Intergenerational Correlation and Social Interactions in Education

Sebastian Bervoets† Yves Zenou‡

January 28, 2016

Abstract

We propose a dynastic model where individuals are born in an educated or uneducated environment that they inherit from their parents. We study the impact of social interactions on the correlation in the parent-child educational status, independently of any parent-child interaction. When the level of social interactions is decided by a social planner, we show that the correlation in education status between generations decreases very fast as social interactions increase. In turn, when the level of social interactions is decided by the individuals themselves, we show that the intergenerational correlation still decreases, although less rapidly than with exogenous social interactions.

Key words: Social mobility, strong and weak ties, intergenerational correlation, education.

1 Introduction

Explaining the educational outcomes of children is one of the most challenging questions faced by economists. Most studies have found that school quality (e.g., Card and Krueger, 1992) and family background (e.g., Ermisch and Francesconi, 2001, Plug and Vijverberg, 2003) have a significant and positive impact on the level of education of children. Parents obviously influence their children’s school performance by transmitting their genes to their children, but also influence them directly, via, for example, their parenting practices and the type of schools to which they send their children (Björklund et al., 2006; Björklund and Salvanes, 2011). Neighborhood and peer effects have also an important impact on educational outcomes of children (Durlauf, 2004; Ioannides and Topa, 2010; Sacerdote 2010; Patacchini and Zenou, 2011; Topa and Zenou, 2015).

We study the intergenerational relationship between parents’ and offspring’s long-run educational outcomes. It is well-documented that the students’s educational achievement is positively correlated with their parents’ education or with other indicators of their parents’ socio-economic status (Björklund and Salvanes, 2011). In the present paper, we illustrate how this correlation can result from peer effects, abstracting from any direct parental influence. We also analyze how variations in social interactions translate into variations in intergenerational correlation.

There have been many attempts in the literature analyzing how intergenerational correlation could be reduced, by focusing on the direct influence that parents have on their children (Björklund and Salvanes, 2011). However, in their paper, Calvó-Armengol and Jackson (2009) show that the correlation between the parent’s and the child’s outcomes can be explained without any direct influence of the parent on the child, but by considering that they share a common environment, which affects their decisions. This provides us with a new channel of intervention that reduces the parent-child correlation, which we want to examine in this paper.

We develop a dynastic model where, at each period of time, with some probability, a person (the parent) dies and is simultaneously replaced by a new born (the child). The child thereby never interacts with his parent and does not inherit any of his idiosyncratic characteristic. Instead, newborns inherit the environment (local community) where their
parents lived in. In our paper, the environment is modelled as a *dyad* in which the newborn interacts with a partner, called his strong tie. This strong tie represents the environment with who the parent interacted before he died. Using the language of the cultural transmission literature (Bisin and Verdier, 2000, 2001), in our model, there is no vertical transmission (i.e. socialization inside the family) but only horizontal transmission (i.e. socialization outside the family).

In our model, when individuals are born, they discover the environment they face, i.e. the educational status of their strong tie, and then decide whether to get educated or not.

We start with a benchmark model where newborns only interact with their strong tie. We show that, even if a parent never interacts with his child, there is still a significant positive correlation between the educational achievement of the father and the son. Indeed, a parent has a higher (lower) probability of being educated if he lived in a favorable (unfavorable) environment. Because the child shares the same environment, the probability that the child will get educated is also higher (lower). This benchmark model allows us to derive simple expressions for both the average level of education at steady state and the level of intergenerational correlation.

We then extend this model to introduce the possibility for individuals to interact with peers outside their local community (*weak ties*). Since individuals start interacting with weak ties, strong ties have a lower influence on their education and this mechanically decreases the parent-child correlation. We model social mixing as the fraction of time a newborn spends with weak ties and consider two cases: first, the level of social mixing is exogenously fixed by a social planner and, second, it is decided by the newborns themselves. In both cases, we show that peer effects, defined here as the interactions with both types of ties, have a great importance in terms of public policies that aim at reducing social inertia.

Indeed, when the socializing decisions are exogenously chosen by the social planner, we show that the correlation in education status between generations decreases very fast as social interactions increase. More precisely, the decrease in correlation is a power four of the increase in social interactions. Hence, a social planner who promotes social mobility will choose to mix individuals as much as possible. By doing so, we also show that the average level of education does not change compared to the benchmark case. However, this
policy decreases social welfare. This is because, while individuals born in an unfavorable environment benefit from such a policy, those born in a favorable environment are penalized, as they now interact with potentially uneducated individuals. The net effect turns out to be negative since the losses of the former are not totally compensated by the gains of the latter. This illustrates a common trade-off faced by a social planner between equity (i.e. decreasing intergenerational correlation) and efficiency.

When the socializing decisions are chosen by the individuals, those with uneducated strong ties always want to meet weak ties while the reverse occurs for individuals with educated strong ties. This is simply because the former will, in the worse case, meet another uneducated person and, at best, meet someone educated, and the reverse will happen for the latter. We show that when individuals can escape their inherited environment, the intergenerational correlation still decreases with respect to the benchmark case, but the extent to which it decreases depends on the average level of education in the population. The higher this share, the higher the impact of social mixing.

We finally show that when socializing decisions are chosen by the individuals, the average education level at steady state increases, contrary to the case where interaction choices are exogenous. Therefore, a social planner who lets individuals choose their own social-mixing levels should promote education. This has indeed two effects: the direct effect of increasing the education level in the population, and the indirect effect of decreasing the intergenerational correlation through the first effect.

The rest of the paper unfolds as follows. In the next section, we relate our model to the relevant theoretical literatures. Section 3 exposes the model without social mixing while Section 4 focuses on exogenous levels of social mixing. In Section 5, the level of social mixing is endogenous since individuals choose how much time they spend with weak and strong ties. Finally, Section 6 concludes. All proofs of propositions, lemmas and remarks can be found in Appendix 1.
2 Related literature

Apart from the paper by Calvó-Armengol and Jackson (2009),¹ our model is related to different literatures. First, it is related to the literature on peer effects in education. De Bartolome (1990) and Benabou (1993) are the standard references for peer and neighborhood effects in education. In this multi-community approach, individuals can acquire high or low skills or be unemployed. The costs of acquiring skills are decreasing in the proportion of the community that is highly skilled but this decrease is larger for those acquiring high skills. This leads to sorting although ex ante all individuals are identical. While there is an extensive empirical literature on the intergenerational transmission of income and education that focuses on the correlation of parental and children’s permanent income or education (Björklund and Jäntti, 2009; Black and Devereux, 2011; Björklund and Salvanes, 2011), there are very few theoretical models studying this issue. Ioannides (2002, 2003) analyses the intergenerational transmission of human capital by explicitly developing a dynamic model of human capital formation with a neighborhood selection. The idea here is to study the impact of parental education and the distribution of educational attainment within a relevant neighborhood on child educational attainment. From a theoretical viewpoint, Ioannides obtains a complete characterization of the properties of the intertemporal evolution of human capital. From an empirical viewpoint, he finds that there are strong neighboring effects in the transmission of human capital and that parents’ education and neighbors’ education have non linear effects that are consistent with the theory.

Using a cultural transmission model à la Bisin and Verdier (2000, 2001), Patachini and Zenou (2011) analyze the intergenerational transmission of education focusing on the interplay between family and neighborhood effects. They develop a theoretical model suggesting that both neighborhood quality and parental effort are of importance for the education attained by children. Their model proposes a mechanism explaining why and how they are of importance, distinguishing between high- and low-educated parents. Empirically, they find that the better is the quality of the neighborhood, the higher is the parents’ involvement in their children’s education.

¹Contrary to their paper, we consider the impact of social interactions through weak and strong ties on the intergenerational correlation in education.
Second, our paper is also related to the social network literature. There is a growing interest in theoretical models of peer effects and social networks (see e.g. Ballester et al., 2006; Calvó-Armengol et al., 2009; Jackson, 2008; Jackson and Zenou, 2015; Ioannides, 2012). There is, however, nearly no theoretical model that looks at the impact of social networks on the intergenerational transmission of education. In the present paper, we model the network as the interaction between strong and weak ties. In his seminal contributions, Granovetter (1973, 1974, 1983) defines weak ties in terms of lack of overlap in personal networks between any two agents, i.e. weak ties refer to a network of acquaintances who are less likely to be socially involved with one another. Formally, two agents A and B have a weak tie if there is little or no overlap between their respective personal networks. Vice versa, the tie is strong if most of A’s contacts also appear in B’s network. In this context, Granovetter (1973, 1974, 1983) develops the idea that weak ties are superior to strong ties for providing support in getting a job. \(^2\) In our model, we stress the role of strong ties as an important mean for the transmission of education. In other words, even though there is no direct influence from the parents, their indirect influence through the inheritance of strong ties affects positively the correlation between the parent and the child.

3 The benchmark model without social interactions

3.1 Model

There are \(n\) individuals in the economy.\(^3\) We assume that individuals belong to mutually exclusive two-person groups, referred to as dyads. We say that two individuals belonging to the same dyad hold a strong tie to each other. We assume that dyad members do not change over time unless one of them dies. A strong tie is created once and for all and can never be broken. Thus, we can think of strong ties as links between members of the same family, or between very close friends. In this section and only here, we assume that different dyads do

\(^2\)For other models on weak and strong ties, see Montgomery (1994), Calvó-Armengol et al. (2007), Sato and Zenou (2015) and Zenou (2013, 2015)

\(^3\)We assume throughout that \(n\) is large, and all the propositions in the paper should be understood as limiting propositions
not interact.

We consider a dynamic model, where, at each period, each individual in the dyad can die with probability $1/n$. When a person dies, he is automatically replaced by a new born who is his child. The child is then matched with the individual who was previously in the same dyad (strong tie) as his parent. The only aspect that the son inherits from his father is his father’s social environment or local community, here the father’s strong tie. There is no other interaction between the father and the son. In particular, the father and the son never live at the same time. This is because we want to analyze the effect of the environment (peer effects) on the child’s education outcomes, independent of any parent-child interaction.

When individual $i$ is born, he discovers the type of his strong tie: $j = 0$ (non-educated) or $j = 1$ (educated). He also discovers his own idiosyncratic ability for education, given by some $\lambda_i$ randomly drawn for a uniform distribution defined on $[0, 1]$. Education is costly and the return to education effort depends both on the individual’s ability to learn and on the type of the strong tie. Lastly, individuals who do not get educated are guaranteed a minimal wage from which they derive utility $\bar{U}$. The utility $U_{ij}$ of individual $i$ with strong tie $j$, exerting effort $e_{ij}$, is given by:

$$U_{i0} (\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - \alpha e_{i0}$$

$$U_{i1} (\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2$$

where $\alpha > 0$ is the penalty incurred when living in an unfavorable environment (i.e. being born with a low-educated strong tie). Each newborn decides how much effort he puts in education. First-order conditions yield:

$$e_{i0} = \max\{0, \lambda_i - \alpha\}$$

$$e_{i1} = \lambda_i$$

Quite naturally, individual $i$ will be educated if $U_{ij}(\lambda_i) > \bar{U}$. This provides us with a threshold level $\tilde{\lambda}_0$ (resp. $\tilde{\lambda}_1$), such that individuals with an uneducated (resp. educated) strong tie and with ability above $\tilde{\lambda}_0$ (resp. $\tilde{\lambda}_1$) will get educated, while those with an ability lower than $\tilde{\lambda}_0$ (resp. $\tilde{\lambda}_1$) will not. These two thresholds are defined as:

$$\tilde{\lambda}_0 = \sqrt{2\bar{U}} + \alpha$$
\[
\tilde{\lambda}_1 = \sqrt{2U}
\]

We assume that \( U \) and \( \alpha \) are such that
\[
0 < \sqrt{2U} < \sqrt{2U} + \alpha < 1
\]

Plugging each effort into each utility function, we obtain:
\[
U^*_0(\lambda_i) = \max \left\{ U, \frac{(\lambda_i - \alpha)^2}{2} \right\}
\]
\[
U^*_1(\lambda_i) = \max \left\{ U, \frac{\lambda_i^2}{2} \right\}
\]

As a result, the probability \( p_0 \) (resp. \( p_1 \)) that an individual with an uneducated (resp. educated) strong tie will be educated is given by:
\[
p_0 = 1 - \tilde{\lambda}_0 = 1 - \sqrt{2U} - \alpha \quad (1)
\]
\[
p_1 = 1 - \tilde{\lambda}_1 = 1 - \sqrt{2U} \quad (2)
\]

These probabilities can be understood as the proportion of individuals that will be educated. Figure 1 summarizes how education choices are made.

![Figure 1: The different probabilities of being educated without social networks](image-url)
3.2 Steady-state equilibrium

So far, we have described what happens within a period. Let us now explain the dynamics of the model and determine the steady-state equilibrium. In equilibrium, the share $\eta_1$ of educated individuals is given by:

$$ \eta_1 = 1 - \tilde{\lambda}_0 + (\tilde{\lambda}_0 - \tilde{\lambda}_1)\eta_1 = p_0 + (p_1 - p_0)\eta_1 $$

Indeed, the fraction of educated individuals are either those who have an ability between $\tilde{\lambda}_0$ and 1 since they will be educated whatever the status of their partner (see Figure 1) or those who have an ability between $\tilde{\lambda}_1$ and $\tilde{\lambda}_0$ and who are matched with an educated partner (this happens with probability $\eta_1$). Rearranging this expression, we obtain:

$$ \eta_1 = \frac{p_0}{1 + p_0 - p_1} $$

Using (1) and (2), we get:

$$ \eta_1^{dyad} \equiv \eta_1 = \frac{1 - \sqrt{2\bar{U}} - \alpha}{1 - \alpha} \quad (3) $$

Of course, when $\bar{U} = 0$ (i.e. there is no outside option) then everyone will prefer being educated, while if $\sqrt{2\bar{U}} = 1 - \alpha$ (i.e. $\bar{U} = \frac{(1-\alpha)^2}{2}$ which is the highest possible payoff for obtaining an education), then no one will become educated.

3.3 The correlation in education between parents and children

We would now like to calculate the intergenerational correlation in education between parents and children. Though they do not interact with each other, there is a correlation that goes through the social network (i.e. strong tie) the parent “transmits” to his child. Let $X$ refer to the education status of the parent and $Y$ to the education status of the child. The intergenerational correlation is given by:

$$ Cor(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} $$

where $Cov(X, Y)$ is the covariance between the educational status of the parent and the child while $Var(X)$ and $Var(Y)$ are the variances of the statuses of the parent and the child. We have:

$$ Cov(X, Y)_{dyad} = \mathbb{E}[(X = 1)(Y = 1)] - [\mathbb{E}(X = 1)][\mathbb{E}(Y = 1)] = \eta_{11} - \eta_1^2 $$
\[
\text{Var}(X)_{\text{dyad}} = \mathbb{E}[X = 1] - [\mathbb{E}(X = 1)]^2 = \eta_1 - \eta_1^2
\]

where \(\eta_1\) is the marginal probability that an individual chooses state 1, i.e. the probability of being educated in steady state (it is given by (3)), and \(\eta_{11}\), the joint probability that an individual is in state 1 and his father was in state 1.

Individuals can be in either of two different states: educated (state 1) and uneducated (state 0). Dyads, which consist of paired individuals, are, in steady state, in one of three different states:

(i) both members are educated (11);

(ii) one member is educated and the other is not (01) or (10);

(iii) both members are uneducated (00).

The steady state distribution of dyads is given by \(\mu = \{\mu_{00}, \mu_{10}, \mu_{01}, \mu_{11}\}\), where \(\mu_{ij}\) stands for the fraction of dyads in state \((ij)^{\text{4}}\). Obviously, by symmetry, \(\mu_{10} = \mu_{01}\). We obtain the following result:

**Proposition 1** When dyads do not interact with each other, the parent-child correlation is equal to:

\[
\text{Cor}_{\text{dyad}} = \alpha^2
\]

Thus, the correlation between a father’s and a son’s education status is positive, even though they never interact with each other. The intuition is simple: when the strong tie is educated, the father is more likely to be educated. This is also true for the son who benefits from a favorable environment and, as a result, his chances to be educated are higher.

In our setting, the correlation (4) increases with \(\alpha\), the cost of interacting with an uneducated strong tie. Indeed, the difference in individual efforts \((e_{1i} - e_{0i})\) between meeting an educated and an uneducated strong tie is equal to \(\alpha\). If this difference is small, it is almost the same to be matched with an educated or a non-educated partner and newborns decide whether to become educated or not independently of the status of their strong tie. If this difference is large, individuals’ decisions strongly depend on the status of their partner.

---

4 Alternatively, \(\mu_{ij}\) can be interpreted as the fraction of time a typical dyad spends in state \((ij)\)
Observe that the quadratic form of the correlation is due to the pattern of influences between parents and children, which transit through the community. In some sense, $\alpha$ measures the intensity of the peer effects. In order for the correlation to exist, it is necessary that the father is subject to this peer effect and that the son is also subject to this peer effect. This “two-step” mechanism explains why $\alpha$ appears in a square in (4).

Observe also that $\alpha = p_1 - p_0$ so that $Cor_{dyad} = (p_1 - p_0)^2$. In terms of interpretation, $p_1$ can be seen as the probability of acting in the same way as an educated dyad partner, while $p_0$ is the probability of choosing the opposite of an uneducated dyad partner. For instance, if $p_1 = 1$ and $p_0 = 0$, which means that individuals always act according to their strong tie, then a father and a son sharing the same strong tie will necessarily act the same way (in terms of education) and $Cor_{dyad} = 1$. On the contrary, if $p_1 = p_0$, then individuals act as often in the same way and in the opposite way as their strong tie so that no parent-child correlation, i.e. $Cor_{dyad} = 0$.

This benchmark case illustrates, in a very simple model, how positive correlation can appear as a result of indirect transmission of behavior through peers, as pointed out by Calvó-Armengol and Jackson (2009). However, individuals usually interact with people outside of their local community and this might have a significant impact on this correlation. We explore this in the next section by introducing the role of weak ties in the parent-child correlation.

4 Exogenous social interactions

We assume that individuals are exogenously forced to interact with people outside their own local community. A newborn will spend a fraction $\omega$ of his time with weak ties and a fraction $1 - \omega$ of his time with his strong tie.$^5$

\hspace{1cm} $^5$Observe that strong ties and weak ties are assumed to be substitutes, i.e. the more someone spends time with weak ties, the less he has time to spend with his strong tie.
4.1 Model

The utility of an individual $i$ whose strong tie is uneducated is now given by:\footnote{The following expressions should be understood as expected utilities.}

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - \omega (1 - \eta_1) \alpha e_{i0} - (1 - \omega) \alpha e_{i0}$$

$$= \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - (1 - \omega \eta_1) \alpha e_{i0}$$

where $\eta_1$ is the share of educated weak ties in steady state. Individual $i$, who is born with an uneducated strong tie, spends a fraction $\omega$ of his time with a weak tie. This weak tie can either be uneducated (with probability $1 - \eta_1$), in which case he bears a penalty of $\alpha$ per unit of effort, or educated (with probability $\eta_1$), in which case he does not suffer from any negative peer effect. The rest of his time $(1 - \omega)$ is spent with the uneducated strong tie.

Similarly, the utility of an individual $i$ who is born with an educated strong tie is given by:

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2 - \omega (1 - \eta_1) \alpha e_{i1}$$

Proceeding as in the previous section, in Appendix 1, we show that

$$p_0 = 1 - \sqrt{2U} - (1 - \omega \eta_1) \alpha$$

(5)

$$p_1 = 1 - \sqrt{2U} - (1 - \eta_1) \omega \alpha$$

(6)

and

$$\eta_1 = \frac{p_0}{1 + p_0 - p_1} = \frac{1 - \alpha - \sqrt{2U}}{1 - \alpha}$$

(7)

Thus, $0 \leq \eta_1 \leq 1$ if

$$0 \leq \sqrt{2U} \leq 1 - \alpha$$

(8)

which also guarantees that $p_0$ and $p_1$ are between 0 and 1.

Looking at (7), it is easily verified that the individual probability of being employed, $\eta_1$, is increasing in both $p_0$ and $p_1$ and decreasing in $\alpha$. Furthermore, $p_0$ is increasing in the time spent with weak ties, $\omega$, while $p_1$ is decreasing with $\omega$. Finally, $\eta_1$, $p_0$ and $p_1$ are all decreasing in $U$. 
Note that $\eta_1$, the average level of education in the population, is the same as in the previous section and thus given by $\eta_1^{\text{dyad}}$ (see (3)). A larger share of those individuals who are born in an unfavorable environment will become educated because they bear a lower cost due to the time spent outside their own community. But conversely, a smaller share of those individuals who are born in a favorable environment will be educated because they will meet some uneducated weak ties. The two effects cancel out.

4.2 Steady-state equilibrium and intergenerational correlation

We are now able to determine the intergenerational correlation. We have:

**Proposition 2** Assume (8). With exogenous social mixing, the parent-child correlation is equal to:

$$\text{Cor}_{\text{exo}} = (1 - \omega)^4 \alpha^2$$

Observe that the correlation (9) can also be expressed as $(1 - \omega)^2(p_1 - p_0)^2$ where $p_1 - p_0$ can be written as $(p_1 - \eta_1) - (p_0 - \eta_1)$. As a result, the correlation measures the bias in the probability of being educated induced by the chance of having an educated strong tie. Said differently, it is the difference between the conditional probability and the overall probability of being educated. Contrary to the previous section, these biases only appear as long as individuals interact within their dyad, which happens in proportion $1 - \omega$.

The quadratic form in (9) appears for the same reason as before: parents are influenced by their environment, who, in turn, influences the children. Note, however, that the social mixing acts through two channels. First, for a given $\alpha$, it decreases the difference between $p_1$ and $p_0$ of a factor of $1 - \omega$ because education decisions depend less on the status of the strong tie. Second, the impact of the first channel only occurs for a fraction of $1 - \omega$. The overall effect of social mixing is multiplicative in the effects of both these channels, hence the factor $(1 - \omega)^4$.

When $\omega$ increases, which means that the penalty of meeting someone who is uneducated increases, the correlation increases because the influence of strong ties is higher. This induces an increase in correlation, which takes a quadratic form. On the contrary, when $\omega$ increases,
the correlation is naturally reduced because individuals are more influenced by their weak ties than their strong ties.

This is an interesting result from a policy viewpoint. If the planner wants to encourage social mobility (i.e. decrease intergenerational correlation), he can either decrease $\alpha$ or increase $\omega$. Our simple model shows that encouraging social mixing is more powerful. For instance, policies such as the Moving to Opportunity (MTO) programs,\textsuperscript{7} might have very large effects in the long-run. As an illustration, switching from $\omega = 0$ to $\omega = 0.2$ decreases the intergenerational correlation by about 60%.

Another illustration of a social mixing policy is to force children to spend more time at school, where social mixing occurs, by prolonging schooling at the lower secondary level. Several recent papers have analyzed the impact of these school reforms, particularly in Europe, on aspects related to the persistence of education across generations. Meghir and Palme (2005) and Aakvik et al. (2010) analyze the effect on earnings and educational attainment of the comprehensive school reforms that took place in Sweden in the 1950s and Norway in the 1960s, respectively, where mandatory schooling was extended by two years and all students had to attend the same track. Both studies find support for a weakening of the effect of family background for disadvantaged pupils with parents with low educational attainment.

On the downside, observe that, while individuals with uneducated strong ties are favored by such policies, those with educated strong ties are reluctant to spend time outside their community. This is related to the standard policy debate on desegregation (Guryan, 2004; Rivkin and Welch, 2006) where mixing students of different backgrounds favors the less educated ones but has a negative effect on the more educated students.\textsuperscript{8}

\textsuperscript{7}By giving housing assistance to low-income families, the MTO programs help them relocate to better and richer neighborhoods. The results of most MTO programs (in particular for Baltimore, Boston, Chicago, Los Angeles and New York) show a clear improvement of the well-being of participants and better labor market outcomes (see, in particular, Ladd and Ludwig, 2001, Katz et al., 2001, Rosenbaum and Harris, 2001).

\textsuperscript{8}See Sáez-Martí and Zenou (2012) who obtain a similar result using a model of cultural transmission and statistical discrimination.
4.3 Welfare analysis

As stated above, an increase in $\omega$ has a positive effect on low-educated individuals but can be harmful to high-educated individuals. Because of this trade-off, we would like now to study the welfare consequences of this effect. The total welfare is equal to

$$W = \int_0^{\bar{\lambda}_1} U d\lambda + \int_{\bar{\lambda}_1}^{\bar{\lambda}_0} (1 - \eta_1)U d\lambda + \int_{\bar{\lambda}_1}^{1} \eta_1 U_{i1} (\lambda) d\lambda + \int_{\bar{\lambda}_0}^{1} (1 - \eta_1) U_{i0} (\lambda) d\lambda \tag{10}$$

The social planner can have two objectives. First, he might want to maximize the sum of utilities (10) of all agents. Second, he might want to minimize the impact of family background on the child’s educational attainment, a policy that has been adopted by most democratic societies (Björklund and Salvanes, 2011). However, these two objectives are contradictory.

**Proposition 3**

(i) If the objective of the planner is to maximize total welfare (10), then it is optimal to set the time spent with weak ties to $\omega^* = 0$.

(ii) If the objective is to minimize the intergenerational correlation, then it is optimal to set the time spent with weak ties to $\omega^* = 1$.

This proposition shows that, depending on the objective function, the efficient outcome may be very different. Indeed, when maximizing total welfare (case (i) in the proposition), the planner accounts for both the positive effect on individuals from unfavorable environments and the negative effect on students from favorable environments. Because the latter effect is weaker than the former, the loss of utility of a person with an educated strong tie meeting weak ties is not sufficiently compensated by the gain of utility of a person with an uneducated strong tie meeting educated weak ties. As a result, the planner finds it optimal to set $\omega^* = 0$. When the planner wants to minimize the intergenerational correlation (case (ii)), he does not want people to be stuck in their initial environment of strong ties and thus chooses $\omega^* = 1$.

**Remark 1** Both the aggregate welfare and the correlation are decreasing and convex. Thus increasing $\omega$ decreases both the correlation and the welfare very quickly. There is no room for “intermediate” policies.
5 Endogenous social interactions

We now endogeneize $\omega$ so that individuals choose both educational effort and the time spent with their strong (or weak) tie. The timing is now as follows. At each period of time, a person (the father) chosen at random dies and is replaced by a newborn (the son) who takes his place in the dyad. The son then discovers the type of his strong tie (educated or not educated) as well as his $\lambda_i$. He then optimally decides $\omega_{ij}$, the time spent with weak and strong ties and then $e_{ij}$ the optimal education effort level. As usual, we solve the model backward.

5.1 Model

The utility of individual $i$ who chooses $\omega_{ij}$ and $e_{ij}$ is now given by:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - (1 - \omega_{i0}\eta_1) \alpha e_{i0}$$

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2 - \omega_{i1} (1 - \eta_1) \alpha e_{i1}$$

Denote:

$$\bar{\lambda}_0 \equiv \sqrt{2U} + (1 - \eta_1) \alpha \quad \text{and} \quad \bar{\lambda}_1 \equiv \sqrt{2U}$$

Proposition 4

(i) For individuals who inherited an uneducated strong tie from their father, their choice of meeting weak ties depend on their initial ability $\lambda_i$. If $\lambda_i < \bar{\lambda}_0$, they choose to never meet weak ties, i.e. $\omega^*_{i0} = 0$, while those for which $\lambda_i \geq \bar{\lambda}_0$ always want to meet weak ties $\omega^*_{i0} = 1$.

(ii) Individuals who inherited an educated strong tie from their father never want to meet weak ties, i.e. $\omega^*_{i1} = 0$.

This result is quite intuitive and is in line with what should be expected: individuals born in a favorable environment do not want to interact with weak ties who are potentially uneducated. On the contrary, individuals born in an unfavorable environment, with an initial ability that is high enough want to spend as much time as possible outside their community.
to avoid the penalty $\alpha$. Observe that, if we introduce a cost of socialization, $-\frac{1}{2} \omega^2$, in the utility function, then we will obtain interior instead of $(0, 1)$ solutions for the choice of weak ties but the results and intuitions are mainly unchanged (see Appendix 2).

5.2 Steady-state equilibrium and intergenerational correlation

Proceeding as in the previous sections, we show in Appendix 1 that $\eta_1$ is the solution to

$$F(\eta_1) \equiv \alpha \eta_1^2 + \eta_1 (1 - 2\alpha) - 1 + \sqrt{2U} + \alpha = 0 \quad (12)$$

**Proposition 5** If $\sqrt{2U} < 1 - \alpha$, there exists a unique solution $\eta_1^* \in [0, 1]$ to (12). It is such that

$$\eta_1^* > \eta_1^{\text{dyad}} = \frac{1 - \sqrt{2U} - \alpha}{1 - \alpha}$$

Furthermore,

$$\frac{\partial \eta_1^*}{\partial U} < 0 \quad \text{and} \quad \frac{\partial \eta_1^*}{\partial \alpha} < 0$$

The average level of education in the population is larger in the model with endogenous choices of interactions than in the previous models where $\eta_1 \equiv \eta_1^{\text{dyad}}$ was defined by (3). It is higher than in the model without social interactions (benchmark model of Section 3) because the individuals with uneducated strong ties can get out and reduce the penalty $\alpha$ of the negative peer effect. It is also higher than in the model with exogenous social interactions (Section 4) because the individuals with educated strong ties do not suffer from the negative peer effect imposed by the planner.

**Proposition 6** Assume $\sqrt{2U} < 1 - \alpha$. Then the correlation between the educational status of the father and the son is equal to:

$$Cor_{net} = (1 - \eta_1^*)^2 \alpha^2 \quad (13)$$

Furthermore,

$$\frac{\partial Cor_{net}}{\partial U} > 0 \quad \text{and} \quad \frac{\partial Cor_{net}}{\partial \alpha} > 0$$
Contrary to the previous sections, the correlation $\text{Cor}_{\text{net}}$ (positively) depends on $\overline{U}$. This is due to the fact that a change in $\overline{U}$ affects both $p_0$ and $p_1$. In Sections 3 and 4 both probabilities were affected in the same way and cancelled out in the quantity $p_1 - p_0$. Here, it is no longer the case because of the asymmetry in behavior of individuals, depending on the status of their strong tie.

When interactions are chosen by the individuals, there is also a reduction in intergenerational correlation, but it is less important than when social mixing is imposed. However, it is worth noting that when the planner implements a policy reducing the cost $\alpha$ of unfavorable environments, this has two effects. First, it directly reduces the correlation through the term $\alpha^2$. Second, it also reduces it indirectly through the increase of $\eta_1^*$ that it triggers.

6 Conclusion

In this paper, we developed a dynastic model where, at each period of time, with some probability, a person (the parent) dies and is replaced by a new born (the child). The new born takes exactly the same position as the father in the dyad and thus interacts with the same person (strong tie), i.e. the local community of his father. There is therefore no vertical transmission but only horizontal transmission via peer and neighborhood effects.

We show that there is a substantial intergenerational correlation between parent and child outcomes that transits through the environment that both the parent and the child share. While policies aiming at increasing social mobility usually rely on the individuals’ idiosyncratic characteristics, this model provides us with an alternative channel for public interventions.

In this very simple framework, we analyze the impact of social interactions on the intergenerational correlation in education and find that it is a very powerful tool for promoting social mobility. When the level of social interactions is centrally decided, we show that the correlation decreases very fast while the average education level remains constant. The prize to pay for this very rapid decrease is that welfare also goes down fast when the level of social interactions increases. When the level of social interactions is decided individually, the correlation decreases, although less rapidly than with exogenous social interactions. In
turn, the average education level increases.

We believe that this paper sheds some light on the effect of the inherited neighborhood and peers on children’s education outcomes. There is a small, but growing literature that considers the impact of ‘initial conditions’ in determining labor market outcomes (see e.g. Ashlund and Rooth, 2007; Almond and Currie, 2011). Recent research has also showed the importance of the birthplace on long-run outcomes (Bosquet and Overman, 2016) and puts forward the role of the geography of intergenerational mobility (Chetty et al., 2016; Chetty and Hendren, 2015; Del Bello et al., 2015) but does not distinguish between direct parental and social influences on education. By ignoring the former and focusing solely on the latter, our model provides some predictions that allow one to understand the impacts of a change in the social environment on education.

References


21


APPENDIX 1: Proofs

**Proof of Proposition 1:** Since \( \eta_1 \) is determined by (3), we need to derive the joint probability \( \eta_{11} \). We get

\[
\eta_{11} = p_1 \mu_{11} + \frac{1}{2} p_0 \mu_{01} + \frac{1}{2} p_0 \mu_{10}
\]

Indeed, in order to have both a newborn and his father in state 1, it is necessary that the father was in state 1 in his dyad. This happens with probability 1 if the individual randomly chosen to die was in a dyad in state \((11)\), and there is a proportion \( \mu_{11} \) of these dyads, or with probability 1/2 if the individual was in a \((01)\) or \((10)\) dyad. These dyads are in proportion \( \mu_{01} \) and \( \mu_{10} \).

In the first case, the son inherits an educated strong tie, he then gets educated with probability \( p_1 \). In the second case, the son inherits an uneducated strong tie, he then gets educated with probability \( p_0 \).

Next, observe that

\[
\mu_{11} = p_1 \eta_1
\]

Indeed, for a new dyad to be in state \((11)\), it has to be that a newborn is born in a dyad with an educated strong tie (with probability \( \eta_1 \)) and gets educated (with probability \( p_1 \)).

Using a similar argument, we also have:

\[
\mu_{10} = \mu_{01} = \frac{1}{2}(1 - p_1) \eta_1 + \frac{1}{2} p_0 (1 - \eta_1) = (1 - p_1) \eta_1
\]

and

\[
\mu_{00} = (1 - p_0)(1 - \eta_1)
\]

From these three expressions, we obtain:

\[
\eta_{11} = p_1 \mu_{11} + p_0 \mu_{01}
\]

\[
= \eta_1 \left[ p_1^2 + p_0 (1 - p_1) \right]
\]

Finally, we have:

\[
Cor(X, Y)_{dyad} = \frac{\eta_{11} - \eta_1^2}{\eta_1 (1 - \eta_1)} = \frac{p_1^2 + p_0 (1 - p_1) - \eta_1}{1 - \eta_1}
\]
which, after some manipulations, leads to (4).

**Derivation of \( p_1 \) and \( p_0 \) given by (5) and (6) and of \( \eta_1 \) given by (7):** The utility functions are given by:

\[
U_{i0} (\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - (1 - \omega \eta_1) \alpha e_{i0}
\]

and

\[
U_{i1} (\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2 - \omega (1 - \eta_1) \alpha e_{i1}
\]

The first-order conditions give:

\[
e_{i0} = \max\{0, \lambda_i - (1 - \omega \eta_1) \alpha\} \quad (14)
\]

\[
e_{i1} = \max\{0, \lambda_i - (1 - \eta_1) \omega \alpha\} \quad (15)
\]

Plugging back \( e_{ij} \) in the utility function and accounting for the outside option \( U \) yields:

\[
U_{i0} (\lambda_i) = \max \left\{ U, \frac{[\lambda_i - (1 - \omega \eta_1) \alpha]^2}{2} \right\} \quad (16)
\]

\[
U_{i1} (\lambda_i) = \max \left\{ U, \frac{[\lambda_i - (1 - \eta_1) \omega \alpha]^2}{2} \right\} \quad (17)
\]

We can determine the threshold values \( \tilde{\lambda}_0 \) and \( \tilde{\lambda}_1 \) as follows:

\[
\tilde{\lambda}_0 = \sqrt{2U} + (1 - \omega \eta_1) \alpha \quad (18)
\]

\[
\tilde{\lambda}_1 = \sqrt{2U} + (1 - \eta_1) \omega \alpha \quad (19)
\]

The probability \( p_0 \) that an individual with an uneducated strong tie will get educated and the probability \( p_1 \) that an individual with an educated strong tie will get educated are given by:

\[
p_0 = 1 - \tilde{\lambda}_0 = 1 - \sqrt{2U} - (1 - \omega \eta_1) \alpha
\]

\[
p_1 = 1 - \tilde{\lambda}_1 = 1 - \sqrt{2U} - (1 - \eta_1) \omega \alpha
\]

which are (5) and (6). In order to close the model, we determine the value of \( \eta_1 \) as follows:
\[
\eta_1 = \eta_1 \left\{ (1 - \omega)p_1 + \omega [\eta_1 p_1 + (1 - \eta_1)p_0] \right\} \\
+ (1 - \eta_1) \left\{ (1 - \omega)p_0 + \omega [\eta_1 p_1 + (1 - \eta_1)p_0] \right\}
\]

Indeed, in equilibrium, a newborn gets educated if either (i) he meets an educated strong tie (probability \( \eta_1 \)), spends a fraction \( 1 - \omega \) of his time with this strong tie and gets educated (probability \( p_1 \)) and spends a fraction \( \omega \) of his time with a weak tie who can be either educated and the newborn gets educated (probability \( \eta_1 p_1 \)) or who can be uneducated and the newborn gets educated (probability \( 1 - \eta_1 p_0 \)) or (ii) he meets an uneducated strong tie (probability \( 1 - \eta_1 \)), spends a fraction \( 1 - \omega \) of his time with this strong tie and gets educated with probability \( p_0 \) and spend a fraction \( \omega \) of his time with a weak tie who can be either educated and the newborn gets educated (probability \( \eta_1 p_1 \)) or who can be uneducated and the newborn gets educated (probability \( 1 - \eta_1 p_0 \)).

This expression can be simplified and we easily obtain:

\[
\eta_1 = \frac{p_0}{1 + p_0 - p_1}
\]

(20)

By replacing \( p_0 \) and \( p_1 \) by their values in (5) and (6), and solving in \( \eta_1 \), we easily obtain (7).

\[ \blacksquare \]

**Proof of Proposition 2:** The joint probability to have both a newborn and his father educated, \( \eta_{11} \), is given by

\[
\eta_{11} = (1 - \omega)(\mu_{11} p_1 + \mu_{10} p_0) + \omega(\mu_{11} + \mu_{10})[\eta_1 p_1 + (1 - \eta_1)p_0]
\]

Indeed, in order to have both a newborn and his father in state 1, there are two possibilities:

(i) either the son interacts within his dyad (probability \( 1 - \omega \)). In that case, the father has to be in state 1, which is the case with probability 1 if it is a (11) dyad (\( \mu_{11} \)) and with probability 1/2 if it is a (10) or (01) dyad (\( \mu_{10} \) or \( \mu_{01} \)). The son will get educated with probability \( p_1 \) if the father was in the dyad 11 and with probability \( p_0 \) if the father was in a dyad (10) or (01).

(ii) or the son interacts with a weak tie (with probability \( \omega \)). In that case, the father has to be educated (with probability \( \mu_{11} + \frac{1}{2}(\mu_{10} + \mu_{01}) \)) and then the son gets educated with
probability $p_1$ if he meets an educated individual (with probability $\eta_1$) and with probability $p_0$ if he meets an uneducated individual (with probability $(1 - \eta_1)$).

In this framework, $\mu_{11}$ is given by

$$\mu_{11} = \eta_1[(1 - \omega)p_1 + \omega(\eta_1 p_1 + (1 - \eta_1)p_0)]$$

and $\mu_{10}$ is given by

$$\mu_{10} = \frac{1}{2}(1 - \eta_1) \{(1 - \omega)p_0 + \omega[\eta_1 p_1 + (1 - \eta_1)p_0]\} + \frac{1}{2}\eta_1 \{(1 - \omega)(1 - p_1) + \omega[\eta_1(1 - p_1) + (1 - \eta_1)(1 - p_0)]\}$$

Indeed, for a $(10)$ dyad to form, either an individual meets a type--0 individual (with probability $(1 - \eta_1)$) and gets educated (either by staying within the dyad ($(1 - \omega)p_0$) or outside the dyad $(\omega(\eta_1 p_1 + (1 - \eta_1)p_0))$, or an individual meets a type--1 individual (with probability $\eta_1$) and decides not to educate (either by staying within the dyad ($(1 - \omega)(1 - p_1)$) or outside the dyad $(\omega(\eta_1(1 - p_1) + (1 - \eta_1)(1 - p_0)))$.

Observing that $\eta_1 p_1 + (1 - \eta_1)p_0 = \eta_1$, that $\eta_1(1 - p_1) = (1 - \eta_1)p_0$ and that $\eta_1(1 - p_1) + (1 - \eta_1)(1 - p_0) = 1 - \eta_1$, we have:

$$\mu_{11} = \eta_1[(1 - \omega)p_1 + \omega \eta_1]$$

$$\mu_{10} = \frac{1}{2}(1 - \eta_1)\{(1 - \omega)p_0 + \omega \eta_1\} + \frac{1}{2}\eta_1\{(1 - \omega)(1 - p_1) + \omega(1 - \eta_1)\}$$

$$\mu_{10} = \frac{1}{2}(1 - \eta_1)(2\omega \eta_1) + \frac{1}{2}(1 - \omega)[\eta_1(1 - p_1) + (1 - \eta_1)p_0]$$

$$\mu_{10} = \omega(1 - \eta_1)\eta_1 + (1 - \omega)\eta_1(1 - p_1)$$

Furthermore, we have:

$$\mu_{11}p_1 + \mu_{10}p_0 = \eta_1[(1 - \omega)(p_1^2 + p_0(1 - p_1)) + \omega(p_1\eta_1 + p_0(1 - \eta_1))]$$

$$\mu_{11}p_1 + \mu_{10}p_0 = \eta_1[(1 - \omega)(p_1^2 + p_0(1 - p_1)) + \omega \eta_1]$$

This implies that

$$\eta_{11} = (1 - \omega)^2 \eta_1(p_1^2 + p_0(1 - p_1)) + \omega(1 - \omega)\eta_1^2 + \omega \eta_1^2$$
and
\[
\frac{\eta_{11} - \eta_1^2}{\eta_1} = (1 - \omega)^2 \left[ p_0^2 + p_0(1 - p_1) \right] + (2\omega - \omega^2 - 1)\eta_1 \\
= (1 - \omega)^2 \left[ p_0^2 + p_0(1 - p_1) - \eta_1 \right]
\]

Finally
\[
Cor_{exo} = \frac{\eta_{11} - \eta_1^2}{\eta_1(1 - \eta_1)} \\
= \frac{(1 - \omega)^2 p_0^2 + p_0(1 - p_1) - \eta_1}{1 - \eta_1} \\
= (1 - \omega)^2 (p_1 - p_0)^2 \\
= (1 - \omega)^4 \alpha^2
\]

which is (9).

**Proof of Proposition 3:** Let us first analyze (i). The total welfare is given by (10), which is
\[
W = \int_{\lambda_1}^{\lambda_0} U d\lambda + \int_{\lambda_0}^{\lambda_1} (1 - \eta_1) U d\lambda + \int_{\lambda_1}^{1} \eta_1 U_{i1} (\lambda) d\lambda + \int_{\lambda_0}^{1} (1 - \eta_1) U_{i0} (\lambda) d\lambda
\]

The first two terms can be calculated and it is easily shown that:
\[
\int_{\lambda_0}^{\lambda_1} U d\lambda + \int_{\lambda_1}^{\lambda_0} (1 - \eta_1) U d\lambda = \tilde{\lambda}_0 U - \eta_1 U \left( \tilde{\lambda}_0 - \tilde{\lambda}_1 \right)
\]
which using (18) and (19) gives
\[
K = \int_{\lambda_0}^{\lambda_1} U d\lambda + \int_{\lambda_1}^{\lambda_0} (1 - \eta_1) U d\lambda = \tilde{U} \left[ \sqrt{2U} + (1 - \eta_1) \alpha \right]
\]
which is independent of \( \omega \) (see (7)) and thus we can ignore these first two terms. So the planner maximizes
\[
\int_{\lambda_1}^{1} \eta_1 U_{i1} (\lambda) d\lambda + \int_{\lambda_0}^{1} (1 - \eta_1) U_{i0} (\lambda) d\lambda
\]

Using (16) and (17), we have:
\[
\int_{\lambda_1}^{1} \eta_1 U_{i1} (\lambda) d\lambda + \int_{\lambda_0}^{1} (1 - \eta_1) U_{i0} (\lambda) d\lambda \\
= \frac{1}{6} \left\{ \eta_1 \left[ (\lambda_1 - \omega \alpha + \omega \eta_1 \alpha)^{\frac{1}{3}}_{\lambda_1} + (1 - \eta_1) \left[ (\lambda_1 - \alpha + \omega \eta_1 \alpha)^{\frac{1}{3}}_{\lambda_0} \right] \right] \right\}
\]

29
Using (18) and (19), we see that
\[
\tilde{\lambda}_1 - \omega \alpha + \omega \eta_1 \alpha = \sqrt{2U} = \tilde{\lambda}_0 - \alpha + \omega \eta_1 \alpha
\]
As a result, we obtain:
\[
\mathcal{W} = K - \frac{1}{6} \left( \sqrt{2U} \right)^3 + \frac{1}{6} \left[ \eta_1 (1 - \omega \alpha + \omega \eta_1 \alpha)^3 + (1 - \eta_1) (1 - \alpha + \omega \eta_1 \alpha)^3 \right]
\]
where \( K' \equiv K - \frac{1}{6} \left( \sqrt{2U} \right)^3 \). We have:
\[
\frac{\partial \mathcal{W}}{\partial \omega} = \frac{1}{2} (1 - \eta_1) \alpha \eta_1 \left[ (1 - \alpha + \omega \eta_1 \alpha)^2 - (1 - \omega \alpha + \omega \eta_1 \alpha)^2 \right]
\]
\[
= -\frac{1}{2} (1 - \eta_1) \alpha \eta_1 (1 - \omega) (2 \omega \eta_1 \alpha + 1 - \alpha + 1 - \omega \alpha)
\]
Since, according to (8), \( \alpha < 1 \), then
\[
\frac{\partial \mathcal{W}}{\partial \omega} \leq 0
\]
As a result, the optimal solution is \( \omega^* = 0 \).

Let us now analyze (ii). The correlation is given by (9), that is
\[
Cor_{exo} = (1 - \omega)^2 (p_1 - p_0)^2 = (1 - \omega)^4 \alpha^2
\]
Since \( \frac{\partial Cor_{exo}}{\partial \omega} < 0 \), it is should be clear that if the planner wants to minimize the correlation, then the solution to this program is \( \omega^* = 1 \).

**Proof of Remark 1:** We have shown in the proof of Proposition 3 that both \( \frac{\partial \mathcal{W}}{\partial \omega} < 0 \) and \( \frac{\partial Cor_{exo}}{\partial \omega} < 0 \). It is straightforward to verify that \( \frac{\partial^2 Cor_{exo}}{\partial \omega^2} > 0 \). For \( \mathcal{W} \), we have:
\[
\frac{\partial^2 \mathcal{W}}{\partial \omega^2} = \eta_1 \alpha^2 (\eta_1 - 1) \frac{[-1 + \eta_1 \alpha - 2 \omega \eta_1 \alpha + \omega \alpha]}{2}
\]
which has a constant sign over \([0, 1]\) and
\[
\left. \frac{\partial^2 \mathcal{W}}{\partial \omega^2} \right|_{\omega = 0} > 0
\]
This proves the result.
Proof of Proposition 4:

First-order conditions on efforts yield:

\[ e_{i0}^* = \max \{ 0, \lambda_i - (1 - \omega_{i0} \eta_1) \alpha \} \] (21)

\[ e_{i1}^* = \max \{ 0, \lambda_i - (1 - \eta_1) \omega_{i1} \alpha \} \]

which imply:

\[ U_{i0}^* (\lambda_i) = \frac{1}{2} (e_{i0}^*)^2 = \max \left\{ \bar{U}, \frac{[\lambda_i - (1 - \omega_{i0} \eta_1) \alpha]^2}{2} \right\} \] (22)

\[ U_{i1}^* (\lambda_i) = \frac{1}{2} (e_{i1}^*)^2 = \max \left\{ \bar{U}, \frac{[\lambda_i - \omega_{i1} (1 - \eta_1) \alpha]^2}{2} \right\} \] (23)

Given these expressions, point (ii) is obvious. Let us show (i). Individuals can get $\bar{U}$ if they exert no effort (in which case we assume they set $\omega_{i0}^* = 0$). Those who can get more than $\bar{U}$ by exerting an effort will get

\[ U_{i0}^* (\lambda_i) = \frac{[\lambda_i - (1 - \omega_{i0} \eta_1) \alpha]^2}{2} \]

which is maximized at $\omega_{i0}^* = 1$.

These individuals get

\[ U_{i0}^* (\lambda_i) = \frac{[\lambda_i - (1 - \eta_1) \alpha]^2}{2} \]

which is greater than $\bar{U}$ only if $\lambda_i > \sqrt{2\bar{U}} + (1 - \eta_1) \alpha \equiv \tilde{\lambda}_0$. Therefore, $\omega_{i0}^* = 1$ if $\lambda_i > \tilde{\lambda}_0$, and $\omega_{i0}^* = 0$ otherwise.

**Derivation of $\eta_1$ given by (12):** We have:

\[ p_0 = 1 - \sqrt{2\bar{U}} - (1 - \eta_1) \alpha \] (24)

and

\[ p_1 = 1 - \sqrt{2\bar{U}} \] (25)

We also have that:

\[ U_{i0} (\lambda_i) = \max \left\{ \bar{U}, \frac{[\lambda_i - (1 - \eta_1) \alpha]^2}{2} \right\} \]
\[ U_{i1}(\lambda_i) = \max \left\{ \lambda_i^0, \frac{\lambda_i^2}{2} \right\} \]

Let us compute the value of \( \eta_1 \). Again, individuals whose ability exceeds \( \tilde{\lambda}_0 \) will get educated whatever the status of their strong tie. They represent a mass of size \( p_0 \). Those whose ability is lower than \( \tilde{\lambda}_1 \) will never get educated while those such that \( \tilde{\lambda}_0 > \lambda_i > \tilde{\lambda}_1 \) will get educated only if they meet an educated strong tie. There is a mass \( p_1 - p_0 \) of these individuals. As a result,

\[ \eta_1 = p_0 + (p_1 - p_0)\eta_1 \]

Using the values of \( p_0 \) and \( p_1 \), we obtain:

\[ F(\eta_1) \equiv \alpha \eta_1^2 + \eta_1 (1 - 2\alpha) - 1 + \sqrt{2U} + \alpha = 0 \]

which is (12).

**Proof of Proposition 5:** We have \( F(0) = \sqrt{2U} - 1 + \alpha < 0 \) and \( F(1) = \sqrt{2U} > 0 \), so there is one solution \( \eta_1^* \) between 0 and 1. Furthermore, for \( \eta_1 \in [0, 1] \), \( F(\eta_1) < 0 \) if and only if \( \eta_1 < \eta_1^* \). It is then enough to check that \( F\left(1 - \frac{\sqrt{2U} - \alpha}{1 - \alpha}\right) < 0 \). After some manipulations we get

\[ Sgn \left[ F\left(1 - \frac{\sqrt{2U} - \alpha}{1 - \alpha}\right) \right] = Sgn \left[ \frac{\alpha(\sqrt{2U} - 1)}{1 - \alpha} \right] < 0 \]

Thus \( \eta_1^* < \frac{1 - \sqrt{2U} - \alpha}{1 - \alpha} \).

As for the comparative statics, we know that around \( \eta_1^* \), \( \frac{\partial F}{\partial \eta_1} > 0 \). Furthermore, \( \frac{\partial F}{\partial U} > 0 \), and \( \frac{\partial F}{\partial \alpha} = (1 - \eta_1)^2 > 0 \), so we get the desired conclusion.

**Proof of Proposition 6:** Let us calculate the correlation between the father and son. This correlation is given by:

\[ Cor_{net} = \frac{\eta_{11} - \eta_1^2}{\eta_1(1 - \eta_1)} \tag{26} \]

We have

\[ \eta_1 = \eta_1 p_1 + (1 - \eta_1) p_0 \]

The steady-state distributions are given by

\[ \mu_{11} = \eta_1 p_1 \]
Indeed, for a $\mu_{11}$ dyad to be formed, it must be that a newborn meets an educated strong tie (probability $\eta_1$) and that he gets educated. But since $\omega_{11}^* = 0$ in equilibrium, he only interacts with his strong tie and gets educated with probability $p_1$.

Accordingly, for a $\mu_{10}$ (or a $\mu_{01}$) dyad to be formed, it must be that either a newborn meets an educated strong tie (he then sets $\omega_{11} = 0$) and does not get educated, with probability $(1 - p_1)$ or the newborn meets a non educated strong tie (probability $1 - \eta_1$, he then sets $\omega_{10} = 1$) and will get educated if $\lambda_i > \tilde{\lambda}_0$. This happens with probability $p_0$.

Thus we obtain:

\[
\mu_{10} =\frac{1}{2}\eta_1(1 - p_1) + \frac{1}{2}(1 - \eta_1)p_0
\]

In turn $\eta_{11}$ is given by

\[
\eta_{11} = \mu_{11}p_1 + \mu_{10}p_0
\]

Indeed, for the father and the son to be both educated, it must be the case that the father was educated and that the son gets educated. Either the father was part of a $\mu_{11}$ dyad and then the son meets an educated strong tie, in which case he gets educated with probability $p_1$, or the father was in a $\mu_{10}$ dyad and then the son meets an uneducated strong tie, in which case he only interacts with weak ties and gets educated with probability $p_0$. Replacing for $\mu_{11}$ and $\mu_{10}$, we obtain

\[
\eta_{11} = \eta_1p_1^2 + (1 - p_1)p_0\eta_1
\]

Hence

\[
Cor_{net} = \frac{\eta_{11} - \eta_1^2}{\eta_1(1 - \eta_1)}
\]

\[
= \frac{\eta_1p_1^2 + (1 - p_1)p_0\eta_1 - \eta_1^2}{\eta_1(1 - \eta_1)}
\]

\[
= \frac{p_1^2 + (1 - p_1)p_0 - \eta_1}{1 - \eta_1}
\]
Because

\[ \eta_1 = \frac{p_0}{1 - (p_1 - p_0)} \]

we have

\[ 1 - \eta_1 = \frac{1 - p_1}{1 - (p_1 - p_0)} \]

Plugging these values in the expression above, we have:

\[
Cor_{net} = \frac{p_1^2 + (1 - p_1) p_0 - \eta_1}{1 - \eta_1} \\
= \frac{p_1^2 + (1 - p_1) p_0 [1 - (p_1 - p_0)] - p_0}{1 - p_1} \\
= \frac{p_1 (p_1 - p_0) + p_0 [1 - (p_1 - p_0)] - p_0}{1 - p_1} \\
= (p_1 - p_0) \frac{p_1 [1 - (p_1 - p_0)] - p_0}{1 - p_1} \\
= (p_1 - p_0)^2
\]

Using the values of \( p_1 \) and \( p_0 \), we finally obtain:

\[ Cor_{net} = (1 - \eta_1)^2 \alpha^2 \]

which is (13).
APPENDIX 2: The model with socialization costs

Let us extend our model by introducing a socialization cost equal to $-\frac{1}{2}\omega_{ij}^2$ so that:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 + \omega_{i0} \left[-(1 - \eta_1)\alpha\right] e_{i0} - (1 - \omega_{i0})\alpha e_{i0} - \frac{1}{2}\omega_{i0}^2$$

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2 + \omega_{i1} \left[-(1 - \eta_1)\alpha\right] e_{i1} - \frac{1}{2}\omega_{i1}^2$$

Let $\tilde{\lambda}_0 \equiv \sqrt{2U(1 - \eta_2^2\alpha^2)} + \alpha$ and $\tilde{\lambda}_1 \equiv \sqrt{2U}$. Then

Proposition 7

(i) For individuals who inherited an uneducated strong tie from their father, their choice of meeting weak ties depend on their initial ability $\lambda_i$. If $\lambda_i < \tilde{\lambda}_0$, they choose to never meet weak ties, i.e. $\omega_{i0}^* = 0$, while, for those for which $\lambda_i \geq \tilde{\lambda}_0$, they set

$$\omega_{i0}^* = \eta_1\alpha \left[\frac{\lambda_i - \alpha}{1 - \eta_1^2\alpha^2}\right] > 0$$

(ii) Individuals who inherited an educated strong tie from their father never want to meet weak ties, i.e. $\omega_{i1}^* = 0$.

The proof of this proposition is similar to that of Proposition 4 so we omit it. Compared to the result of Proposition 4 we see that the only difference is in case (i) when $\lambda_i \geq \tilde{\lambda}_0$. Indeed, in that case, introducing a quadratic socialization cost changes $\omega_{i0}^*$ from one to an interior solution.

As above, the steady-state level of education is given by:

$$\eta_1 = \frac{p_0}{1 - (p_1 - p_0)}$$

where $p_0 = 1 - \tilde{\lambda}_0$ and $p_1 = 1 - \tilde{\lambda}_1$. We obtain $\eta_1^*$ as a solution of:

$$F(\eta_1) = \eta_1(1 + \sqrt{2U} - \alpha) + (1 - \eta_1)\sqrt{2U}\sqrt{1 - \eta_1^2\alpha^2} - 1 + \alpha = 0 \quad (27)$$

and obtain the following proposition.
Proposition 8 If $\sqrt{2U} < 1 - \alpha$, there exists a unique solution $\eta^*_1 \in [0, 1]$ to (27). It is such that

$$\eta^*_1 > \frac{1 - \sqrt{2U} - \alpha}{1 - \alpha}$$

Furthermore, we have:

$$\frac{\partial \eta^*_1}{\partial U} < 0 \quad \text{and} \quad \frac{\partial \eta^*_1}{\partial \alpha} < 0$$

We see that the results are qualitatively unchanged compared to the case when there is no socialization costs.

Proof: We have $F(0) = \sqrt{2U} - 1 + \alpha < 0$ and $F(1) = \sqrt{2U} > 0$, so there is at least one solution $\eta^*_1$ between 0 and 1. Now,

$$\frac{\partial F}{\partial \eta_1} = 1 + \sqrt{2U} - \alpha - \sqrt{2U} \left[ \frac{1}{1 - \eta_1} - \alpha \frac{\eta_1}{\sqrt{1 - \eta_1} \alpha} \right]$$

Because $\sqrt{2U} < 1 - \alpha$,

$$1 + \sqrt{2U} - \alpha - \sqrt{2U} \left[ \frac{1}{1 - \eta_1} - \alpha \frac{\eta_1}{\sqrt{1 - \eta_1} \alpha} \right] > \sqrt{2U}$$

and thus

$$\frac{\partial F}{\partial \eta_1} > \sqrt{2U} \left[ 1 - \left( \frac{1 - \eta_1}{\sqrt{1 - \eta_1}} - \alpha \frac{\eta_1}{\sqrt{1 - \eta_1} \alpha} \right) \right]$$

Using

$$\sqrt{1 - \eta_1} = \frac{1 - \eta_1}{\sqrt{1 - \eta_1}} > \frac{1 - \eta_1}{\sqrt{1 - \eta_1} \alpha}$$

we get

$$\frac{\partial F}{\partial \eta_1} > \sqrt{2U} \left[ 1 - \sqrt{1 - \eta_1} \frac{\eta_1 \alpha^2}{\sqrt{1 + \eta_1 \alpha}} \right]> 0 \quad \text{for} \quad \eta_1 \alpha^2 < 1$$

, which proves the uniqueness of $\eta^*_1 \in [0, 1]$.

To show that $\eta^*_1 > \frac{1 - \sqrt{2U} - \alpha}{1 - \alpha}$, we use the fact that $\frac{\partial F}{\partial \eta_1} > 0$ and check that $F(\frac{1 - \sqrt{2U} - \alpha}{1 - \alpha}) < 0$. Some manipulations lead to

$$F(\frac{1 - \sqrt{2U} - \alpha}{1 - \alpha}) < 0 \iff 2U \left( -1 + \sqrt{1 - \left( \frac{1 - \sqrt{2U} - \alpha}{1 - \alpha} \right)^2 \alpha^2} \right) < 0$$

36
and this inequality is always true.

As for the comparative statics, using $\frac{\partial F}{\partial \eta_1} > 0$ and $\frac{\partial F}{\partial \eta_2} > 0$, we get $\frac{\partial \eta_1}{\partial U} < 0$.

Next, to see that $\frac{\partial \eta_1}{\partial \alpha} < 0$, we need to show that $\frac{\partial F}{\partial \alpha} > 0$. We have

$$\frac{\partial F}{\partial \alpha} = (1 - \eta_1) \left[ 1 - \sqrt{2\eta_1} \frac{\alpha \eta_1^2}{\sqrt{1 - \eta_1^2 \alpha^2}} \right]$$

We also have

$$\sqrt{2U} \alpha \eta_1^2 < \sqrt{2U} < 1 - \alpha < 1 - \eta_1 \alpha < 1 - \eta_1^2 \alpha^2 < \sqrt{1 - \eta_1^2 \alpha^2}$$

which proves that $\frac{\partial F}{\partial \alpha} > 0$. □