EC7104 The Climate & the Economy
Spring 2016
August 2016

Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. **A climate model**

Consider the following simple climate model:

$$\begin{aligned} T_t &= T_{t-1} + \sigma_1 \left(F_{t-1} - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T^L_{t-1} \right) \right) \\ T^L_t &= T^L_{t-1} + \sigma_3 \left(T_{t-1} - T^L_{t-1} \right) \end{aligned}$$

where T_t and T^L_t, respectively, denote the atmospheric and ocean temperature in period t, both measured relative to a pre-industrial steady state. F_t is forcing in period t (change in energy balance relative to pre-industrial steady state) and κ and the $\sigma' s$ are positive parameters.

(a) Suppose we observe that if forcing is constant at the level $F = 4$, the atmospheric temperature T eventually stabilizes at 3 degrees. Will also the ocean temperature stabilize? If so, at which temperature?

(b) Which parameter(s) if any is determined by the information in a., i.e., that $F_t = 4$ for all t, implies that T_t converges to 3? Use the information to calibrate (put a number to) relevant parameter(s).

(c) Suppose $F_0 = 4$ and $T_0 = T^L_0 = 0$. What are T_1 and T^L_1 in terms of the parameters? Suppose we observe that $T_1 = 1$. Use this to calibrate the relevant parameter(s).

(d) Now consider the case of tipping points created by temperature dependent feedbacks. Specifically, in the model above we have assumed that the energy outflow is proportional to temperature. Assume now instead that the proportionality factor falls discretely whenever $T_t > 2$. Which parameter is affected and no longer a temperature independent constant? Draw a stylized graph with time on the x-axis and T_t on the y-axis showing the dynamics of the atmospheric temperature in the case of tipping points. Assume as above that at the start of your graph, $T_0 = T^L_0 = 0$ and that $F_t = 4$ for all $t \geq 0$.

2. **A Solow model with technology growth**

(a) Solow's model with technology (but not population) growth involves the following assumptions: (i) investment is a fraction s of output; (ii) output is a constant-returns to scale function of capital and labor measured in efficiency (or human-capital) units; (iii) the efficiency per labor unit grows at a constant rate g; and (iv) capital next year is capital this year less depreciation—which is a constant fraction of this year's capital stock—plus investment. Represent these four assumptions by equations and derive an equal that gives you capital next year as a function of capital this year and the efficiency level per unit of labor (no other endogenous variables than capital should be in the equation).

(b) What is the long-run growth rate of capital in this model? What is the long-run growth rate of investment? What is the long-run growth rate of output?

(c) Explain what a balanced growth path is and derive an expression for the capital/output ratio—in terms of parameters (and not endogenous variables)—on a balanced growth path.

(d) Suppose the economy is on a balanced growth path and there is a sudden, large jump upwards in technology—such as when a new technology for generating energy would be invented, thus raising the efficiency per labor unit by more than the usual, smooth year-to-year growth. Immediately after the big jump, the efficiency per labor unit again grows at the same constant rate as prior to the technology jump. Explain, using a diagram, how capital would grow subsequent to the technology jump. Would it grow at a constant, a decreasing, or an increasing rate?

3. **A static IAM with utility damages**

Consider the following static IAM model: the representative consumer has preferences $u(c) = \log c - \gamma S$, the production of the consumption good occurs according to

$$c = Dk^{1/3} n_c^{1/3} E^{1/3},$$

(coal) energy is produced according to

$$E = \chi n_e,$$

there is a constraint $n_e + n_c = 1$ for the labor input, the carbon cycle satisfies $S = \varphi E$, and TFP does not involve any damages (climate damages only appear in utility). Finally, firms maximize profits and do not take the utility damage into account in their decisions. Temperature is determined by a standard, logarithmic forcing equation à la Arrhenius.

(a) Derive an equation determining market coal use (in the absence of any taxes).

(b) Derive the first-order condition for a social planner maximizing consumer welfare.

(c) Find an expression for the optimal tax on carbon in this economy.

(d) Given any outcome for coal production, E, what is the outcome for temperature (relative to its preindustrial level)?

(e) If one were to use this model to compare quotas to carbon taxes, how would the two policy instruments compare in terms of consumer welfare?

4. **Hotelling with an alternative energy source**

Assume that an economy exists for two periods and that final output is produced with capital and energy. In the first period, only fossil fuel (E) is available as an energy source. However, in the second period there is also an alternative energy source S_1. Specifically, production in the two periods are given by

$$Y_0 = A_0 K_0^{\alpha} E_0^{1-\alpha} \quad \text{and} \quad Y_1 = A_1 K_1^{\alpha} \left(R_0 - E_0 + S_1\right)^{1-\alpha},$$

where K is capital A_0 and A_1 are the exogenously given technology levels in periods 0 and 1, respectively. Capital is assumed to depreciate fully between the periods. There is a fixed supply of fossil energy so the resource constraint for fossil fuel is given by

$$E_0 + E_1 = R_0.$$

The production of final output can be used for consumption or savings which implies two more aggregate resource constraints:

$$C_0 = A_0 K_0^{\alpha} E_0^{1-\alpha} - K_1 \quad \text{and} \quad C_1 = A_1 K_1^{\alpha} \left(E_1 + S_1\right)^{1-\alpha}.$$

The utility function is logarithmic in consumption and households maximize the discounted present value of utility:

$$U\left(C_0, C_1\right) = \log\left(C_0\right) + \beta \log\left(C_1\right),$$

where β is the discount factor.

(a) Set up the social planning problem.

(b) Derive the Euler equation for capital as well as the Hotelling equation.

(c) Solve explicitly for K_1, E_0, and E_1.

(d) Show how the amount of alternative energy source, S_1, affects the allocation of fossil fuel between the periods.

B. Short questions

1. Suppose the CCR (Carbon Climate Response) as defined in class is 1.5 degrees Celsius per 1000 GtC. Given that we have already released around 500 GtC, what is the maximum amount we can additionally emit in order to stay below 2.0 degrees heating?

2. Dell, Jones and Olken (NBER WP 14132) use data on climate variation and GDP to analyze the effect of climate change on the level and growth rate of output in different types of countries. A summary of the key findings is as follows: An increase in temperature has a substantial ...... effect on ...... but only for ...... countries. Fill in the correct words.

3. Explain how a discount factor (denoted β) is defined mathematically and briefly discuss its role for investment decisions. Is there a parameter in the Solow model that would be influenced by the discount factor? If you answer yes, specify one such parameter; whether you answer yes or no, defend your answer very briefly.

4. Empirical studies strongly suggest that households want to "smooth" their consumption both over time and across (random) states of nature. How do economists model this preference for smoothing? If household A has a stronger desire for smoothing than does household B, how is this modeled?

5. In the public debate you sometimes hear the claim that we are consuming the earth's resources at too fast a pace. Critically discuss this claim. Provide at least one argument for when that could be true and one argument that could speak against it. No formal analysis is required but you should base your reasoning on economic theory.