quad (6)$$

where

$$d^*_c = \frac{1}{2} - d^*_2 - d^*_1, \quad (7)$$

$$e^* = 2d^*_2 + d^*_c, \quad (8)$$

$$u^* = 1 - e^*. \quad (9)$$

**Definition 1.** A steady-state labor market equilibrium is a four-tuple $(d^*_2, d^*_1, d^*_0, e^*, u^*)$ such that equations (5)–(9) are satisfied.

Define $Z = (1 - \omega)/\omega$, $B = \delta/(\lambda\omega)$. We have the following result.

**Proposition 1.** (i) There always exists a steady-state equilibrium $U$ where all individuals are unemployed and only $d_0$-dyads exist, that is, $d^*_2 = d^*_1 = e^* = 0$, $d^*_0 = 1/2$, and $u^* = 1$. (ii) If

$$\frac{\delta}{\lambda} < \omega + \sqrt{\omega(4 - 3\omega)}, \quad (10)$$

there exists a steady-state equilibrium $I$, where $0 < e^* < 1$ is defined by

$$e^* = \frac{B^2}{2d^*_c} - B - Z > 0, \quad (11)$$

$0 < u^* < 1$ by (9), and $0 < d^*_c < 1/2$ is the unique (feasible) solution of the following equation:

$$-\frac{Z}{B} d^*_c^2 - \frac{1 + Z}{2} d^*_c + \left(\frac{B}{2}\right)^2 = 0. \quad (12)$$
Also, the other dyads are given by

\[ d_1^* = \frac{2e^*}{B} d_0^*, \]  
\[ d_2^* = \frac{(Z + e^*) e^*}{B^2} d_0^*. \]  

If condition (10) holds, then an interior equilibrium always exists. Indeed, if the job-destruction rate \( \delta \) is sufficiently low and/or the job-contact rate \( \lambda \) is sufficiently high, then an interior equilibrium exists. As a result, we are in a “reasonable” economy, where jobs are not destroyed too fast and jobs are created at a sufficiently high rate (otherwise we will end up with the steady-state equilibrium \( U \) where all workers are unemployed). Following Shimer (2005) and Pissarides (2009), the monthly transitions data in the United States give a mean value for 1960–2004 of 0.594 for the job-finding rate and 0.036 for the job-separation rate, that is, \( \lambda = 0.594 \) and \( \delta = 0.036 \). In that case, condition (10) is always satisfied even for very low values of \( \omega \), like, for example, \( \omega = 0.01 \).

The steady-state equilibrium \( U \) is obviously uninteresting, and, from now on, we will only focus on the labor market equilibrium \( I \). For this equilibrium, we can, in fact, calculate explicitly \( e^* \) and \( d_0^* \). We obtain

\[ e^* = \sqrt{\lambda[\lambda + 4\delta(1 - \omega)]} - 2\delta + 2\lambda \omega - \lambda \frac{2\lambda \omega}{2\lambda \omega}, \]  
\[ d_0^* = \frac{\delta^2}{\lambda \omega + \lambda \omega \sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}} . \]  

B. Comparative Statics Results

We have the following comparative-statics results:

**Proposition 2.** Consider the steady-state equilibrium \( I \) and assume (10). Then, an increase in the job-destruction rate \( \delta \) decreases

\[ e^* = \left[ (1 - \omega + \omega e^*) \lambda \delta \right] \]  
\[ d_0^* = \left[ 2\omega e^* \lambda \delta \right] \]  

To obtain (15) and (16), we proceed as follows. First, we plug the values of \( d_2^* \) and \( d_1^* \) from (5) and (6) into \( e^* = 2d_2^* + d_1^* \) to obtain

\[ e^* = \left[ (1 - \omega + \omega e^*) \lambda \delta \right] + 1 \frac{2\omega e^* \lambda \delta}{d_0^*} \]  

Then, we plug the values of \( d_2^* \) and \( d_1^* \) from (5) and (6) into \( d_0^* = \frac{1}{2} - d_2^* - d_1^* \) to obtain

\[ d_0^* + \left[ (1 - \omega + \omega e^*) \lambda \delta \right] + 1 \frac{2\omega e^* \lambda \delta}{d_0^*} \frac{d_0^*}{d_0^*} = \frac{1}{2}. \]  

By solving simultaneously these two equations, we get (15) and (16).
the employment rate $e^*$ and the time spent in the $d_1$-dyad but increases the time spent in a $d_0$-dyad. If the elasticity of $d^*_0$ with respect to $\delta$ is lower in absolute value than the elasticity of $e^*$ with respect to $\delta$, that is,

$$\left| \frac{\partial d^*_0}{\partial \delta} \right| < \left| \frac{\partial e^*}{\partial \delta} \right|,$$

(17)

then an increase in $\delta$ reduces the time spent in a $d_1$-dyad.

The effect of $\delta$, the job-destruction rate, on the different endogenous variables is interesting and not straightforward. Consider, first, the effect of $\delta$ on the different dyads. When $\delta$ increases, workers lose their job at a faster rate and thus spend more time in the $d_0$-dyad (where both workers are unemployed) and less time in the $d_2$-dyad (where both workers are employed). The effect of $\delta$ on the time spent in the $d_1$-dyad is, however, less clear. Indeed, when $\delta$ increases, this can either increase or decrease the time spent in a $d_1$-dyad because there will be flow in $d_1$ from $d_2$ and flow out from $d_1$ to $d_0$.

Let us now focus on $\lambda$, the job-information rate. We have the following proposition.

**Proposition 3.** Consider the steady-state equilibrium $I$ and assume (10). An increase in the job-information rate $\lambda$ increases the employment rate $e^*$ but decreases the time spent in a $d_0$-dyad. Moreover, if the elasticity of $e^*$ with respect to $\lambda$ is higher in absolute value than the elasticity of $d_0^*$ with respect to $\lambda$, that is,

$$\left| \frac{\partial e^*}{\partial \lambda} \right| > \left| \frac{\partial d^*_0}{\partial \lambda} \right|,$$

then an increase in $\lambda$ increases the time spent in the $d_1$-dyad. Finally, if

$$d_0^* > \frac{\delta^0}{(1-\omega)\omega\lambda(\lambda-2)(\delta^2 - 4\lambda^2)},$$

(18)

an increase in raises the time spent in a $d_2$-dyad.

Interestingly, the effect of the job-information rate $\lambda$ on the different endogenous variables is quite different than that of the job-destruction rate $\delta$. When $\lambda$ increases, people spend less time in a $d_0$-dyad, since both unemployed workers have a higher chance of meeting a weak tie with job information. This increases employment as long as $\lambda$ is not too small. The effects on the time spent in a $d_0$-dyad is less clear because unemployed workers tend to find a job at a faster rate, which both increases (from a $d_0$-dyad to a $d_1$-dyad) and decreases (from a $d_1$-dyad to a $d_2$-dyad) the...
time spent in a \( \delta_2 \)-dyad. Finally, the effect of \( \lambda \) on the time spent in a \( \delta_2 \)-dyad is positive only if the size of the \( \delta_2 \)-dyads is sufficiently large, because there should be enough individuals moving first from a \( \delta_2 \)-dyad to a \( \delta_1 \)-dyad and then from a \( \delta_1 \)-dyad to a \( \delta_2 \)-dyad.

C. Social Interactions

Let us study the impact of social interactions (captured by \( \omega \)) on the different endogenous variables. We have the following important result.

**Proposition 4.** Consider steady-state equilibrium \( I \) and assume (10). Then, increasing the percentage of weak ties \( \omega \) decreases the number of \( \delta_1 \)-dyads and increases the employment rate \( e^* \) in the economy, that is,

\[
\frac{\partial \delta_1^*}{\partial \omega} < 0, \quad \frac{\partial e^*}{\partial \omega} > 0.
\]

The effects of \( \omega \) on \( \delta_1 \) and on \( \delta_2 \) are, however, ambiguous.

I show here that by increasing the probability of meeting new workers (i.e., weak ties), the steady-state employment rate increases. This is not a trivial result, since, by increasing \( \omega \), we have different and opposite effects on the job formation/destruction process. On the one hand, we increase the probability of getting out of unemployed dyads, while, on the other hand, we potentially give up the information of an employed partner in favor of a link with an unemployed one. In my model, it is indeed better to meet weak ties because a strong tie does you no good in state \( \delta_2 \) since your best friends are all unemployed. But a weak tie can do you good in any state because that person might be employed.

Observe that, if we take as given \( \omega \) and assume that you spend the same amount of time with weak and strong ties, then it is clear that, in a \( \delta_1 \)-dyad, a strong tie is more valuable than a weak tie in order to find a job, because the strong tie is employed (with probability 1), while the weak tie that you meet will be employed with probability \( e < 1 \), and thus \( (1 - \omega) \lambda > \omega e \lambda \). This is not, however, enough to compensate for the fact that in a \( \delta_2 \)-dyad strong ties play no role at all, while only the weak ties help the unemployed workers to leave a \( \delta_2 \)-dyad. Indeed, since \( e^* \) is the fraction of time you spend employed over your lifetime (or equivalently the probability of being employed in steady state), what is shown in proposition 4 is that increasing marginally the time spent with weak ties \( \omega \) increases the total the time you spend employed over your lifetime. In other words, since \( e^* = 2d_1^* + d_2^* \) and \( n^* = 1 - e^* = 2d_2^* + d_1^* \), this means that, when \( \omega \) increases, you spend relatively more time in the \( \delta_2 \)-dyads over your lifetime and relatively less time in the \( \delta_2 \)-dyad. This result is also interesting, since it solves a trade-off between status-quo relations and new relations.
It also formally demonstrates Granovetter’s informal idea of the strength of weak ties in finding a job.

Observe that our model does not directly incorporate the idea that a strong tie indicates a stronger personal relationship than a weak tie, and therefore a strong tie might be more willing to help you out than a weak tie. Indeed, whenever a weak tie has information about a job she is always willing to help you, exactly like a strong tie. A simple way to model the fact that a strong tie might be more willing to help you out than a weak tie is to add a parameter \( a < 1 \) so that the probability of finding a job via a weak tie is always multiplied by \( a \), while that of finding a job via a strong tie is not. In that case, the probability of leaving a \( d_0 \)-dyad, or a dyad for an unemployed worker, is now \( \omega e \lambda a \) instead of \( \omega e \lambda \), which means that, to find a job, you need to meet a weak tie (probability \( q \)) who is employed (probability \( e \)) who has heard about a job (probability \( l \)) and who is willing to help you (probability \( a \)). In that case, strong ties are more willing to help you than weak ties. It is readily checked that the net flow of dyads from each state between \( t \) and \( t + dt \) is then given by

\[
\begin{align*}
\dot{d}_2(t) &= (1 - \omega + \omega e(t) a) \lambda d_1(t) - 2 \delta d_2(t), \\
\dot{d}_1(t) &= 2 \omega e(t) \lambda a d_0(t) - (\delta + [1 - \omega + \omega e(t)] \lambda) d_1(t) + 2 \delta d_2(t), \\
\dot{d}_0(t) &= \delta d_1(t) - 2 \omega e(t) \lambda a d_0(t).
\end{align*}
\]

We can derive the steady-state equilibrium, as in proposition 1, and show that, if

\[
\frac{\delta}{\lambda} < \frac{\omega e + \sqrt{\omega e + \omega \lambda}}{2},
\]

there exists an interior steady-state equilibrium \( I \), where \( 0 < m^* < 1 \) is defined by

\[
e^* = \frac{\sqrt{\lambda [4 \delta (1 - \omega) + \lambda (1 - \omega)^2 + \lambda \omega \lambda (2 - 2 \omega + \omega e)] - 2 \delta - \lambda (1 - \omega) + \lambda \omega \lambda}}{2 \lambda \omega \lambda},
\]

and the dyads are equal to

\[
\begin{align*}
d_2^* &= \frac{\delta^2}{\omega \lambda a (1 - \omega + \omega e) + \omega \lambda \lambda \sqrt{\lambda [4 \delta (1 - \omega) + \lambda (1 - \omega)^2 + \lambda \omega \lambda (2 - 2 \omega + \omega e)]}}, \\
d_1^* &= \frac{2 e^* \omega \lambda a}{\delta - d_0^*}, \\
d_0^* &= \frac{\omega \lambda a (1 - \omega + \omega e) e^*}{\delta^2} d_0^*.
\end{align*}
\]
In that case, however, the effect of $\omega$ on $e^*$ is now ambiguous and depends on $\alpha$. At the extreme, if $\alpha$ is very small (close to zero), strong ties would be more useful in finding a job than weak ties, and there will be a positive effect of strong ties (and not weak ties) on employment. This is an extreme case where most job information stemming from weak ties is unreliable. On the contrary, if $\alpha$ is close to 1, then, by continuity, the result of proposition 4, that is, $\partial e^*/\partial \omega > 0$, should hold. For intermediate values of $\alpha$, the effect of $\omega$ on $e^*$ is clearly ambiguous. This is because there is still an asymmetry between weak and strong ties since only weak ties help in leaving a dyad, but it is compensated by the fact that strong ties have even a bigger role in helping unemployed workers leave a $d_1$-dyad. Indeed, when $\alpha = 1$, for someone spending the same amount of time with his weak and strong ties, the rate at which workers left a $d_1$-dyad via a weak tie was $(1 - \omega)\lambda$ while, via a strong tie, it was $\omega e\lambda$, with $(1 - \omega)\lambda > \omega e\lambda$. When $0 < \alpha < 1$, this inequality becomes $(1 - \omega)\lambda > \omega e\lambda\alpha$, which means that the difference in rates at which unemployed workers leave unemployment has increased.

D. Correlation in Employment Status

In order to better understand the model, let us determine the correlation in employment status between workers in the same dyad. Denote this by

$$\Delta = \sqrt{\lambda(\lambda + 4\delta(1 - \omega))}.$$  \hspace{1cm} (19)

We have seen that the employment rate was given by

$$e^* = 2d_1^* + d_2^* = \frac{\Delta - 2\delta + 2\lambda\omega - \lambda}{2\lambda\omega}$$  \hspace{1cm} (20)

and $d_2^*$ by

$$d_2^* = \frac{\delta^2}{\lambda\omega(\lambda + \Delta)}.$$

We can easily calculate $d_1^*$ and $d_2^*$ as follows:

$$d_1^* = \frac{\delta(\Delta - 2\delta + 2\lambda\omega - \lambda)}{\lambda\omega(\lambda + \Delta)},$$

$$d_2^* = \frac{(\Delta - 2\delta + \lambda)(\Delta - 2\delta + 2\lambda\omega - \lambda)}{4\lambda\omega(\lambda + \Delta)}.$$

Denote by $s = (s_1, s_2)$ the state of the dyad and observe that $d_2^*$ corresponds to $\{s_1 = 0, s_2 = 0\}$, $d_2^*$ to $\{s_1 = 1, s_2 = 1\}$, and $d_1^*$ to either $\{s_1 = 0, s_2 = 1\}$ or $\{s_1 = 1, s_2 = 0\}$. The latter implies that, in steady state, the fraction of
people who will be in state \{s_1 = 0, s_2 = 1\} is \(d_1^* / 2\), and the fraction of people who will be in state \{s_1 = 1, s_2 = 0\} is \(d_2^* / 2\). As a result, the steady-state joint distribution is given as shown in table 1.

From table 1, we can calculate the marginal probability of being employed. Indeed, to be employed one needs either to belong to a \(d_1\)-dyad or a \(d_2\)-dyad. This probability is thus equal to

\[
\frac{\delta(\Delta - 2\delta + 2\lambda \omega - \lambda)}{2\lambda \omega (\lambda + \Delta)} + \frac{\delta(\Delta - 2\delta + 2\lambda \omega - \lambda)}{2\lambda \omega (\lambda + \Delta)} + \frac{2(\lambda - 2\delta + \Delta)(\Delta - 2\delta + 2\lambda \omega - \lambda)}{4\lambda \omega (\lambda + \Delta)} = \frac{\Delta - 2\delta + 2\lambda \omega - \lambda}{2\lambda \omega},
\]

which is (20).

Let us now calculate the correlation in employment status between two individuals in a dyad.\(^{10}\) We have

\[
\text{Cor}(s_1 = 1, s_2 = 1) = \frac{\text{Cov}(s_1 = 1, s_2 = 1)}{\sqrt{\text{Var}(s_1 = 1)} \text{Var}(s_2 = 1)},
\]

where

\[
\text{Cov}(s_1 = 1, s_2 = 1) = \mathbb{E}(s_1 = 1, s_2 = 1) - [\mathbb{E}(s_1 = 1)] [\mathbb{E}(s_2 = 1)] = 2d_2^* - (e^*)^2
\]

\[
= \frac{(\Delta - 2\delta + 2\lambda \omega - \lambda)[(\lambda + \Delta)(2\delta + \lambda - \Delta) - 4\lambda \omega \delta]}{4\lambda^2 \omega^2 (\lambda + \Delta)}
\]

and

\[
\text{Var}(s_1 = 1) = \mathbb{E}(s_1 = 1) - [\mathbb{E}(s_1 = 1)]^2 = e^* - (e^*)^2
\]

\[
= \frac{(\Delta - 2\delta + 2\lambda \omega - \lambda)(2\delta + \lambda - \Delta)}{4\lambda^2 \omega^2}
\]

\[
= \text{Var}(s_2 = 1) = \mathbb{E}(s_2 = 1) - [\mathbb{E}(s_2 = 1)]^2.
\]

\(^{10}\) The other correlations between different employment statuses can be calculated in a similar way.
Using the value of $\Delta$ in (19), we obtain

$$\text{Cor}(s_1 = 1, s_2 = 1) = \frac{\sqrt{\lambda(\lambda + 4\delta(1 - \omega))} - \lambda}{\sqrt{\lambda(\lambda + 4\delta(1 - \omega))} - \lambda + 2\lambda\omega} > 0.$$  

The employment status of two employed individuals belonging to the same dyad is thus positively correlated: if one agent is employed, the probability that the other agent is employed is positive. This is because, in this model, workers obtain a job through social interactions only, and two workers belonging to the same dyad have a strong tie relationship with each other and thus share information about jobs $1 - \omega$ percent of their time. As a result, since they help each other find a job, their employment statuses are correlated.

To better understand this result, consider the case when $\lambda = 0$, that is, when employed workers are not informed about jobs. In that case, the correlation is equal to zero since all workers end up in a $d_0$-dyad and stay there forever. Similarly, when $\delta = 0$, that is, when there is no job-destruction rate, the correlation is also equal to zero since, as soon as $\lambda > 0$, everybody will end up employed independently of her partner in the dyad. When $\omega = 0$, workers only interact with their strong ties, and their employment statuses are perfectly correlated. Finally, the correlation in employment status between two employed workers is decreasing with $\omega$ and $\lambda$ and increasing in $\delta$. Indeed, when the job-destruction rate $\delta$ increases, workers tend to be more often unemployed, and thus they help each other more to find a job. As a result, their correlation in employment status increases. The same reasoning applies for a decrease in $\lambda$. When $\omega$ increases so that social interactions with weak ties rise at the expense of the social interactions with strong ties, workers in the same dyad help each other less, and thus their correlation in employment status decreases.

E. Choosing Social Interactions

We would like now to extend the model so that $\omega$ is chosen by individuals and not exogenously defined as in the previous sections. The timing is as in the previous section. We assume that there is some cost of interacting with weak ties. Let $c$ denote the marginal cost of these interactions. The expected utility is now given by

$$\text{EV}(\omega) = e^\ast(\omega)y + [1 - e^\ast(\omega)]b - c\omega,$$

where $e^\ast(\omega)$ is defined by (11) or (15). Each individual optimally chooses $\omega$ that maximizes $\text{EV}(\omega)$. The first-order condition yields

$$\frac{\partial \text{EV}(\omega)}{\partial \omega} = \frac{\partial e^\ast(\omega)}{\partial \omega}(y - b) - c = 0. \quad (21)$$
We have seen (see proposition 4) that if \( (10) \) holds, then \( \partial e^*(\omega)/\partial \omega > 0 \).

We have the following result in proposition 5.

**Proposition 5.** Assume \( (10) \), and consider steady-state equilibrium \( \mathcal{I} \). Then there exists a unique interior \( 0 < \omega^* < 1 \) that maximizes \( EV(\omega) \).

Higher wages or lower unemployment benefits or lower interaction costs will increase the interactions with weak ties, that is,

\[
\frac{\partial \omega^*}{\partial y} > 0, \quad \frac{\partial \omega^*}{\partial b} < 0, \quad \frac{\partial \omega^*}{\partial c} < 0.
\]

Furthermore, if \( \lambda \geq 2 \), an increase in \( \delta \), the job-destruction rate, or an increase in \( \lambda \), the job-information rate, induces workers to spend more time with their weak ties, that is,

\[
\frac{\partial \omega^*}{\partial \delta} > 0, \quad \frac{\partial \omega^*}{\partial \lambda} > 0.
\]

There is a clear trade-off between the benefits of interacting with weak ties and the costs associated with it (see [21]). Indeed, workers want to interact with weak ties because it increases their probability of being employed (or, equivalently, the time they spend employed during their lifetime), that is, \( \partial e^*(\omega)/\partial \omega > 0 \). Concerning the wage \( y \) and the unemployment benefit \( b \), a higher \( y \) or \( b \) increases the value of employment and, since \( e^*(\omega) \) and \( \omega \) are positively related, workers will interact more with weak ties. Quite naturally, increasing the cost \( c \) of social interactions reduces the time spent with weak ties.

What is interesting and new here is the effect of the aggregate labor market variables on social interactions. When \( \delta \) or \( \lambda \) increases, workers spend more time with their weak ties because the cross effect of \( \delta \) or \( \lambda \) on employment is positive, that is, \( \partial e^2(\omega)/\partial \omega \delta > 0 \) and \( \partial e^2(\omega)/\partial \omega \lambda > 0 \). Indeed, when \( \delta \) or \( \lambda \) increases, the positive effect of weak ties \( \omega \) on employment \( e^* \) is even stronger, and thus workers rely more on their weak ties. This is an interesting result, since it shows that, in downturn periods where jobs are destroyed at a faster rate, workers tend to spend more time with their weak ties because they know they will help them exit unemployment. In an economy where the flow of job information is faster, the same results occur.

### IV. Finding a Job Outside the Social Network

While we do know that lots of jobs are obtained through networks, lots of jobs are not, and, as a result, one limitation of our model is that there is no role for job finding outside of the social network. Let us now consider the possibility of an unemployed worker learning of a vacancy directly from an employer as well as via a social interaction. To be more
precise, we now assume that the unemployed workers can also hear directly about a job at rate $\lambda$. For example, employers put “help wanted” signs on their windows or have adds in newspapers. In that case, anybody (employed or unemployed) can have access to this information. As in the previous section, the job information that only goes through employed workers is when the employers do not advertise their vacancies publicly but only tell their own employed workers about them.

In that case, the unemployed workers can now find a job either directly or through their social network. It is readily verified that the net flow of dyads from each state between $t$ and $t + dt$ is now given by

$$\begin{align*}
\dot{d}_2(t) &= [2 - \omega + \omega e(t)]\lambda d_1(t) - 2\delta d_2(t), \\
\dot{d}_1(t) &= 2\lambda[\omega e(t) + 1]d_0(t) - (\delta + [2 - \omega + \omega e(t)]\lambda)d_1(t) + 2\delta d_2(t), \\
\dot{d}_0(t) &= \delta d_1(t) - 2\lambda[\omega e(t) + 1]d_0(t).
\end{align*}$$

(22)

Indeed, the only difference with the previous section where only the employed workers could hear about a job (see [4]) is that now the unemployed workers can also hear about a job at rate $\lambda$. For example, the rate at which a dyad $d_0$ becomes a dyad $d_1$ is not anymore $2\omega e\lambda$ but $2\lambda + 2\omega e\lambda$, since now each individual in the dyad can also hear directly about a job. As a result, a person in a dyad $d_0$ will find a job either directly at rate $\lambda$ or through her social network at rate $\omega e\lambda$, since her weak tie has to be employed and has to hear about a job. Observe that jobs obtained through the social network are only transmitted by employed workers, because if an unemployed worker hears directly about a job, she will not give this information to someone in her network but will take it herself.

In a steady-state $(d^*_2, d^*_1, d^*_0)$, each of the net flows in (22) is equal to zero, and we obtain

$$d^*_2 = \frac{\lambda(2 - \omega + \omega e^*)d^*_1}{2\delta},$$

(23)

$$d^*_1 = \frac{2\lambda(\omega e^* + 1)d^*_2}{\delta},$$

(24)

This is for simplicity. Below, we relax this assumption and assume that the rate at which the employed and the unemployed workers hear about a job is not the same.
where
\[ d_2^* = \frac{1}{2} - d_2^* - d_1^*, \]  
\[ u^* = 1 - e^*, \]  
\[ e^* = 2d_2^* + d_1^*. \]

We have a first result.

**Lemma 1.** When the unemployed workers can directly hear from a job, neither a full-unemployment equilibrium \( \mathcal{U} \) for which \( e^* = 0 \) and \( u^* = 1 \) nor a full-employment equilibrium \( \mathcal{E} \) for which \( e^* = 1 \) and \( u^* = 0 \) can exist. This is a first interesting result, saying that a full-unemployment equilibrium \( \mathcal{U} \) such that \( e^* = 0 \) and \( u^* = 1 \) is not anymore possible here while it always existed in the benchmark model. This is because when workers are stuck in a \( d_0 \)-dyad, they can always leave it without using their network since there is always a positive probability that one of the unemployed worker belonging to a \( d_0 \)-dyad hears directly about a job. As a result, nobody gets "stuck" in a \( d_0 \)-dyad.

**Proposition 6.** When the unemployed workers can directly hear from a job, there exists a unique steady-state interior equilibrium \( \mathcal{I} \) where \( 0 < e^* < 1 \) is (implicitly) given by

\[ e^* = \frac{\lambda (\omega e^* + 1)[\lambda (2 - \omega + \omega e^*) + \delta]}{\delta^2 + \lambda (\omega e^* + 1)[\lambda (2 - \omega + \omega e^*) + 2\delta]}, \]  

\( 0 < d_2^* < 1/2 \) by

\[ d_2^* = \frac{\delta^2/2}{\delta^2 + \lambda (\omega e^* + 1)[\lambda (2 - \omega + \omega e^*) + 2\delta]}, \]  

and \( d_2^* \) and \( d_1^* \) by (23) and (24), respectively.

Even though the model is much more complicated, we are able to prove that there exists a unique steady-state interior equilibrium and to give the equilibrium values of all variables. Their values are, however, more cumbersome and only implicitly defined. We would like now to see if one of our main results, namely, the positive relationship between employment and weak ties, still holds in this more general model.
Define
\[ \Phi(e) \equiv -2\lambda \omega e^3 - (3\lambda - 4\lambda \omega + 2\delta)e^2 + [\delta + 2\lambda(2 - \omega)]e - \lambda. \] (30)

We have the following result.

**Proposition 7.** When the unemployed workers can directly hear from a job, an increase in the percentage of weak ties \( \omega \) increases the employment rate \( e^* \) in the economy only when \( e^* \) takes intermediate values. On the contrary, when \( e^* \) is low or high, then an increase in \( \omega \) reduces \( e^* \). Formally, if \( \underline{e} \) and \( \overline{e} \) are the positive real roots of \( \Phi(e) \), then (i) when \( \underline{e} < e^* < \overline{e} \), then \( \partial e^*/\partial \omega > 0 \); (ii) when \( 0 < e^* < \underline{e} \) or \( \overline{e} < e^* < 1 \), then \( \partial e^*/\partial \omega < 0 \).

This is an interesting result, showing that the positive relationship between \( e^* \) and \( \omega \) only holds when employment is neither too low nor too high. Indeed, when the employment rate \( e^* \) is very low, then weak ties do not help a lot, because there is little chance that they are employed. This is because, when someone is stuck in a \( d_0 \)-dyad, she can leave it without the help of a weak tie, since she can directly hear from a job at rate \( \lambda \). In the previous model, only a weak tie could help her find a job, because neither her strong tie nor she herself could find a job. When the employment rate \( e^* \) is very high, then weak ties are less important, because strong ties are also more likely to be employed.

This new result provides a new perspective on the notion of “weak ties” proposed by Granovetter. As highlighted in the Introduction, Granovetter was postulating that weak ties were always superior to strong ties in helping workers find a job. Proposition 4 says that when the unemployed workers can only find jobs through their social contacts (which is the case in developing countries for unskilled workers, who are often illiterate and thus have no access to newspapers,\(^{12}\) or in developed countries for very specific jobs that are not advertised formally), then weak ties are superior to strong ties for finding a job. However, when the unemployed workers can find a job both directly and through their social contacts, proposition 7 states that strong ties can be superior to weak ties when the employment rate in the economy is either low or high.

To illustrate this result and to have some sense of what is a high and a low employment rate, let us perform some simple numerical simulations. As in Section III.C, following Shimer (2005) and Pissarides (2009), we take \( \lambda = 0.594 \) and \( \delta = 0.036 \). For the time spent with weak ties, we take \( \omega = 0.2 \). As stated in proposition 6, there is a unique solution, which

\(^{12}\) See, e.g., Wahba and Zenou (2005, 2012) for Egypt.
is here given by $e^* = 0.9699$ and $d^*_0 = 0.00073$. Table 2 gives all the equilibrium values.

The unemployment rate is equal to 3%. Workers do not stay that much in a $d_0$-dyad (only 0.15% of their time),\textsuperscript{13} because they hear directly of a job and have their social network. On the contrary, they spend most of their time in a $d_2$-dyad (94.13% of their time).

We can then determine the relationship between $e^*$ and $\omega$. In the proof of proposition 7, we showed that the sign of $\partial e^*/\partial \omega$ is the same as that of $\Phi(e)$, which is given by (30). It is easily verified that $\Phi(e)$ has three real solutions, one being negative and equal to $e = -2.56$, and two being positive and equal to $e = 0.3609$ and $\bar{e} = 0.9709$. Here, since $e^* = 0.9699$, it is between $e = 0.3609$ and $\bar{e} = 0.9709$. Then, following proposition 7, an increase in $\omega$ always increases $e^*$, that is, $\partial e^*/\partial \omega > 0$. It is interesting to notice that, when we say that $e^*$ has to take an intermediate value for $\Phi(e)$, it is relative to the values of $e$ and $\bar{e}$. In the present example, the equilibrium employment rate is quite high (96.99%), and we still have that $\partial e^*/\partial \omega > 0$.

Let us now assume that the rate at which the employed (denoted by $\lambda_1$) and the unemployed workers (denoted by $\lambda_2$) hear about a job is not the same. In that case, the flows in the labor market are now given by

\begin{align*}
\dot{d}_2(t) &= (\lambda_0 + [1 - \omega + \omega e(t)]\lambda_1)d_1(t) - 2\delta d_2(t), \\
\dot{d}_1(t) &= 2[\omega e(t)\lambda_1 + \lambda_0]d_2(t) - (\delta + \lambda_2 + [1 - \omega + \omega e(t)]\lambda_1)d_1(t) + 2\delta d_2(t), \\
\dot{d}_0(t) &= \delta d_1(t) - 2[\omega e(t)\lambda_1 + \lambda_2]d_2(t).
\end{align*}

(31)

\textsuperscript{13} For the interpretation of the results, it is better to use $2d^*_0$ than $d^*_0$, since the former is normalized and gives the time spent in a $d_0$-dyad. The same applies for $d^*_1$ and $d^*_2$. 

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-State Equilibrium</td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>$e^*$ (%)</td>
</tr>
<tr>
<td>$u^*$ (%)</td>
</tr>
<tr>
<td>$d^*_0$</td>
</tr>
<tr>
<td>$2d^*_0$ (%)</td>
</tr>
<tr>
<td>$d^*_1$</td>
</tr>
<tr>
<td>$2d^*_1$ (%)</td>
</tr>
<tr>
<td>$d^*_2$</td>
</tr>
<tr>
<td>$2d^*_2$ (%)</td>
</tr>
</tbody>
</table>
In a steady-state \((d_2^*, d_1^*, d_0^*)\), each of the net flows in (31) is equal to zero, and we obtain
\[
d_2^* = \lambda_0 + (1 - \omega + \omega^*)\lambda_1 d_1^*,
\]
\[
d_1^* = \frac{2(\lambda_0 + \omega^*\lambda_1)d_0^*}{\delta}.
\]
Since \(e^* = 2d_0^* + d_1^*\), we have
\[
e^* = \left[\lambda_0 + (1 - \omega + \omega^*)\lambda_1 + \delta\right] \frac{2(\lambda_0 + \omega^*\lambda_1)d_0^*}{\delta^2}.
\]
Since \(d_0^* = 1/2 - d_1^* - d_2^*\), we easily obtain
\[
2d_0^* = \frac{\delta^2}{\delta^2 + (\lambda_0 + \omega^*\lambda_1)[\lambda_0 + (1 - \omega + \omega^*)\lambda_1 + 2\delta]}. 
\]
Using the same arguments as in the proof of proposition 6, it is straightforward to show that there exists a unique interior steady-state equilibrium \(I\), where \(e^*\) and \(d_0^*\) are given by (34) and (35) and \(d_2^*\) and \(d_1^*\) by (32) and (33). Plugging the value of \(d_0^*\) from (35) into (34), we obtain
\[
e^* = \frac{(\lambda_0 + \omega^*\lambda_1)[\lambda_0 + (1 - \omega + \omega^*)\lambda_1 + \delta]}{\delta^2 + (\lambda_0 + \omega^*\lambda_1)[\lambda_0 + (1 - \omega + \omega^*)\lambda_1 + 2\delta]}.
\]
Proceeding as in the homogeneous case (when \(\lambda_0 = \lambda_1 = \lambda\)), we have
\[
\frac{\partial e^*}{\partial \omega} \geq 0 \iff \Phi(e) \geq 0,
\]
where
\[
\Phi(e) = -2\lambda_1 \omega^3 - \left[2\lambda_0 + 2\delta + \lambda_1(1-4\omega)\right] e^2 + \left[\delta + 3\lambda_0 + \lambda_1(1-2\omega)\right] e - \lambda_0.
\]

We obtain exactly proposition 7 with the only difference that \(\epsilon\) and \(\bar{\epsilon}\) are now the positive real roots of \(\Phi(e)\) and not of \(\Phi(e)\). Let us now run some numerical simulations by assuming that the average rate is the same as in the homogeneous case so that \((\lambda_0 + \lambda_1)/2 = \lambda = 0.594\). Let us take \(\lambda_0 = 0.5\) and \(\lambda_1 = 0.688\), so that \(\lambda_1 - \lambda_0\) is relatively small. In that case, the results are nearly the same as in the homogeneous case, since the employment rate is \(e^* = 0.9698\) and \(\epsilon = 0.3286\) and \(\bar{\epsilon} = 0.9709\).

Let us keep the same average \((\lambda_0 + \lambda_1)/2 = 0.594\), but let us now increase the difference between \(\lambda_0\) and \(\lambda_1\). We take \(\lambda_0 = 0.2\) and \(\lambda_1 = 0.988\). In that case, we obtain a similar employment rate, that is, \(e^* = 0.9687\), but the lower bound in employment rate is much smaller, since \(\epsilon = 0.1830\).
and $\bar{e} = 0.9710$. So we see that the lower is $\lambda$, the lower is $\bar{e}$ and the larger is the interval between $\bar{e}$ and $e$, and therefore the more likely there is a positive relationship between $\omega$ and $e^\rho$.

V. Discussion

In order to highlight the contribution of our paper, let us compare it with that of Calvó-Armengol and Jackson (2004), a prominent paper in the theoretical literature on social networks in the labor market.\footnote{As stated in the Introduction, there are other theoretical papers dealing with social networks in the labor market. They are usually not dynamic and, if they are, they are just a particular case of Calvó-Armengol and Jackson (2004). That is why here we mainly compare our model to that of Calvó-Armengol and Jackson (2004).} Contrary to the present model, where only a very specific network structure (i.e., the dyad) is assumed, Calvó-Armengol and Jackson (2004) explicitly model a social network (which can have any possible structure), using graph theory (Jackson 2008), where information flows between individuals having a link with each other. They show that an equilibrium with a clustering of workers with the same status is likely to emerge, since, in the long run (i.e., steady state), employed workers tend to be friends with employed workers.

The main differences between our model and the Calvó-Armengol and Jackson model (2004) are as follows. First, Calvó-Armengol and Jackson (2004) model strong ties in a richer way, since each individual may have many strong ties (not only one as in our model) and thus must be in competition with friends of friends. This leads to the fact that, if my (employed) strong tie hears about a job, I will not necessarily obtain this formation, since my strong tie will share it with all her unemployed strong ties. This is not the case in our model. Second, we model weak ties in a richer way than the Calvó-Armengol and Jackson because the unemployed workers meet the, directly at every period of time, while in the Calvó-Armengol and Jackson model, workers never directly meet weak ties. The latter seems to be a strong assumption and not consistent with evidence on weak ties as a direct of source of information about jobs, as suggested by Granovetter. In the Calvó-Armengol and Jackson model, my weak ties will never directly provide me with information about jobs. They will help my strong ties get a job, which eventually will help me in finding a job. Third, the Calvó-Armengol and Jackson model assumes that workers spend all their time with strong ties (in the language of our model, this means that $\omega = 0$), while in our model, workers choose how much time to spend with weak and strong ties. This helps us to formally demonstrate Granovetter (1973, 1974, 1983)’s idea that weak ties are superior to strong ties for providing support in getting a job by showing that $\omega$ has a positive impact on the employment rate in the economy. This result cannot be shown in the Calvó-Armengol and
Jackson model, because it is based on the asymmetric role of weak and strong ties in helping the unemployed workers find a job. Finally, the main advantage of our approach is that it leads to a very tractable model. We obtain closed-form solutions for the equilibrium values, and it is easy to incorporate new aspects (e.g., explain racial differences in employment) and still obtain clear-cut results.

VI. Conclusion

In this paper, we have developed a simple and tractable model of weak and strong ties in the labor market. We have formally demonstrated Granovetter’s informal idea of the strength of weak ties in finding a job. In our model, it is better to meet weak ties, because a strong tie does not help in the state where all best friends are unemployed. But a weak tie can help leaving unemployment in any state, because that person might be employed. So there is an asymmetry that is key to the model and that explains why weak ties are superior to strong ties in helping unemployed workers find a job. This can also explain why some workers may be stuck in unemployment traps by having little contact with weak ties that can help them escape unemployment. When we allow unemployed workers to learn of a vacancy directly from an employer as well as via a social interaction, we show that the previous result is not always true. It is only valid when the employment rate in the economy is not too high, because now the unemployed workers can escape from the unemployment traps by finding a job by themselves.

One of the virtues of our model is that it is simple and tractable. In particular, it is easy to extend it and to incorporate new aspects. For example, Zenou (2013) uses this model to explain racial differences in employment. Indeed, if minority workers have little interaction with weak ties, especially white workers, they will end up experiencing a high unemployment rate. This is because, as highlighted in this paper, weak ties are an important source of job information, and when ethnic minorities miss it, they end up having a higher unemployment rate than whites. There is indeed a vicious circle, because, if ethnic minorities are discriminated against in the labor market, they will experience a higher unemployment rate than whites and will mostly rely on other ethnic minority workers (strong ties) who also experience a high unemployment rate. Since jobs are mainly found through social networks via employed friends, ethnic

15 For example, in Sec. III.D, we were able to explicitly calculate the correlation in employment status between different workers in a dyad and to see how this correlation varies with the different parameters of the model.

16 See, e.g., Battu, Seaman, and Zenou (2011), who show that networks are a popular method for the ethnic minorities in the United Kingdom, even though they are not necessarily the most effective in terms of gaining employment.
minorities will then be stuck in a “bad” labor market situation. This result will be true even if unemployed workers can find a job directly, as in Section IV. This may be because minority workers are more likely to reside in segregated areas (Ihlanfeldt and Sjoquist 1998). As a result, if black workers do not have access to weak ties (especially whites), in particular because they are segregated and separated from them, then their main source of information about jobs will be provided by their strong ties. But if the latter are themselves unemployed, this might partially explain unemployment differences by race.

Appendix

Proof of Proposition 1

We establish the proof in two steps. First, lemma 2 characterizes all steady-state dyad flows. Lemma 3 then provides conditions for their existence.

Lemma 2. There exists at most two different steady-state equilibria: (i) a full-unemployment equilibrium $U$ such that $e^* = 0$ and $u^* = 1$, (ii) an interior equilibrium $I$ such that $0 < e^* < 1$ and $0 < u^* < 1$.

Proof. By combining (5)–(8), we easily obtain

$$e^* = \left[ (1 - \omega + \omega e^*)\lambda + \delta \right] \frac{2\omega e^*\lambda}{\delta} - d^*_2.$$  \hspace{1cm} (A1)

We consider two different cases.

(i) If $e^* = 0$, then equation (A1) is satisfied. Furthermore, using (5) and (6), this implies that $d^*_1 = d^*_2 = 0$ and, using (7) and (9), we have $d^*_3 = 1/2$ and $u^* = 1$. This is referred to as steady-state $U$ (full unemployment).

(ii) If $e^* > 0$, then solving equation (A1) yields

$$e^* = \frac{1}{\lambda \omega} \left[ \frac{\delta^2}{2\omega \lambda d^*_3} - \delta \right] - \frac{(1 - \omega)}{\omega}.$$  \hspace{1cm} (A2)

Define $Z = (1 - \omega)/\omega$, $B = \delta/(\lambda \omega)$. This equation can now be written as

$$e^* = \frac{B^2}{2d^*_3} - B - Z.$$  \hspace{1cm} (A3)

Moreover, by combining (5) and (6), we obtain

$$d^*_1 = \frac{2e^*}{B} d^*_3, \quad d^*_2 = \frac{Z + e^*}{B^2} d^*_3.$$  \hspace{1cm} (A3)
• Let us first focus on the case where $e^* = 1$. In that case, it has to be that only $d^*-dyads$ exist, and thus $d^* = d^*_1 = 0$, which, using (A3), implies that $d^* = 0$. So this case is not possible.

• Let us now thus focus on the case $0 < e^* < 1$ (which implies that $0 < u^* < 1$).

By plugging (A2) and (A3) in (7), and after some algebra, we obtain that $d^*_0$ solves $\Phi(d^*_0) = 0$, where $\Phi(x)$ is the following second-order polynomial:

$$\Phi(d^*_0) = -\frac{Z}{B}x^2 - \frac{(1 + Z)}{2}x + \left(\frac{B^2}{2}\right). \quad (A4)$$

QED

**Lemma 3.**  
(i) The steady-state equilibrium $U$ always exists.  
(ii) The steady-state equilibrium $I$ exists when $\delta < \lambda[\omega + \sqrt{\omega(4-3\omega)}]/2$.

**Proof.** (i) In this equilibrium $e^* = 0$, which implies that $b(e) = (1 - \omega)\lambda$ and $q(e) = 0$. There are only $d^*_0$-dyads, so all workers are unemployed and will never receive a job offer since $q(e) = 0$. So when a $d^*_0$-dyad is formed, it is never destroyed, and thus this equilibrium is always sustainable.

(ii) We know from lemma 2 that a steady-state $I$ exists and that $e^* \neq 1$. We now have to check that $e^* > 0$ and $0 < d^*_0 < 1/2$. Let us thus verify whether there exists some $0 < d^*_0 < 1/2$ such that $\Phi(d^*_0) = 0$, where $\Phi(\cdot)$ is given by (A4). We have $\Phi(0) = (B/2)^2 > 0$ and $\Phi(0) = -(1 + Z)/2 < 0$. Therefore, (A4) has a unique positive root smaller than $1/2$ if and only if

$$\Phi(1/2) = \frac{1}{4} \left[B^2 - (1 + Z) - \frac{Z}{B}\right] = \frac{1}{4} \left(1 + \frac{1}{B}\right)(B^2 - B - Z) < 0.$$  

The unique positive solution to $x^2 - x - Z = 0$ is $[1 + \sqrt{(4-3\omega)/\omega}]/2$. Then $d^*_0 < 1/2$ if and only if $B < [1 + \sqrt{(4-3\omega)/\omega}]/2$, equivalent to

$$\frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2}.$$  

Observe that $d^*_0 < 1/2$ guarantees that $e^* > 0$. QED

**Proof of Proposition 2**

By differentiating (15), we obtain

$$\frac{\partial e^*}{\partial \delta} = \frac{1 - \omega}{\omega\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}} - \frac{1}{\lambda \omega}. \quad (A5)$$
We have
\[
\frac{\partial e^*}{\partial \delta} < 0 \iff \frac{1 - \omega}{\omega \sqrt{\lambda + 4\delta(1 - \omega)}} < \frac{1}{\lambda \omega},
\]
which is equivalent to
\[
(1 - \omega)\lambda < \lambda + 4\delta(1 - \omega).
\]
Simplifying further this inequality leads to
\[
\lambda \omega(\omega - 2) < 4\delta(1 - \omega),
\]
which is always true since \( \omega < 1 \).

By differentiating (12), we get
\[
\frac{\partial d^*_c}{\partial \delta} = \frac{1}{\lambda \omega} \left[ \frac{B^2}{4zd^*_c + (1 + Z)B} \right] > 0. \quad (A6)
\]

By differentiating (13), we get
\[
\frac{\partial d^*_i}{\partial \delta} = \frac{2\lambda \omega}{\delta^2} \left[ \frac{\partial e^*}{\partial \delta} d^*_c \delta + e^* \frac{\partial d^*_c}{\partial \delta} \delta - e^* d^*_c \right]. \quad (A7)
\]

Using (A5), (A6), and the fact that \( B \equiv \delta/(\lambda \omega) \), we obtain
\[
\frac{\partial d^*_i}{\partial \delta} = \frac{2\lambda \omega}{\delta} \left[ \frac{(1 - \omega)d^*_c \delta}{\omega \sqrt{\lambda + 4\delta(1 - \omega)}} - \frac{d^*_c \delta}{\lambda \omega} + \frac{2zd^*_c e^* + B^2 e^*}{4zd^*_c + (1 + Z)B} - e^* d^*_c \right].
\]

This is clearly ambiguous. However, if we go back to (A7), observe that if \( (\partial e^*/\partial \delta)d^*_c \delta + e^*(\partial d^*_c/\partial \delta)\delta < 0 \), then \( \partial d^*_i/\partial \delta < 0 \). This is equivalent to
\[
\frac{\partial e^* \delta}{\partial \delta e^*} + \frac{\partial d^*_c \delta}{\partial \delta d^*_c} < 0
\]
\[
\iff \frac{\partial d^*_c \delta}{\partial \delta d^*_c} < -\frac{\partial e^* \delta}{\partial \delta e^*}
\]
\[
\iff \left| \frac{\partial d^*_c \delta}{\partial \delta d^*_c} \right| < \left| \frac{\partial e^* \delta}{\partial \delta e^*} \right|.
\]

Finally, by differentiating (14), we get
\[
\frac{\partial d^*_i}{\partial \delta} = \frac{(\lambda \omega)^2}{\delta^3} \left\{ (Z + 2e^*) \frac{\partial e^*}{\partial \delta} d^*_c \delta + (Ze^* + e^* \frac{\partial e^*}{\partial \delta} d^*_c \delta - 2(Ze^* + e^* \frac{\partial e^*}{\partial \delta} d^*_c \delta \right\}. 
\]
Since \((Z + 2e^*)(\partial e^*/\partial \delta)\delta < 0\), let us show that \((Ze^* + e^{*z})(\partial d^*_\delta/\partial \delta)\delta - 2(Ze^* + e^{*z})d^*_\delta < 0\). Using (A6) and the fact that \(B = \delta/(\lambda \omega)\), this last inequality is equivalent to

\[
\frac{2Zd^{*z}}{4d^*_\delta Z + (1 + Z)B} - 2d^*_\delta < 0,
\]

which is equivalent to

\[
-\frac{3Z}{2B}d^{*z} - \frac{(1 + Z)}{2}d^*_\delta + \left(\frac{B}{2}\right)^2 < 0.
\]

Since \(d^*_\delta\) is defined as (see [12])

\[
-\frac{Z}{B}d^{*z} - \frac{(1 + Z)}{2}d^*_\delta + \left(\frac{B}{2}\right)^2 = 0,
\]

the inequality above is thus equivalent to

\[
-\frac{1Z}{2B}d^{*z} < 0,
\]

which is always true. QED

**Proof of Proposition 3**

Differentiate first (12). We obtain

\[
\frac{\partial d^*_\delta}{\partial \lambda} = -\frac{Z\lambda \omega}{\delta}d^{*z} - \frac{\delta^2}{2}d^*_\delta + \frac{2\lambda \omega}{1 + Z} < 0.
\] (A8)

By differentiating (15), we have

\[
2\omega \lambda^2 \partial e^*/\partial \lambda = \frac{\lambda^2 + 2\delta \lambda (1 - \omega)}{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}} - \frac{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}}{\lambda[\lambda + 4\delta(1 - \omega)]} + 2\delta
\]

\[
= 2\delta - \frac{2\delta \lambda (1 - \omega)}{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}}.
\]

As a result,

\[
\frac{\partial e^*/\partial \lambda} > 0 \iff 2\delta > \frac{2\delta \lambda (1 - \omega)}{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}}
\]

\[
\iff 4\delta(1 - \omega) > \lambda \omega (\omega - 2),
\]

which is always true since \(\omega < 2\).
By differentiating (14), we obtain

\[ B^3 \frac{\partial d^e_0}{\partial \lambda} = \frac{\partial e^e}{\partial \lambda} (Z + 2e^e) B d^e_\omega + (Z e^e + e^{e^2}) \frac{\partial d^e_0}{\partial \lambda} B + 2(Z e^e + e^{e^2}) \frac{\delta}{\lambda^2 \omega} d^e_0 \]

\[ = \frac{\partial e^e}{\partial \lambda} (Z + 2e^e) \frac{\delta}{\lambda \omega} d^e_\omega + 2(Z e^e + e^{e^2}) \frac{\delta}{\lambda^2 \omega} d^e_\omega + (Z e^e + e^{e^2}) \frac{\delta}{\lambda \omega} \frac{\partial d^e_0}{\partial \lambda}. \]

Using (A8), we have

\[ B^3 \frac{\partial d^e_\omega}{\partial \lambda} = \frac{\partial e^e}{\partial \lambda} (Z + 2e^e) \frac{\delta}{\lambda \omega} d^e_\omega + (Z e^e + e^{e^2}) \frac{\delta}{\lambda^2 \omega} d^e_\omega - \frac{(1 - \omega) \lambda d^e_\omega}{\delta} + \frac{2 \lambda^2 \omega}{\delta} (Z e^e + e^{e^2}) \]

\[ = \frac{\partial e^e}{\partial \lambda} (Z + 2e^e) \frac{\delta}{\lambda \omega} d^e_\omega + (Z e^e + e^{e^2}) \left[ \frac{2 \delta d^e_\omega}{\lambda^2 \omega} + \frac{2(1 - \omega) \lambda d^e_\omega}{\delta} \right] \]

\[ = \frac{\partial e^e}{\partial \lambda} (Z + 2e^e) \frac{\delta}{\lambda \omega} d^e_\omega + (Z e^e + e^{e^2}) \left[ \frac{2 \delta d^e_\omega (1 - \omega) d^e_\omega (4 - \lambda) + 2 \delta^2 \lambda d^e_\omega - \frac{\delta^3}{\lambda \omega}}{\lambda^2 \omega \lambda^1 \omega (1 - \omega) d^e_\omega + \lambda \delta} \right]. \]

We know from (12) that

\[- \frac{(1 - \omega) \lambda}{\delta} d^{e_2} - \frac{1}{2 \omega} d^e_\omega + \frac{\delta^2}{4 \lambda^2 \omega^2} = 0, \]

and thus we have

\[ B^3 \frac{\partial d^e_\omega}{\partial \lambda} = \frac{\partial e^e}{\partial \lambda} (Z + 2e^e) \frac{\delta}{\lambda \omega} d^e_\omega - \frac{2 \delta (Z e^e + e^{e^2}) (1 - \omega) \lambda (\lambda - 2) d^e_\omega}{\lambda^2 \omega (1 - \omega) d^e_\omega + \lambda \delta}. \]

Using (A9), which we demonstrate below, we obtain

\[ B^3 \frac{\partial d^e_\omega}{\partial \lambda} = \frac{\delta}{\lambda^2 \omega} - \frac{\delta^3}{\lambda^2 \omega} (Z + 2e^e) \left( \frac{2 \lambda \omega (1 - \omega) d^e_\omega (4 - \lambda) + 2 \delta d^e_\omega - \frac{\delta^3}{\lambda^2 \omega}}{4 \lambda^2 \omega^4 \lambda (1 - \omega) d^e_\omega + \delta} \right) \]

\[- \frac{2 \delta (Z e^e + e^{e^2}) (1 - \omega) \lambda (\lambda - 2) d^e_\omega}{\lambda^2 \omega (1 - \omega) d^e_\omega + \lambda \delta}. \]

Using (12), we get

\[ B^3 \frac{\partial d^e_\omega}{\partial \lambda} = \frac{\delta}{\lambda^2 \omega} \frac{\delta^3 (Z + 2e^e) (1 - \omega) \lambda (\lambda - 2)}{\delta - \frac{d^e_\omega}{\omega}} + \frac{2 \lambda^2 \omega^2 (4 \lambda \omega (1 - \omega) d^e_\omega + \delta)}{4 \lambda^2 \omega^4 \lambda (1 - \omega) d^e_\omega + \delta} \]

\[ \therefore B \frac{\partial d^e_\omega}{\partial \lambda} = \frac{\delta}{\lambda^2 \omega} + \frac{(1 - \omega) \lambda (\lambda - 2) \delta^3 (Z + 2e^e) - 4 \lambda^2 \omega^2 (Z e^e + e^{e^2}) (1 - \omega) \lambda (\lambda - 2) d^e_\omega}{2 \lambda^2 \omega^4 \lambda (1 - \omega) d^e_\omega + \delta}. \]
A sufficient condition for \( \partial d_0^*/\partial \lambda > 0 \) is

\[
(1 - \omega)\omega(\lambda - 2)[\delta^3(Z + 2e^*) - 4\lambda^2\delta\omega^2(Z + e)e^*]d_0^2 - \delta^3(Z + 2e^*)d_0^* > 0,
\]

which is equivalent to

\[
d_0^* > \frac{\delta^3}{(1 - \omega)\omega(\lambda - 2)[\delta^2 - 4\lambda^2\omega^2(Z + e)e^*]}.
\]

An upper bound for \( [(Z + e)/(Z + 2e^*)]e^* \) is 1, and thus this condition can be written as

\[
d_0^* > \frac{\delta^3}{(1 - \omega)\omega(\lambda - 2)(\delta^2 - 4\lambda^2\omega^2)^*},
\]

which is condition (18).

Finally, by differentiating (13), we get

\[
\frac{\partial d_0^*}{\partial \lambda} = \frac{2\omega}{\delta} \left[ \frac{\partial e^*}{\partial \lambda} d_0^* \lambda + e^* \frac{\partial d_0^*}{\partial \lambda} \lambda + e^* d_0^* \right].
\]

This is clearly ambiguous. However, since \( e^* d_0^* > 0 \), a sufficient condition for \( \partial d_0^*/\partial \lambda > 0 \) is

\[
\frac{\partial e^*}{\partial \lambda} d_0^* \lambda + e^* \frac{\partial d_0^*}{\partial \lambda} \lambda > 0 \Leftrightarrow \frac{\partial e^*}{\partial \lambda} \frac{\partial d_0^*}{\partial \lambda} > -\frac{\partial d_0^*}{\partial \lambda} \frac{\partial d_0^*}{\partial \lambda}
\]

\[
\Leftrightarrow \left| \frac{\partial e^*}{\partial \lambda} \right| > \left| \frac{\partial d_0^*}{\partial \lambda} \right|.
\]

QED

**Proof of Proposition 4**

(i) By totally differentiating (12), we obtain

\[
\frac{\partial d_0^*}{\partial \omega} = \frac{\lambda}{\delta} d_0^2 + \frac{1}{2\omega^2} d_0 - \frac{\delta^2}{2\lambda^2\omega^3} d_0,
\]

and thus
\[ \text{sgn} \frac{\partial d_0^*}{\partial \omega} = \text{sgn} \left[ \frac{\lambda}{\delta} d_0^2 + \frac{1}{2\omega^2} d_0 - \frac{\delta^2}{2\lambda^2 \omega^3} \right]. \]

Let us study
\[ \Phi(d_0) = \frac{\lambda}{\delta} d_0^2 + \frac{1}{2\omega^2} d_0 - \frac{\delta^2}{2\lambda^2 \omega^3}, \]
\[ \Phi(0) = -\frac{\delta^2}{2\lambda^2 \omega^3} < 0, \]
\[ \Phi'(d_0) = 2 \frac{\lambda}{\delta} d_0 + \frac{1}{2\omega^2} > 0 \text{ when } d_0 \geq 0, \]
\[ \Phi''(d_0) = 2 \frac{\lambda}{\delta} > 0. \]

We have a quadratic function that crosses only once the positive orthant. Let us calculate \( \hat{d}_0 > 0 \), the value for which \( \Phi(d_0) \) crosses the \( d_0 \)-axis. For that, we have to solve \( \Phi(\hat{d}_0) = 0 \). It is easy to verify that
\[ \hat{d}_0 = \frac{\delta}{4\lambda \omega^3} \left( \sqrt{1 + \frac{8\delta \omega}{\lambda}} - 1 \right) > 0. \]

It should be clear that if \( \hat{d}_0 < 1/2 \), then \( \Phi(d_0) < 0 \) for \( 0 < d_0 < 1/2 \) and thus \( \partial d_0^*/\partial \omega < 0 \). Let us thus check that \( \hat{d}_0 < 1/2 \), which is equivalent to
\[ \Omega\left( \frac{\delta}{\lambda} \right) = 2 \left( \frac{\delta}{\lambda} \right)^3 - \omega \frac{\delta}{\lambda} - \omega^3 < 0. \]

We have
\[ \Omega(0) = -\omega^3 < 0, \]
\[ \Omega\left( \frac{\delta}{\lambda} \right) = 6 \left( \frac{\delta}{\lambda} \right)^2 - \omega, \]
with
\[ \Omega\left( \frac{\delta}{\lambda} \right) < 0 \Leftrightarrow \frac{\delta}{\lambda} < \sqrt[3]{\frac{\omega}{6}}. \]

As a result, when \( \delta/\lambda < \sqrt[3]{\omega/6}, \hat{d}_0 < 1/2 \) and thus \( \partial d_0^*/\partial \omega < 0 \). Since we are in equilibrium \( I \), condition (10) has to hold, that is,
\[ \frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2}. \]
Let us show that
\[ \sqrt{\frac{\omega}{6}} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2}. \]

This inequality is equivalent to
\[ 2\omega < 3\omega^2 + 3\omega(4-3\omega) + 6\omega \sqrt{\omega(4-3\omega)} \]
\[ \Leftrightarrow 3\omega^2 + 9\omega(1 - \omega) + \omega + 6\omega \sqrt{\omega(4-3\omega)} > 0, \]

which is always true since \( \omega < 1. \)

(ii) By totally differentiating (15), we obtain
\[ e^* = \frac{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]} - 2\delta + 2\lambda\omega - \lambda}{2\lambda\omega}, \]
\[ 2\lambda\omega^2 \frac{de^*}{d\omega} = 2\delta + \lambda - \left[ \frac{\lambda^2 + 4\delta\lambda - 2\lambda\delta\omega}{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}} \right]. \]

Thus,
\[ (2\delta + \lambda)^2(\lambda + 4\delta - 4\delta\omega) > \lambda(\lambda + 4\delta - 2\delta\omega)^2, \]
\[ (2\delta + \lambda)^2 > \delta\lambda(1 - \omega) + \lambda(\lambda + 2\delta), \]
\[ 2(\lambda + 2\delta) > \lambda(1 - \omega). \]

\[ \frac{de^*}{d\omega} > 0 \]
\[ \Leftrightarrow (2\delta + \lambda)^2[\lambda + 4\delta(1 - \omega)] > \lambda(\lambda + 4\delta - 2\delta\omega)^2 \]
\[ \Leftrightarrow \lambda + 4\delta > -\lambda\omega, \]

which is always true.

Finally, from (13) and (14), it is easy to see that \( \partial d_i^*/\partial\omega \) and \( \partial d_z^*/\partial\omega \) cannot be signed. QED

\[ \text{Proof of Proposition 5} \]

First, it is easily verified that, using (15), that
\[ \frac{de^2}{d^2\omega} = \frac{\sqrt{\lambda}}{2\lambda\omega^3} \left[ \frac{2\delta\sqrt{\lambda + 4\delta(1 - \omega)} - (\lambda + 4\delta - 2\delta\omega)4\delta}{\lambda + 4\delta(1 - \omega)} \right] < 0, \]

(A11)
so that the second-order condition is always satisfied. As a result, \( \omega^* \), the solution to (21), is unique.

Let us show that \( 0 < \omega^* < 1 \). The optimal \( \omega^* \) is given by

\[
\frac{\partial e^*(\omega)}{\partial \omega} = \frac{c}{y - b}.
\]

By differentiating (15), we obtain

\[
2\lambda \omega^2 \frac{\partial e}{\partial \omega} = \frac{(\lambda + 2\delta)\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]} - \lambda[\lambda + 2\delta(2 - \omega)]}{\sqrt{\lambda[\lambda + 4\delta(1 - \omega)]}}.
\] (A12)

Using (A12), we have

\[
\lim_{\omega \to 0} \frac{\partial e(\omega)}{\partial \omega} = \frac{1}{\omega^2} \left[ \frac{\lambda + 2\delta - \sqrt{\lambda^2 + 4\lambda\delta}}{2\lambda} \right] = +\infty
\]

since \( \lambda + 2\delta > \sqrt{\lambda^2 + 4\lambda\delta} \). We also have

\[
\lim_{\omega \to 1} \frac{\partial e(\omega)}{\partial \omega} = 0.
\]

Furthermore, since \( \frac{\partial^2 e}{\partial \omega^2} < 0 \), we know that \( \frac{\partial e(\omega)}{\partial \omega} \) is a decreasing function, and since \( c/(y - b) \) is a constant, it has to be that the intersection between \( \frac{\partial e(\omega)}{\partial \omega} \) and \( c/(y - b) \) lies between 0 and 1. As a result, there is a unique \( \omega^* \), which is between 0 and 1.

Second, by differentiating (21), it is straightforward to show that

\[
\frac{\partial \omega^*}{\partial y} > 0, \quad \frac{\partial \omega^*}{\partial b} < 0, \quad \frac{\partial \omega^*}{\partial c} < 0.
\]

Third, by differentiating (21), we have

\[
\frac{\partial \omega^*}{\partial \delta} = \frac{\partial^2 e^*(\omega)}{\partial \omega \partial \delta} \quad \text{and} \quad \frac{\partial \omega^*}{\partial \lambda} = \frac{\partial^2 e^*(\omega)}{\partial \omega \partial \lambda}.
\]
Since $\partial_\omega^2 e^*/\partial^2 \omega < 0$ (see [A11]), the sign of $\partial_\omega^*/\partial \delta$ is the same as of $\partial_\omega^2 e^*(\omega) / \partial^2 \omega \delta$ and the sign of $\partial_\omega^*/\partial \lambda$ is the same as of $\partial_\omega^2 e^*(\omega) / \partial \omega \lambda$. Let us now calculate these cross-derivatives.

By differentiating (15), we have

$$2\lambda \omega \frac{\partial^2 e^*}{\partial \omega} = 2\delta + \lambda \left[ \frac{\lambda + 2\delta(2 - \omega)}{\sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}} \right].$$  \hfill (A13)

Thus, by differentiating again this equation with respect to $\delta$, we obtain

$$2\lambda \omega \frac{\partial^3 e^*}{\partial \omega \delta} = 2 - \frac{2\lambda^2 + 4\lambda \delta(2 - \omega)(1 - \omega)}{|\lambda + 4\delta(1 - \omega)| \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}}$$

$$\Leftrightarrow \lambda \omega \frac{\partial^3 e^*}{\partial \omega \delta} = \frac{|\lambda + 4\delta(1 - \omega)| \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|} + 2\lambda \delta(2 - \omega)(1 - \omega) - \lambda^2}{|\lambda + 4\delta(1 - \omega)| \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}}.$$

Let us show that

$$\lambda \omega^3 \frac{\partial^3 e^*}{\partial^3 \omega} > 0.$$

This is equivalent to

$$\frac{|\lambda + 4\delta(1 - \omega)| \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|} + 2\lambda \delta(2 - \omega)(1 - \omega) - \lambda^2}{|\lambda + 4\delta(1 - \omega)| \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}} > \lambda^2$$

$$\Leftrightarrow \lambda \sqrt{\frac{\lambda + 4\delta(1 - \omega)}{\lambda}} + 4\delta(1 - \omega) \sqrt{\frac{\lambda + 4\delta(1 - \omega)}{\lambda}} + 2\delta(2 - \omega)(1 - \omega) > \lambda.$$

Since

$$\lambda \sqrt{\frac{\lambda + 4\delta(1 - \omega)}{\lambda}} > \lambda \Leftrightarrow \sqrt{1 + \frac{4\delta(1 - \omega)}{\lambda}} > 1$$

is always true, then $\lambda \omega^3 (\partial^3 e^*/\partial \omega \delta) > 0$, and thus $\partial^3 e^*/\partial \omega \delta > 0$. As a result, $\partial_\omega^*/\partial \delta > 0$.

Now, by differentiating (A13) with respect to $\lambda$, we obtain

$$2\lambda^2 \frac{\partial^3 e^*}{\partial \omega \delta} = \frac{4\lambda \delta + 8\delta^2(2 - \omega)(1 - \omega)}{\sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}}$$

$$\Leftrightarrow 2\lambda^2 \frac{\partial^3 e^*}{\partial \omega \delta} = \frac{4\delta \lambda + 8\delta^2(2 - \omega)(1 - \omega) - \delta \lambda + 4\delta(1 - \omega) \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}}{\lambda \sqrt{\lambda|\lambda + 4\delta(1 - \omega)|}}.$$
Let us show that
\[ 4\delta \lambda^4 + 8\delta^2 \lambda^3 (2 - \omega)(1 - \omega) > \delta (\lambda + 4\delta (1 - \omega)) \sqrt{\lambda + 4\delta (1 - \omega)}. \]

This is equivalent to
\[ 16\lambda^7 + 64\lambda^5 (2 - \omega)^2 (1 - \omega)^2 + 64\delta^2 (2 - \omega)(1 - \omega) > [\lambda + 4\delta (1 - \omega)]^3 \]
\[ \Leftrightarrow 16\lambda^7 + 64\lambda^5 \delta^2 (2 - \omega)^2 (1 - \omega)^2 + 64\delta^2 (2 - \omega)(1 - \omega) \]
\[ > \lambda^3 + 12\lambda^2 \delta (1 - \omega) + 48\lambda \delta^2 (1 - \omega)^2 + 64\delta^3 (1 - \omega)^3 \]
\[ > \lambda^3 (\lambda^4 - 1) + 12\lambda \delta (1 - \omega) [(2 - \omega) \lambda^4 - 1] [\lambda + 4\delta (1 - \omega)] \]
\[ + 15\lambda^7 + 52\delta (1 - \omega) \lambda^6 (2 - \omega) + 16\delta (1 - \omega)^2 \lambda^5 \delta (2 - \omega)^2 - 64\delta^3 (1 - \omega)^3 > 0. \]

If \( \lambda \geq 2 \), then all terms are positive but the last one. Observe that (10), that is,
\[ \delta < \frac{\lambda}{2} \left[ \omega + \sqrt{\omega (4 - 3\omega)} \right], \]
implies that (by taking the upper bound of \( \omega \) on the right-hand side),
\[ \delta < \frac{\lambda}{2} \left[ \frac{1 + \sqrt{4}}{2} \right] = \frac{3}{2} \lambda. \]

As a result,
\[ 64\delta^3 (1 - \omega)^3 < 64 \left( \frac{3}{2} \right)^3 \lambda^3 = 216\lambda^3. \]

Since \( \lambda \geq 2 \),
\[ 15\lambda^7 = 15\lambda^4 \lambda^3 \geq 15 \times 2^4 \lambda^3 = 240\lambda^3, \]
and thus the inequality above is always true. As a result, \( 2\omega^2 (\partial^2 e^* / \partial \omega \partial \lambda) > 0 \), and thus \( \partial^2 e^* / \partial \omega \partial \lambda > 0 \) and \( \partial \omega^* / \partial \lambda > 0 \). QED

**Proof of Lemma 1**

By combining (23) to (27), we easily obtain
\[ e^* = [(2 - \omega + \omega e^*) \lambda + \delta] \frac{2(\omega e^* + 1) \lambda d^*_\omega}{\delta^2}. \quad (A14) \]

First, a full-unemployment equilibrium \( \mathcal{U} \) such that \( e^* = 0 \) and \( \nu^* = 1 \) is not anymore possible here. Indeed, if \( e^* = 0 \), then for equation (A1) to be
satisfied it has to be that \( d_0^* = 0 \). Using (23) and (24), this implies that \( d_1^* = d_2^* = 0 \). But, in that case, equation (25) can never be satisfied since it implies that \( 0 = 1/2 \), which is impossible. As a result, a full-unemployment equilibrium \( \mathcal{U} \) cannot exist.

Second, let us see if full-employment equilibrium \( \mathcal{E} \) such that \( e^* = 1 \) and \( u^* = 0 \) can exist. In that case, it has to be that only \( d_2 \)-dyads exist and thus \( d_0^* = d_1^* = 0 \), which, using (23) implies that \( d_2^* = 0 \). So this case is also not possible. QED

Proof of Proposition 6

In the proof of lemma 1, we have shown that the equilibrium employment rate is now given by (A14), which we rewrite for the ease of exposition as

\[
e^* = [(2 - \omega + \omega e^*)\lambda + \delta \frac{2(\omega e^* + 1)\lambda d_0^*}{\delta^2}].
\]  

(A15)

By combining (23), (24), and (25), we easily obtain the equilibrium value of \( d_0^* \) as follows:

\[
2d_0^* = \frac{\delta^2}{\delta^2 + \lambda(\omega e^* + 1)\lambda(2 - \omega + \omega e^*) + 2\delta}.
\]

(A16)

As a result, the equilibrium values of and \( d_0^* \) are determined by equations (A15) and (A16).

Let us first study (A16), which we write as

\[
d_0^*(e) = \frac{\delta^2}{2\delta^2 + \lambda(\omega e + 1)\lambda(2 - \omega + \omega e) + 2\delta}.
\]

We have

\[
d_0^*(0) > 0, \quad \frac{\partial d_0^*}{\partial e} < 0.
\]

Let us now study equation (A15), which can be written as

\[
d_0^*(e) = \frac{e\delta^2}{2\lambda(\omega e + 1)[(2 - \omega + \omega e)\lambda + \delta]}.
\]

We have

\[
d_0^*(0) = 0.
\]
We also have

\[
\frac{2\lambda}{\delta^2} \frac{d_0}{de} = \frac{(\omega + 1)[(2 - \omega + \omega e)\lambda + \delta] - \omega e[(2 - \omega + \omega e)\lambda + \delta] - \omega e(\omega + 1)}{[(\omega + 1)[(2 - \omega + \omega e)\lambda + \delta]]^2}.
\]

Since the denominator is strictly positive, let us focus on the numerator and show that it is also positive. We want to show that

\[
(\omega + 1)[(2 - \omega + \omega e)\lambda + \delta] > \omega e[(2 - \omega + \omega e)\lambda + \delta] + \lambda(\omega e + 1)
\]

\[
\iff (\omega + 1)[(2 - \omega + \omega e)\lambda + \delta] > \omega e[(2 - \omega)\lambda + \delta + 2\lambda e] + \lambda
\]

\[
\iff \omega e[(2 - \omega + \omega e)\lambda + \delta] + (2 - \omega + \omega e)\lambda + \delta > \omega e[(2 - \omega)\lambda + \delta + 2\lambda e] + \lambda
\]

\[
\iff (2 - \omega + \omega e)\lambda + \delta > \lambda(\omega e + 1)
\]

\[
(2 - \omega)\lambda + \delta > \lambda\omega e e^2.
\]

Since \(\lambda\omega e^2 < \lambda\) (both \(\omega\) and \(e\) are less than 1) and since \((2 - \omega)\lambda > \lambda\), this inequality is always true. As a result, \(\frac{d_0}{de} > 0\).

Since the first function always decreases starting from a strictly positive number while the other always increases starting from zero, there is a unique intersection between these two curves and thus a unique equilibrium where \(e^* > 0\) and \(d_0 > 0\). We need now to show that and \(e^* < 1\). From equation (A16), we see straight off that \(d_0' < 1/2\) whenever \(e^* > 0\). Let us now show that \(e^* < 1\). Plugging the value of \(d_0\) from (A16) into equation (A15), we obtain

\[
e^* = \frac{\lambda(\omega e^* + 1)[\lambda(2 - \omega + \omega e^*) + \delta]}{\delta^2 + \lambda(\omega e^* + 1)[\lambda(2 - \omega + \omega e^*) + 2\delta]}.
\]

It is clear from this last equation that, for any \(e > 0\), the equilibrium value of \(e\) has to be less than 1. We can also show it in the following way. By developing this equality, we easily obtain

\[
\lambda^2\omega^2 e^3 + [\lambda(3 - 2\omega) + 2\delta]\lambda\omega e + [\delta^2 + \delta\lambda(2 - \omega) + \lambda^2(2 - \omega)(1 - \omega) - \lambda^2\omega]e
\]

\[
= \lambda^2(2 - \omega) + \delta\lambda.
\]

If we denote by

\[
\Theta(e) = \lambda^2\omega^2 e^3 + [\lambda(3 - 2\omega) + 2\delta]\lambda\omega e + [\delta^2 + \delta\lambda(2 - \omega) + \lambda^2(2 - \omega)(1 - \omega) - \lambda^2\omega]e
\]

\[+ \lambda^2(2 - \omega)(1 - \omega) - \lambda^2\omega]e - (2 - \omega)\lambda^2 - \delta\lambda,
\]
then it is easily verified that $\Theta(0) < 0$. Let us show that $\Theta'(e) > 0$. We have

$$\Theta'(e) = 3\lambda^2 e^2 + 2[\lambda(3-2\omega) + 2\delta]\lambda e + \delta^2 + \delta\lambda(2-\omega) + \lambda^2(2-\omega)(1-\omega) - \lambda^2\omega.$$ 

The only negative term is $-\lambda^2\omega$. Let us show that

$$2[\lambda(3-2\omega)]\lambda e > \lambda^2\omega.$$ 

This is equivalent to

$$e > \frac{1}{2(3-2\omega)}.$$ 

Since $1/[2(3-2\omega)]$ is increasing in $\omega$, let us take the upper bound of $\omega$, which is $\omega = 1$. In that case, this inequality rewrites as $e > 0.5$, which is always true for reasonable value of employment $e$. Thus $\Theta'(e) > 0$. This confirms the uniqueness and positivity of $e$ (because $\Theta(0) < 0$ and $\Theta'(e) > 0$). To show that the solution of this equation is less than 1, that is, that $e^* < 1$, it suffices to show that $\Theta(1) > 0$ (since we know that $\Theta(0) < 0$ and $\Theta'(e) > 0$). This is equivalent to

$$\lambda^2 e^2 + [\lambda(3-2\omega) + 2\delta]\lambda e + \delta^2 + \delta\lambda(2-\omega) + \lambda^2(2-\omega)(1-\omega) - \lambda^2\omega - (2-\omega)e^2 - \delta\lambda > 0,$$

which is always true. Since there is a unique $0 < e^* < 1$ and a unique $0 < d^*_1 < 1$, $d^*_2$ is uniquely determined by (23) and $d^*_1$ by (24). QED

**Proof of Proposition 7**

The equilibrium employment rate is given by (28), or equivalently by the following:\(^{17}\)

$$\Theta(e) = \lambda^2 e^2 + [\lambda(3-2\omega) + 2\delta]\lambda e + \delta^2 + \delta\lambda(2-\omega) + \lambda^2(2-\omega)(1-\omega) - \lambda^2\omega - (2-\omega)e - (2-\omega)\lambda^2 - \delta\lambda.$$ 

By totally differentiating this equation, we obtain

$$\frac{\partial e^*}{\partial \omega} = -\frac{2\lambda e + (3\lambda - 4\lambda e + 2\delta)e^2 - [\delta + 2\lambda(2-\omega)]e + \lambda}{\Theta'(e)}.$$ 

\(^{17}\) See the proof of proposition 6 in the appendix.
We have shown in the proof of proposition 6 that \( \Theta'(e) > 0 \). Thus the sign of \( \partial e^* / \partial \omega \) is the same as the sign of

\[
\Phi(e) \equiv -2\lambda \omega e^3 - (3\lambda - 4\lambda \omega + 2\delta)e^2 + [\delta + 2\lambda(2 - \omega)]e - \lambda,
\]

so that

\[
\frac{\partial e^*}{\partial \omega} \geq 0 \iff \Phi(e) \geq 0.
\]

Let us study the cubic function \( \Phi(e) \). We have \( \Phi(0) < 0 \) and

\[
\Phi(1) = -2\lambda \omega - (3\lambda - 4\lambda \omega + 2\delta) + [\delta + 2\lambda(2 - \omega)] - \lambda = -\delta < 0.
\]

Let us now calculate \( \Phi'(e) \) since its roots will provide the critical points where the slope of the cubic function is zero. We have

\[
\Phi'(e) = -6\lambda \omega e^2 - 2(3\lambda - 4\lambda \omega + 2\delta)e + [2\lambda(2 - \omega) + \delta].
\]

The discriminant of \( \Phi'(e) \) is equal to

\[
\Delta = 4\{(3\lambda - 4\lambda \omega + 2\delta)^2 + 6\lambda \omega [2\lambda(2 - \omega) + \delta]\} > 0.
\]

Since the discriminant \( \Delta > 0 \), the cubic function has a local maximum and a local minimum. It is readily verified that one root of \( \Phi'(e) \) is negative and the other one is positive and given by

\[
e^* = \frac{- (3\lambda - 4\lambda \omega + 2\delta) + \sqrt{(3\lambda - 4\lambda \omega + 2\delta)^2 + 6\lambda \omega [2\lambda(2 - \omega) + \delta]}}{6\lambda \omega} > 0.
\]

Let us show that \( e^* < 1 \). This is equivalent to

\[
\sqrt{(3\lambda - 4\lambda \omega + 2\delta)^2 + 6\lambda \omega [2\lambda(2 - \omega) + \delta]} < 6\lambda \omega + (3\lambda - 4\lambda \omega + 2\delta)
\]

\[
\iff 2\lambda + 3\delta > 0,
\]

which is always true. Figure A1 plots the curve of \( \Phi(e) \), where \( e \) and \( \bar{e} \) are the positive real roots of \( \Phi(e) \). As a result, since \( \Phi(e) \) determines the sign of \( \partial e^* / \partial \omega \), we have that (i) when \( e < e^* < \bar{e} \), then \( \partial e^* / \partial \omega > 0 \); (ii) when \( 0 < e^* < e \) or \( \bar{e} < e^* < 1 \), then \( \partial e^* / \partial \omega < 0 \). QED.
References


Weak and Strong Ties in the Labor Market


