Read each question carefully. Motivate and explain your answers clearly. The number of points for each question is indicated below the question. The maximum number of points is 30.

1. Suppose you are examining the conditional expectation model for $(y, \boldsymbol{x})$, with y a scalar and $\boldsymbol{x}$ a $k \times 1$ vector of random variables:

$$y = \boldsymbol{x}'\beta + e, \mathrm{E}[e|\boldsymbol{x}] = 0, \mathrm{E}[e^2|\boldsymbol{x}] = \sigma^2(\boldsymbol{x}) < \infty.$$

Note that the CEF error is assumed to be a function of $\boldsymbol{x}$ so it is heteroscedastic. You should assume you have access to a sample of n independently and identically distributed observations $(y_i, \boldsymbol{x}_i)_{i=1}^n$ and that $\mathrm{E}[|y|] < \infty$ and $\mathrm{E}[||\boldsymbol{x}||] < \infty$.

(a) Derive the least squares estimator for the parameters β.

(b) Is the estimator unbiased? Is the estimator consistent?

(c) Derive the variance matrix of the least-squares estimator. (It is sufficient to derive the variance matrix conditional on the observed data.)

(6 points)

2. Let the regression model of y on two scalar random variables $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ (ignoring the intercept) be

$$y = \boldsymbol{x}'\beta + e = \beta_1\boldsymbol{x}_1 + \beta_2\boldsymbol{x}_2 + e,\ \mathrm{E}[\boldsymbol{x}e] = 0,\ \mathrm{E}[e^2|\boldsymbol{x}] = \sigma^2 < \infty.$$

You are interested in testing the hypothesis

$$H_0 : \beta_1 + \beta_2 = 1.$$

Derive the test-statistic for testing the hypothesis H_0. What is the appropriate reference distribution for the test statistic?

(6 points)

3. You are interested in testing the hypothesis H_0 that the coefficient $\beta_1 = 0$ in the linear regression

$$y = \beta_1\boldsymbol{x}_1 + \beta_2\boldsymbol{x}_2 + \beta_3\boldsymbol{x}_3 + \beta_4 + e,$$

where $(y, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$ are scalar random variables. However, your computational capacity is constrained so you can only estimate regressions with one explanatory variable and an intercept at a time. Discuss whether or not it is possible case to test the hypothesis H_0.

(6 points)

4. Let the regression equation of interest and the population moment be

$$y = \boldsymbol{x}'\beta + e,\ \mathrm{E}[\boldsymbol{z}e] = 0,$$

where $(y, \boldsymbol{x}, \boldsymbol{z})$ are 1×1, $k \times 1$ and $l \times 1$, $l > k$ random variables for which we have an iid random sample. The regression error e is heteroscedastic, i.e., $\mathrm{E}[e^2|\boldsymbol{x}] = \sigma^2(\boldsymbol{x})$. $\boldsymbol{z}$ consists of valid instruments. We want to estimate β using generalized method of moments.

(a) Construct the sample analog of the moment condition that is used for estimation and formulate the quadratic function of the sample analog of the moment condition that is used to solve for the estimator. Motivate your choice of weight matrix.

(b) Derive the estimator from the minimization problem. What need you to assume about the variables to ensure that the estimator is consistent?

(c) Derive the asymptotic variance of the estimator. What do you need to assume to be able to estimate the variance consistently?

(6 points)

5. Let the univariate time series process be

$$y_t = \mu + \beta y_{t-1} + e_t;\ t = 1, \ldots, T; \mathrm{E}[e_t] = 0; \mathrm{E}[e^2] = \sigma^2; |\beta| \leq 1; \mathrm{E}[e_t e_s] = 0 \forall t \neq s.$$

You can assume $Y_0 = \mu$.

(a) What is the expectation, variance, and autocorrelation function of y_t?

(b) Is this process weakly stationary? Is this process ergodic?

(6 points)