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Exam for EC 2110, Uncertainty and asymmetric information, 10 January 2016

1. Short questions:

a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
(i) $v(c) = \ln(e^{4c+2})$
(ii) $v(c) = c + c^3$
(iii) $v(c) = \sqrt{1 + c^2}$
(iv) $v(c) = 1 + c - e^{-c}$

b) Consider an economy consisting of two individuals (A and B), who will end up in state 1 with probability π and in state 2 with probability $1 - \pi$. Individual A is endowed with state claims $(\bar{c}_1^A, \bar{c}_2^A)$, and individual B is endowed with state claims $(\bar{c}_1^B, \bar{c}_2^B)$. Both individuals are risk averse. Their elementary utility functions are $v_A(c)$ and $v_B(c)$. It is possible for A and B to trade state claims. Let (c_1^{A*}, c_2^{A*}) and (c_1^{B*}, c_2^{B*}) denote the market equilibrium amounts of state claims. State all six conditions that need to be satisfied to obtain the market equilibrium under uncertainty. (Note: You are not supposed to calculate the equilibrium outcome. Just state the conditions that have to be satisfied.)

c) Consider a factory owner who needs to hire staff. The value of output produced is given by $S(q) = \ln(1 + q)$, where q is the amount of effort exerted by a worker. The factory owner pays a wage t to workers. There are two types of workers who differ with respect to their cost of effort. Workers of type $\underline{\Theta} = 1$ incur cost $C(q, \underline{\Theta}) = q$, while workers of type $\overline{\Theta} = 4$ incur cost $C(q, \overline{\Theta}) = 4q$ when exerting effort q. Workers' utilities are given by $u_\Theta(q, t) = t - C(q, \Theta)$. The factory owner has no information regarding the efficiency of workers, but knows that the share of efficient workers is ν. State the factory owner's optimization problem and all constraints that need to be satisfied. Which constraints are relevant? Explain why the other constraints are not relevant.

d) Consider a market where there are two types of workers. Type 0 has marginal product $\Theta_0 = 2$ and an outside opportunity wage of $w_0(\Theta_0) = 1$. Type 1 has marginal product $\Theta_1 = 5$ and an outside opportunity wage of $w_0(\Theta_1) = 3$. The cost of education z is given by $C(z, \Theta_0) = \frac{z}{\Theta_0} = \frac{z}{2}$ for type 0 and $C(z, \Theta_1) = \frac{z}{\Theta_1} = \frac{z}{5}$ for type 1. A worker's utility function is given by $U(w, z, \Theta) = w - C(z, \Theta)$. Workers know their own type but employers cannot tell the high from the low productivity workers. Illustrate in a figure, with wage on the y-axis and the amount of education on the x-axis, which contracts will make it possible to separate type 1 from type 0 workers. Explain what contract will be offered to type 1 workers when there is perfect competition among employers. (Assume that, if two contracts yield the same level of utility, a type 0 worker prefers the one which requires less education.)

2. There are two states of the world, state 1 and state 2. The probability for state 1 occurring is $\pi = \frac{1}{4}$ (and the probability for state 2 occurring is $1 - \pi = \frac{3}{4}$). It is not possible to directly trade in state claims. However, there exists a complete asset market, where two assets, asset A_1 and asset A_2, can be traded.

   a) State the conditions that have to be satisfied for a complete asset market.

   The price of A_1 is given by $P_1^A = 3$, and the price of A_2 is given by $P_2^A = 2$. The following yield matrix indicates how much each asset yields in each state (e.g. A_1 yields $z_{11} = 4$ in state 1):

	State 1	State 2
Asset A_1	$z_{11} = 4$	$z_{12} = 2$
Asset A_2	$z_{21} = 2$	$z_{22} = 2$

   Consider an individual whose preference-scaling function is given by $v(c) = \ln c$ and who is endowed with $\overline{q_1} = 4$ units of A_1 and $\overline{q_2} = 2$ units of A_2.

   b) What are the implicit prices of state claim 1 (P_1) and state claim 2 (P_2)? (Hint: $P_1^A = z_{11}P_1 + z_{12}P_2$ and $P_2^A = z_{21}P_1 + z_{22}P_2$.)

   c) What will the individual's portfolio of assets (i.e. the endowment $\overline{q_1}$ and $\overline{q_2}$) yield in the two different states? (That is, what is the individual's implicit endowment of state claims $\overline{c_1}$ and $\overline{c_2}$?)

   d) State the von-Neumann-Morgenstern expected utility function.

   e) To obtain the optimal amounts of implicit state claims, two conditions need to be satisfied. State these two conditions. Then calculate the optimal amounts of state claims.

   f) Given the optimal amounts of state claims, what are the optimal amounts of assets A_1 and A_2? (Note that it is possible to go short, i.e. either q_1^* or q_2^* can be negative!)

3. Sven owns a bicycle which is worth 100 SEK. By undertaking precautionary measures at cost $\Psi = 1$ he can reduce the likelihood of his bike being stolen next year to $\pi = \frac{1}{2}$. Sven's elementary utility function is given by $v(c, \Psi) = \sqrt{c} - \Psi$, where c is the value of Sven's fortune which consists of his bike, i.e. $c = 100$ if his bike is not stolen and $c = 0$ if it is stolen.

   a) What is the expected value of Sven's fortune in one year's time? What is Sven's expected utility?

   Sven has an option of purchasing insurance at a premium of 64 SEK, in which case he would receive 100 SEK if his bike is stolen. The insurance company is able to perfectly verify that Sven undertakes precautionary measures to keep the probability of his bike being stolen at $\pi = \frac{1}{2}$ after he has bought insurance. Hence, the insurance contract is offered under complete information.

   b) What is the expected value of Sven's fortune in one year's time if he buys insurance? What is Sven's expected utility if he buys insurance?

   c) Explain why Sven will buy insurance although the expected value of his fortune will be smaller if he buys insurance than if he does not. (Hint: Relate your answer to Sven's attitude to risk.)

   d) Is the insurance premium actuarily fair? If not, what is the actuarily fair insurance premium?

   Now assume that the insurance contract is offered under asymmetric information. Thus, after Sven has bought insurance he becomes more careless, thereby saving the cost for undertaking precautionary measures, which increases the probability of his bike being stolen to $\widehat{\pi} = 1$.

   e) Explain in words how the insurance contract has to be designed to induce Sven to undertake precautionary measures. State the constraints that need to be satisfied.

   f) What contract will a monopolistic insurance company offer in the presence of hidden action?