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Exam for EC7112, Game theory, 27 April 2016

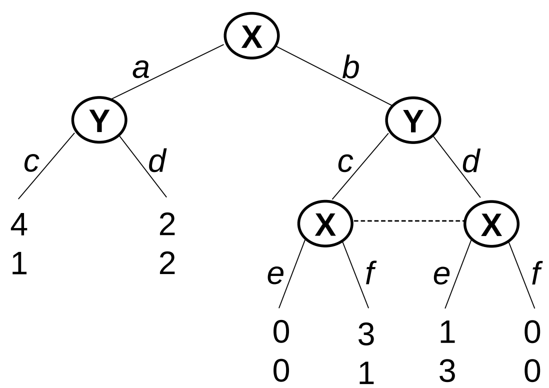
1. (30 points) Consider the following three-player game, where payoffs in each cell are presented in the following order: u_A, u_B, u_C.

		Player **C**					
		c_1			c_2		
		Player **B**			Player **B**		
		b_1	b_2	b_3	b_1	b_2	b_3
	a_1	$1,2,1$	$0,0,0$	$0,1,2$	$2,0,1$	$1,1,1$	$1,2,1$
Player **A**	a_2	$2,1,3$	$2,2,2$	$1,0,3$	$3,3,2$	$3,3,1$	$2,2,0$
	a_3	$3,2,1$	$0,1,4$	$2,0,1$	$3,1,0$	$0,3,2$	$3,2,2$

a) Which strategy profile(s) survive IDSDS (iterated deletion of strictly dominated strategies)? Use the answer sheet to illustrate and explain how you apply IDSDS.

b) Given the outcome of IDSDS, identify all (pure and mixed strategy) Nash equilibria of this game.

2. (35 points) Consider the following extensive form game between players **X** and **Y**, where player **X**'s payoffs are presented above player **Y**'s payoffs.



a) Identify all subgames of the game. State the strategy sets of both players.

b) Apply backward induction to identify all subgame perfect Nash equilibria strategy profiles of the game. Note that there may be more than one Nash equilibrium in a subgame - in this case derive the best response functions and illustrate these in a figure to obtain all Nash equilibria of the subgame.

c) Illustrate this game on normal form. Identify all pure strategy Nash equilibria.

3. (35 points) Two firms consider releasing a new product (the same product for both firms). However, there is uncertainty regarding market demand for this new product. With probability π demand is strong enough for making the product release profitable; being the only firm releasing the product yields a higher profit in this case. With probability $1 - \pi$ releasing the product will lead to a loss due to weak demand.

   Interaction between the two firms takes place over two periods, with simultaneous decision-making in both periods. In period 1, both firms simultaneously decide whether to *release* the product or to *wait*. There are three different cases to consider:

   (i) Both firms release the product in the first period. With probability π they both make a profit of 1 in both periods, i.e. both firms make a total profit of 2. With probability $1 - \pi$, both incur a loss of -1 and obviously leave the market in period 2.

   (ii) Only one of the firms releases the product, while the other firm chooses to wait in the first period. With probability π, the releasing firm makes a profit of 2 in the first period, whereupon the other firm enters the market and both make a profit of 1 in the second period. With probability $1 - \pi$, the releasing firm incurs a loss of -1 and quits the market, and the other firm obviously does not release the product in the second period.

   (iii) Both firms decide to wait. In this case they have to simultaneously decide on releasing the product in the second period. If they both release the product in the second period, they make a profit of 1 each with probability π and incur a loss of -1 each with probability $1-\pi$. If only one firm releases the product in the second period, it makes a profit of 2 with probability π, and incurs a loss of -1 with probability $1 - \pi$, while the other firm makes zero profits. If neither firm releases the product in the second period, both make zero profits.

   In what follows, the pure strategy Nash equilibria for different values of π are to be determined. For simplicity it is assumed that releasing the product is chosen whenever the expected payoff from releasing is at least as large as the expected payoff from not releasing the product.

   a) Given that neither firm has released the product in the first period, the two firms strategically interact in period 2. State both firms' expected payoffs for all pure strategy profiles in period 2. Illustrate the interaction in period 2 in a payoff matrix. Determine the pure strategy Nash equilibria in period 2 for different values of π.

   b) Model the interaction in the first period as a strategic game (that is, state the players, their actions and their expected payoffs in period 1). Note: you have to take the different pure strategy Nash equilibrium outcomes in period 2 for different values of π into account when determining the expected payoff when no firm releases the product in the first period.

   c) Determine all pure strategy Nash equilibria of the first period interaction for different values of π.

Answer sheet question 1a)

		Player **C**					
		c_1			c_2		
		Player **B**			Player **B**		
		b_1	b_2	b_3	b_1	b_2	b_3
	a_1	$1, 2, 1$	$0, 0, 0$	$0, 1, 2$	$2, 0, 1$	$1, 1, 1$	$1, 2, 1$
Player **A**	a_2	$2, 1, 3$	$2, 2, 2$	$1, 0, 3$	$3, 3, 2$	$3, 3, 1$	$2, 2, 0$
	a_3	$3, 2, 1$	$0, 1, 4$	$2, 0, 1$	$3, 1, 0$	$0, 3, 2$	$3, 2, 2$

		Player **C**					
		c_1			c_2		
		Player **B**			Player **B**		
		b_1	b_2	b_3	b_1	b_2	b_3
	a_1	$1, 2, 1$	$0, 0, 0$	$0, 1, 2$	$2, 0, 1$	$1, 1, 1$	$1, 2, 1$
Player **A**	a_2	$2, 1, 3$	$2, 2, 2$	$1, 0, 3$	$3, 3, 2$	$3, 3, 1$	$2, 2, 0$
	a_3	$3, 2, 1$	$0, 1, 4$	$2, 0, 1$	$3, 1, 0$	$0, 3, 2$	$3, 2, 2$

		Player **C**					
		c_1			c_2		
		Player **B**			Player **B**		
		b_1	b_2	b_3	b_1	b_2	b_3
	a_1	$1, 2, 1$	$0, 0, 0$	$0, 1, 2$	$2, 0, 1$	$1, 1, 1$	$1, 2, 1$
Player **A**	a_2	$2, 1, 3$	$2, 2, 2$	$1, 0, 3$	$3, 3, 2$	$3, 3, 1$	$2, 2, 0$
	a_3	$3, 2, 1$	$0, 1, 4$	$2, 0, 1$	$3, 1, 0$	$0, 3, 2$	$3, 2, 2$

		Player **C**					
		c_1			c_2		
		Player **B**			Player **B**		
		b_1	b_2	b_3	b_1	b_2	b_3
	a_1	$1, 2, 1$	$0, 0, 0$	$0, 1, 2$	$2, 0, 1$	$1, 1, 1$	$1, 2, 1$
Player **A**	a_2	$2, 1, 3$	$2, 2, 2$	$1, 0, 3$	$3, 3, 2$	$3, 3, 1$	$2, 2, 0$
	a_3	$3, 2, 1$	$0, 1, 4$	$2, 0, 1$	$3, 1, 0$	$0, 3, 2$	$3, 2, 2$