# Optimal Climate Policy with Household Wealth Inequality

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**Abstract**

Policy makers concerned with setting optimal values for carbon instruments to address climate change externalities often employ integrated assessment models (IAMs). While these models differ on their assumptions of climate damage impacts, discounting and technology, they conform on their assumption of complete markets and a representative household. In the face of global inequality and significant vulnerability of asset poor households, I relax the complete markets assumption and introduce a realistic degree of global household inequality. A simple experiment of introducing a range of global carbon taxes, shows a household's position on the global wealth distribution predicts the identity of their most preferred carbon price. Specifically, poor agents prefer a relatively strong climate change policy. This preference exists even without progressive redistribution of the revenue. However, transfers of the carbon tax revenue is of first order quantitative importance for household policy preference. I find that, parallel to the literature on macroeconomic policy and incomplete markets, the carbon tax can partially fill the role of insurance by reducing the volatility of future welfare.

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# 1 Introduction

To date, models of climate and the economy have calculated optimal carbon policy under the assumption of complete markets and a representative agent. Meanwhile a growing empirical literature on climate impacts highlights the distributional costs of climate change, with the global poor being particularly vulnerable. In order to explore the implications of relaxing these assumptions from the integrated assessment modelling literature, I introduce a standard incomplete markets framework. Thus, in addition to an uncertain global climate state, households also face idiosyncratic productivity shocks. Calibrating the model to the global economy, I find that there are differences in the cost of carbon faced across the wealth distribution. Poor households are vulnerable to future shocks, due to their relative paucity in private insurance. Hence, poor households prefer *ex ante* stronger public action through high carbon taxation, even in the absence of progressive redistribution. However, transfers of the carbon tax revenue are of primary concern in the identity of a household's most preferred carbon policy.

The International Panel on Climate Change (IPCC) details the impact (both realized and potential) on the world's more vulnerable population in its chapter on Livelihoods and Poverty in the 2014 Climate Change Report. In this chapter, they discuss the interaction of climate change and the challenges faced by the poor and economically vulnerable. While climate change implies specific threats related to shifting weather patterns, increased incidence of natural disasters, decreased land arability, etc.; the report also notes that climate change exasperates existing vulnerabilities experienced by the poor. While there will be regional heterogeneity in climate change, the impact will be felt globally, and the poor, in all regions, will suffer from market disruption, declining agricultural yields, reduced access to water, etc. Indeed, while poor households in low income countries (LICs) will incur the greatest costs of climate change impacts, the IPCC notes that inhabitants of some middle income countries (MICs), including urban Chinese, are among the most at risk to climate-related impacts.

One popular tool for policy makers is the integrated assessment model (IAM) (e.g. DICE, Nordhaus and Sztorc (2013); FUND, Tol (1997) and Golosov et al. (2014)) which aims to capture the features of the climate change problem, including: modelling the carbon system; atmospheric carbon's relationship to global temperature; temperature's relationship to welfare loss; and the economic system, including modelling the micro-foundations of savings and fossil fuel use. There are a wide range of IAMs, which differ on the assumptions they make, however a common feature of these models is their reliance on a representative agent assumption for assessing consumer behaviour and welfare impacts. While there has been a trend towards providing regional detail, the unit of analysis remains nation states or regional blocs (e.g. RICE, and WITCH, Bosetti et al. (2006)). In this paper, I change the unit of analysis to individual households, who experience varying degrees of vulnerability in the face of their economic decisions and the threat of climate change.

Climate change impacts are likely to vary significantly across the population, depending on household characteristics, including: location, occupation, wealth, etc. This paper focuses primarily on wealth inequality, and looks to address the question on how a realistic distribution of household wealth changes the optimal carbon taxation problem from the familiar representative agent framework. In this sense, this paper seeks to answer both *how* and *how much* inequality matters for optimal carbon taxation. The primary channel I investigate is the role that the carbon tax plays as public insurance. When capital markets are incomplete, households need to take precautionary action to insure against idiosyncratic shocks. Moving assets to the future then becomes a question of consumption smoothing, aggregate risk mitigation and insurance against idiosyncratic shocks. It is this last component that is absent from the current body of literature on optimal carbon taxation.

Models with incomplete markets and heterogeneous households have become common in more traditional research areas of macroeconomics, allowing a better understanding for the distributional impacts and implications of public policy. These types of models offer insights into the role of public policy as a way of mitigating risks, through, for example, social security and progressive taxation (see Heathcote et al. (2009) for an introduction). Climate policy can play a similar role. In light of poor households' explicit vulnerability to climate change, relaxing the representative agent assumption seems a natural progression for IAMs used for assessing the welfare impacts of climate change, and delivering estimates for the optimal policy response. In general, aggregation will miss the nuances of household behaviour and welfare implications across the distribution.

In order to explore these implications, I present a simple integrated assessment model that encompasses the carbon, climate and economic systems. The model is calibrated to match: a global $CO_2$ emissions path scenario from the IPCC, aggregate risk and damage estimates from the IAM literature, as well as, moments from the global distribution of income and wealth. The model includes both aggregate climate risk and idiosyncratic household productivity shocks, which may be correlated. The primary exercise is to evaluate a range of global carbon taxes, observe household welfare responses, and identify their most preferred policy.

This policy preference depends on both the characteristics of the household and the policy. The carbon tax is determined in advance of the period in which it applies; thus, household welfare is considered *ex ante*. In order to isolate the impact of wealth inequality on the identity of a household's most-preferred tax, I suppress the revenue redistribution channel and equalize idiosyncratic risk-profiles. In this analysis, households differ only on their wealth endowment - a determinant of their ability to self-insure. Clearly, *ex post* redistribution of the tax revenue can be a significant contributor to the distribution of welfare impacts. However, thorough analysis of revenue recycling, double-dividends and interaction with other distortionary taxation is beyond the scope of this study.

My first finding is that when idiosyncratic risk is correlated with the aggregate risk, such that the variance of a household's future labour income increases in a bad aggregate state, the dispersion of the population's tax preference becomes large and quantitatively relevant. This result arises from the way an increase in the risk on labour earnings affects households along the distribution of wealth. Increasing the carbon tax decreases earnings volatility and allows households to reduce precautionary savings. This effect is larger for households who receive a relatively large proportion of their earnings from labour - i.e. the poor.

My second finding, is that the economic vulnerability of poor households in the standard incomplete markets framework creates a role for carbon taxation as a form of public insurance that can substitute for private savings. Carbon taxation reduces the impact of extreme climate realizations on earnings. However, quantitatively, the direction of transfers inherent in the carbon tax rebate is the most important factor in a household's preference for strong climate policy arising from climate related damages.

# 2 Background

The impending threat of climate change is well-documented, and arguably represents the most significant challenge that policy makers have faced to date. As a an externality problem, climate change is characterised by a global commons challenge, whereby private use (combustion of fossil energy) impedes upon the quality of a public good - global climate. The climate change externality is compounded by the diffuse nature of the pollutant, carbon dioxide ($CO_2$) is uniformly mixed, and thus individual contribution to the stock of carbon in the atmosphere has a minuscule impact on one's own welfare. Another challenge is the reliance of our current industrial structure on the combustion of fossil energy. There is no easy substitute for fossil fuel, which makes solutions expensive. Given the severity of climate change's impact on the welfare of current and future generations, there is a clear opportunity for policy intervention and public coordination - preferably on a global scale.

A growing empirical literature on the impacts of climate change identifies significant distributional considerations. As mentioned above, the IPCC notes that the global poor will be especially susceptible to decreasing agricultural yields, access to clean drinking water and global market disruption. Skoufias (2012) summarizes some of the quantitative evidence on the welfare impacts of climate change particularly with respect to global poverty. The authors note that there are sectoral considerations, particularly with respect to decreasing agricultural productivity, however the most vulnerable population may be urban wage-labourers, who are particularly exposed to food price shocks. The global demographic shift towards ubranization also implies that this could be a key driver on climate change's influence on poverty metrics. Dell et al. (2013) review the empirical literature on weather shocks and climate impacts. Several of the channels through which weather can impact welfare include, labour productivity; health and mortality; and industrial and services output. While not addressing household inequality directly, the authors do note that climate impacts are likely heterogeneous, with damage being higher for low income countries. On longer horizons, climate-induced health shocks can create generational links from climate damage. Weather shocks which create better conditions for disease vectors or decrease maternal and infant nutrition can have effects on infant mortality as well as long-run implications for adult outcomes (e.g. education, wealth and health).

### *2.1 Related literature*

Recent work, related to this topic, has looked at environmental taxes in the context of distributional issues for public finance. Bosetti and Maffezzoli (2013) and Fried et al. (2015) use an incomplete markets framework and examine the distributional impacts of various carbon taxation schemes. Neither of these studies present an IAM, or indeed an externality - the aim not being to derive an optimal tax - but rather explore the implications of a potentially regressive environmental tax policy and the potential for double dividends through various revenue recycling schemes. In contrast, I hold constant the fiscal structure of the global economy and only allow climate specific taxation to vary.

This paper is also closely related to the work done on expanding IAMs to account for heterogeneity of impacts. Models with regional heterogeneity, such as RICE and PAGE, account for geographical heterogeneity and can be used to make assessments of the distributional impacts of climate change on poverty-related metrics, as in Skoufias (2012). To my knowledge, however, no study has relaxed the representative agent assumption in an IAM framework, and thus welfare analysis relies on aggregates, such as the elasticity of a poverty count to changes in GDP.

A recent study, Dennig et al. (2015) that acknowledges the need to move beyond regional aggregation, explores an alternative to the standard RICE framework by incorporating income inequality *within* regions. In the likely case that damages are greater for the poor within a region, the authors find that the optimal carbon tax would be well in excess of the case which does not account for intra-regional income inequality. While similar in spirit, my work differs from this in several key ways: I focus on individual household behaviour in the face of incomplete savings markets, rather than representative agents of regional income quintiles; I do not currently explore regional climate damage heterogeneity, or an explicitly regressive climate damage; and in my framework, the savings decision for each household is endogenous to the climate policy (rather than a fixed proportion of income), which ends up being a key channel through which inequality drives policy impacts. Finally, my analysis focuses on *ex ante* impacts caused by uninsurable risk. Therefore, my contribution is from the perspective of today's poor, rather than ex post analysis of the impact on the future poor.

# 3 The Framework

In order to address the question on how optimal carbon policy setting responds to changes in household wealth inequality, I propose the following simple dynamic framework, which adopts much of the structure from Golosov et al. (2014). The model is a dynamic stochastic general equilibrium model, which includes a simple description of climate change mechanics and allows for heterogeneous households. Thus it features a dynamic decision on household consumption and savings and competitive firms use fossil energy as an input in production. This increases the stock of greenhouse gases in the atmosphere, which accumulate over time and increase global mean temperature. The increase in temperature has a negative impact on aggregate production. Finally, there exists an aggregate shock related to the climate change externality, such that today's decision makers don't know the severity of the future temperature increase.

## *3.1 Households*

Each household $i$ chooses a sequence of consumption, $c_{i,t}$ and savings, $k_{i,t+1}$ to maximise their expected lifetime utility taking aggregate prices, $w_t$ and, $r_t$, as given.

$$
\begin{aligned}
&\max_{c_{i,t},k_{i,t+1}} \quad \sum_{t=1}^{T} \beta^t \mathbf{E}[u(c_{i,t})] \\
&\text{s.t.} \quad c_{i,t} + k_{i,t+1} = (1 + r_t - \delta)k_{i,t} + w_t l_{i,t} h_{i,t} + g_{i,t} \\
&k_{i,t+1} \geq -b
\end{aligned}
$$

where the households supply their period t labour endowment, $l_{i,t}$, (normalized to 1) inelastically. As in Bewley-Aiyagari-Hugget-type models, $h_{i,t}$ is an idiosyncratic labour productivity state that modifies an agent's labour income through the *effective* supply. Agents also have different wealth holdings, where $k_0$ is an initial endowment. Markets are incomplete, and households can not borrow beyond the constraint $b$. Households may also receive a government transfer, $g_{i,t}$, financed by the revenue from carbon taxation. Aggregate consumption, labour and capital supply are given by summing individual household contributions.

$$C_t = \sum_{i=1}^{n} c_{i,t} \quad L_t = \sum_{i=1}^{n} l_{i,t} h_{i,t} \quad K_t = \sum_{i=1}^{n} k_{i,t}$$

### *3.2 Production*

The product market is competitive, where representative firms solve a static problem each period by choosing how much capital, $K_t$, labour, $L_t$, and fossil energy, $E_t$, to use in order to maximize profits.

$$\max_{K_t, L_t, E_t} \quad (1 - D(T_t))F(K_t, L_t, E_t) - r_t K_t - w_t L_t - (\kappa + \tau_t)E_t$$

Fossil energy can be produced at constant marginal cost, $\kappa$, and is in large enough supply such that there are no scarcity rents. While scarcity is a feature of oil and gas fuels, coal is in virtual infinite supply from the perspective of the intended model horizon. Since firms are small, they do not recognize the contribution of their own emissions to global mean temperature, $T_t$. However, a regulator can implement a tax, $\tau_t$, in order to impact their energy use. The climate externality manifests itself in the form of a reduction in aggregate production, $1 - D(T_t)$, where "damage", $D(T_t)$ is increasing in temperature. In the model, temperature decreases production for a given set of inputs.

### *3.3 Climate change*

The Greenhouse Effect arises from the growing stock of atmospheric carbon, $S_t$. As the stock of carbon grows, the energy flow out of the earth's atmosphere decreases and results in rising global temperatures. Economic activity contributes to the stock of carbon through the combustion of hydrocarbon energy, $E_t$. While there is a potential to model the complexity of the climate system, including multiple carbon reservoirs, feedback effects, etc., I employ a more concise statement of the climate system. First, I note that the relationship between the atmospheric concentration of carbon and the global mean temperature can be interpreted as roughly linear over relevant ranges for $S_t$.[1] This allows me to abstract from

[1] This observation arises from two counteracting trends in the way that carbon behaves in atmosphere. On one hand the impact of the atmospheric stock of carbon on global mean temperature follows a logarithmic trend, while on the other hand the increase in carbon absorbed by oceans leads to higher acidity levels and a decrease in their absorptive capacity. The combination of these effects yield essentially a linear relationship between temperature and carbon in the atmosphere.

temperature in the model, and instead focus on carbon as the key climate state variable. I also choose to make a simplification on the way that $S_t$ evolves over time. Golosov et al. (2014) propose the following reduced-form carbon depreciation function, which relates facts about the persistence of carbon emissions in the atmosphere to how much of a marginal impulse of emissions remains in the atmosphere after a length of $s$ periods.

$$1 - d(s) = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^s \tag{1}$$

where $\varphi_L$ is the share of $E_t$ that remains in the atmosphere forever, and the remaining parameters are calibrated to account for facts about the life-cycle of carbon in the atmosphere. Thus there are two components of $S_t$ at any point in time, a permanent component $S_{1,t} = S_{1,t-1} + \varphi_L E_t$, and a component that depreciates over time, $S_{2,t} = \varphi S_{2,t-1} + \varphi_0(1 - \varphi_L)E_t$.

As mentioned earlier, damage takes the form of a reduction in aggregate output. This is a large simplification of the negative impacts that a rising global mean temperature would have on human welfare. One could imagine other ways in which climate damage could be represented, such as direct loss to household utility, or an increase in the capital depreciation rate, however many IAMs , including Nordhaus' DICE model, assume a loss of aggregate output. For the sake of comparison to popular formulations of other IAMs, I choose to follow this assumption, and implement the aggregate damage function proposed in Golosov et al. (2014).

$$1 - D(S_t) = \exp -\theta_{k,t} S_t \tag{2}$$

Climate change damage is also a source of aggregate risk, where the eventual realization of atmospheric carbon's potency as a GHG is a source of uncertainty faced by decision makers in the model. For simplicity, I assume there are two possible realizations of the aggregate shock, $\theta_k$, which occurs at some unknown point in the future. $\theta_{high}$ occurs with the probability of $\pi_{high}$ and denotes a high impact the climate externality, while $\theta_{low}$ occurs with probability $1 - \pi_{high}$.

# 4 Representative agent reference case

The solution to the model framework when markets are complete is equivalent to solving the model in the absence of income risk and borrowing constraints. If, in addition, global households are represented by an agent with mean wealth, the optimal tax has the familiar interpretation of the Pigouvian tax, which is set in order to equate the marginal private cost to the marginal social cost (in the case of a negative externality). With the ability to aggregate all agents in an economy to a single representative agent, it is also easy to define a social welfare function to be optimized: to maximize the representative agent's utility. Thus, I turn to the planning solution to identify the optimal level of emissions (which implies the optimal tax value) under complete markets.

$$F_E - \kappa = \tau_t = \mathbf{E}_t \sum_{s=0}^{T} \beta^s \frac{u'(c_{t+s})}{u'(c_t)} F_S(K_{t+s}, L_{t+s}, E_{t+s}, S_{t+s}) S'_{t+s} \tag{3}$$

Where primes denote a function's first derivative with respect to $E_t$. This expression shows that the tax which implements the optimal allocation of fossil fuel use is set equal to the difference between the marginal private benefit of fuel use (marginal product of energy $F_E$) and the marginal private cost, $\kappa$, at the social optimum fuel allocation. The right hand side of this expression is often referred to as the social cost of carbon (SCC) and includes the damage associated with the negative externality from fossil fuel use, both in the current period, as well as, future periods through the persistence of the carbon pollutant.

In the absence of distortionary taxation, the carbon revenue is rebated lumpsum to the representative household. In the representative agent case there are no gains from redistribution. It is also implicit that each household views the climate threat in the same way. There is significantly less uncertainty about one's future welfare under such assumptions.

# 5 Stylized Model

In order to understand how household inequality may impact the setting of an optimal carbon policy, I propose a stylized version of the dynamic model summarized above. The stylized model retains the features that are important for exploring the channels through which inequality and climate vulnerability matter. Dynamics coupled with uncertainty provide the channel through which the current poor are implicitly more vulnerable to climate risk.

## *5.1 Period 1 as an endowment economy*

As a illustrative simplification from the Section 2 framework, I assume that there is no production in the first period, but rather households can consume and save from their initial endowment. Household inequality stems from the initial distribution of assets. An implication of there being no production, is that there is no fossil fuel use in period 1, and thus the stock of carbon is only impacted *endogenously* by firms use of fuel in period 2. Production in period 2 yields factor prices, from which households earn income in period 2.

This stylized model is summarized by the following household and firm problems and their resulting equilibrium conditions.

$$\begin{aligned}
&\max_{c_{i,1},c_{i,2}} \quad u(c_{i,1}) + \beta\mathbf{E}[u(c_{i,2})] \\
&\text{s.t.} \quad c_{i,1} + k_{i,2} = \omega_i \\
&c_{i,2} = (1 + r_2 - \delta)k_{i,2} + w_2 h_{i,2} + g_{i,2} \\
&k_{i,2} \geq -b
\end{aligned}$$

where $\omega_i$ is household $i$'s initial endowment.

The resulting optimal savings condition for household $i$ is given by:

$$\begin{aligned}
&-u'(c_{1,i}) + \beta\mathbf{E}[R_2 u'(c_{2,i})] + \mu_i = 0 \\
&\mu_i[k_{2,i} + (-b)] = 0 \\
&\mu_i \geq 0
\end{aligned}$$

Assuming CRRA utility, an unconstrained household $i$ will save according to:

$$(\omega - k_{2,i})^\sigma = \mathbf{E}\left[\frac{(w_2 h_{i,2} + R_2 k_{2,i} + g_{i,2})^\sigma}{\beta R_2}\right]$$

Assuming Cobb-Douglas production, Period 2 factor prices and firm input demands are given by the solution to the firms problem as stated in the previous section:

$$\begin{aligned}
&r_t = \alpha e^{-\theta_{t,k} S_t} K_t^{\alpha-1} L_t^{1-\alpha-\nu} E_t^\nu \\
&w_t = (1-\alpha-\nu) e^{-\theta_{t,k} S_t} K_t^{\alpha} L_t^{-\alpha-\nu} E_t^\nu \\
&\kappa(1+\tau_t) = \nu e^{-\theta_{t,k} S_t} K_t^{\alpha} L_t^{1-\alpha-\nu} E_t^{\nu-1}
\end{aligned}$$

From this we can see that fossil fuel demand is decreasing in $\tau$ , and thus can be set by the regulator to internalize the climate change externality. Also factor earnings are decreasing in the atmospheric stock of carbon.

### *5.2 Generating inequality*

Household inequality in the stylized model arises from two sources: a random wealth endowment that places the recipient on the global wealth distribution and an idiosyncratic labour productivity draw that adds to the initial endowment resources in the first period, as well as, determines the potential for future earnings. These sources of idiosyncratic uncertainty are potentially correlated in that a household with a higher wealth endowment may be more likely to experience a high labour productivity shock in period 2. Labour is supplied inelastically, so a household's period 2 labour income is dependent on their period 2 productivity realization and the prevailing aggregate wage.

Under this structure, the distribution of wealth is controlled by choosing a distribution for the initial wealth endowment. Income inequality consists of multiple states, which are meant to represent a household's position on the global income distribution. In general, there can be many income states in order to meet more precise income inequality targets, and it is clear that a realistic representation of global income inequality would require many income states - especially to represent the difference between those in poverty in the developing world and those living in poverty in a wealthy nation.

### *5.3 Calibrating the stylized model*

In order to give the stylized model a quantitative grounding, I proceed by calibrating the model to reflect the global interaction of climate and the economy over two periods of fifty years each. The model has three broad categories for calibration: preferences and technology, carbon and climate, and household inequality.

#### Preferences and technology

I adopt fairly standard assumptions for preferences and technology from the macroeconomics literature, including CRRA utility, Cobb-Douglas production and full depreciation. In the short-run, the degree of substitutability between capital-labour and energy should be relatively limited. However, the length of periods in the model allow assumptions that correspond to longer horizon characteristics of the production side. Factor shares, $\alpha$ and $\nu$ are based on averages from historic data, with respective values 0.3 and 0.04 taken from Golosov et al. (2014). The final parameter on the firm side is the constant marginal cost of fossil fuel use, $\kappa$, which I calibrate endogenously to achieve the business-as-usual atmospheric stock of carbon estimates from the most recent IPCC report.

The choice of $\beta$ is an important (and controversial) one in IAMs, as it determines the weight that current decision makers put on future generations, when the bulk of climate change is due to occur. The value of the optimal policy is very sensitive to the selection of this parameter (see e.g. Tol (2009), Saelen et al. (2008) and Anthoff et al. (2009) for discussion). However, in the absence of heterogeneity across households in regards to $\beta$, it is not essential for understanding the question of *intra*-generational inequality[2]. For now I choose 0.98 as an annual rate, which is in a standard range for this parameter in the family of IAMs and allows the opportunity for sensitivity analysis.

**Carbon and climate**

The simplified carbon system requires one parameter, $\varphi$, which denotes the share of carbon that remains in the atmosphere after an emissions pulse. Using a reduced-form carbon depreciation formula from Golosov et al. (2014) allows the model to abstract from multiple carbon stocks (atmosphere and oceans), and rely on a single atmospheric carbon stock. Using this formulation and period length of 50 years, I assume that $\varphi = 0.49$, which means that roughly half the emitted carbon remains in the atmosphere. $S_2$ is the atmospheric carbon stock associated with IPCC predictions for business as usual (laissez faire equilibrium) $4°C$ increase in temperature by 2100. We can find this by using a formula from Arrhenius (1896), which relates an increase in the stock of carbon over pre-industrial levels to global mean temperature [3].

$$4 = \Delta T = \lambda \frac{\ln \frac{S}{S_0}}{\ln 2} = 3 \frac{\ln \frac{S}{600}}{\ln 2}$$

where $\lambda$ denotes the *sensitivity* of temperature to atmospheric carbon concentration (or more precisely denotes the increase in temperature resulting from a doubling of pre-industrial atmospheric carbon concentration, which is here set to $3°C$). This corresponds to an atmospheric carbon stock value of 1,500 (GtC). This is roughly 900 GtC in excess of pre-industrial levels. Thus 900 GtC becomes the calibration target for the business-as-usual (BAU) value of $S_2$ (after normalising $S_0$ to 0). To find out how much carbon is emitted in the second period alone, I return to the IPCC BAU scenario which predicts roughly $2°C$ warming by 2050, and using the same method implies $S_1 = 350$ GtC. Taking the difference between the two periods' stocks implies that our laissez-faire economy has to produce $\phi E_2 = S_2 - S_1$, $E_2 = \frac{900-350}{0.49} \approx 1100 GtC$.

The exponential functional form that climate damage takes requires the calibration of

[2]It is perhaps worth discussing the role of heterogeneity in time preference as a theory of inequality and a means of generating realistic distributions of wealth in equilibrium (see for example Krusell and Smith (1998)). Clearly if households have varying preferences for future outcomes, this opens up another dimension for setting a one-size-fits-all carbon policy.

[3]see macro handbook chapter for further information – citation pending

$\theta$, which can be found by solving the relationship $exp{-}\theta S_2 = 1 - D(T)$. Following the calibration of Golosov et al. (2014), who also include uncertainty in their estimates, I choose $\{\theta_h, \theta_l\} = \{2.046 \cdot 10^{-4}, 1.060 \cdot 10^{-5}\}$. These values imply a loss to aggregate output of roughly 20% and 1% respectively, if $S_2$ reaches 900 GtC by 2100. Assigning probabilities to the two states, I follow the same calibration $\{\pi_h, \pi_l\} = \{0.934, 0.068\}$.

**Household inequality**

The final category for calibration is household inequality. As explained above, there are two sources of household heterogeneity, which arise from two sources of economic inequality. Agents in the model are assigned an initial wealth and labour productivity profile. Initial household wealth is distributed according to the wealth distribution in Davies et al. (2011). According to this study the level of wealth in our base year 2000 is 44,000 per adult (PPP), and the distribution is summarized below in 1.

Table 1: Distribution of Wealth

| Decile | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| World wealth share % | 0.1 | 0.3 | 0.6 | 1.1 | 1.6 | 2.4 | 3.8 | 6.3 | 13.1 | 70.7 | 0.802 |

Agents are also assigned a productivity state in the first period, which corresponds to their position in the income distribution. There are five productivity states calibrated according to the quintiles of the global income distribution (PPP) in Ortiz and Cummins (2011) and shown in the calibration table 2. I assume that whatever causes a household to be productive also causes them to be wealthy, such that the initial wealth endowment is distributed according to a household's position on the income distribution (and vice versa). Since the first period is an endowment economy, a household's first period productivity state determines two things, their belief about their future earnings (through the probability transition matrix) and the total size of their period 1 endowment. Thus, each household receives two endowments, one that represents their initial wealth holdings and one that represents the labour income they earn during the first 50-year period. Since income is a flow, I calculate the income endowment by taking the level of income (PPP) in the base year and grow it at the growth rate of world GDP over the first period and then sum all years. I then take this total amount and divide it in proportion to a quintile's share of total income. Each member within a quintile receives an equal amount of that quintile's share.

Table 2: Parameters Calibrated Exogenously

| Parameter | Value | Description | Source |
|---|---|---|---|
| *Preferences* | | | |
| $\beta$ | 0.98 | Annual discount factors | *Macro literature* |
| *Technology* | | | |
| $\alpha$ | 0.3 | Capital's value share of output | *Macro literature* |
| $\nu$ | 0.04 | Fossil energy's value share of output | (Golosov et al., 2014) |
| $b$ | 0 | Household borrowing limit | Author's choice |
| *Carbon and climate* | | | |
| $\theta_l$ | $1.9341 \cdot 10^{-5}$ | Climate damage elasticity in low state | (Golosov et al., 2014) |
| $\theta_h$ | $2.3780 \cdot 10^{-4}$ | Climate damage elasticity in high state | —"— |
| $[\pi_l, \pi_h]$ | [0.932, 0.068] | Probabilities of aggregate states | —"— |
| $\varphi_L$, $\varphi_0$, $\varphi$ | 0.2, 0.4, 0.0987 | carbon depreciation rates | |
| *Inequality* | | | |
| Income quintiles | [0.827, 0.117, 0.023, 0.019, 0.014] | Share of global income | Ortiz and Cummins (2011) |

# 6 Carbon tax experiment

As an exercise to examine the impact of a carbon tax over the distribution of households, I evaluate the stylized model over a grid of tax values and examine the response of households across the wealth distribution. The idea being that the characteristics of an individual household will lead to varying welfare impacts and thus a most-preferred tax value. As the first period is an endowment economy, the carbon tax is only levied in the second period when production occurs. However the value of the tax is "negotiated" in the first period. Although pessimistic, it is perhaps not an unrealistic assumption that a globally coordinated tax needs to be set in advance of the period in which it becomes active. Choosing the carbon policy in advance implies that welfare analysis is from the perspective of period 1. Thus a household's favourite tax is chosen according to its beliefs about what will happen in the future.

Given the degree of inequality in the world, there is substantial opportunity to increase welfare through redistribution strategies, or pursuing "double dividend"-style tax relief. In the absence of distortionary taxes in the model, I opt for lump sum redistribution of the carbon tax revenue. There are many possible ways to share the tax revenue, and this will have a large impact on the identity of an agent's most preferred tax. For this exercise, I explore two approaches for handing the tax revenue. In the first case, I discard the tax revenue in order to isolate the mechanisms associated with the carbon tax's impact on capital and labour income. Discarding revenue is clearly suboptimal, however it reveals a few channels through which the carbon tax can influence a household's welfare. The second approach is to rebate the tax in such a way as to be non-redistributive. Achieving perfect neutrality of the rebate is difficult in the incomplete markets setting with aggregate risk, so instead I condition the rebate on the first period endowment.

I begin with the experiment where the carbon tax revenue is discarded. Following the calibration described above reveals that a standard calibration of climate damage is insufficient to make any agent better off. Figure 1 shows the difference between gross output and output net of carbon tax revenue. From the gross output curve, we see that the carbon tax increases the availability of resources over a range of the instrument. The difference between the two curves is the revenue raised by the regulator from levying the carbon tax on fossil fuel use.

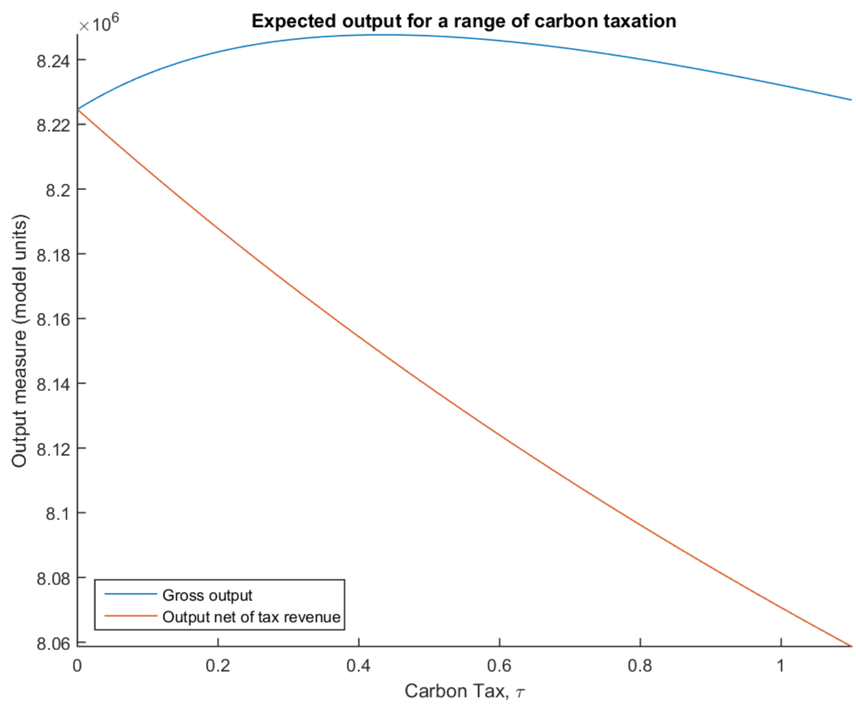


Figure 1: Output response to carbon tax

To proceed I make an adjustment to the damage parametrization, which will become a point of comparison for both the case where tax revenue is discarded, as well as the case where revenue is returned. The extent of damage from climate change is one of the most uncertain aspects of calibration in IAMs. If the climate is more sensitive than the standard calibration, then temperature will rise more quickly, and damage will be more severe. In proceeding I assume that damage to production is twice as bad in expectation as in the previous calibration. Here 10% of production will be lost when atmospheric carbon reaches 1500 GtC by 2100, rather than 5%. To do this, I leave the elasticity of damage in the high damage state unchanged at $\theta_h = 1.9341 * 10^{-5}$, but increase the elasticity of damage in the low state to $\theta_l = 6.0335 * 10^{-5}$. The high and low aggregate state probabilities remain unchanged. The big implication of increasing the elasticity of damage is that the benefit from reducing emissions outweigh the costs even when the tax revenue is discarded. This is a way of understanding how the carbon tax affects households in the absence of redistribution. Taking the carbon tax rebate out of the policy means that household welfare can only be influenced through the tax's impact on labour and capital earnings.

#### High damage, no rebate

The first case for this experiment returns to a world where the state of the climate does not have implications for the idiosyncratic risk. That is, the payoffs and transition probabilities in the idiosyncratic states are the same whether the aggregate state is good or bad. In addition, there is no income persistence across periods, as agents face i.i.d probability. Thus an agent in period 1 is equally likely of transitioning to one of the five income states in period 2. While perhaps unrealistic in a global inequality context, an i.i.d probability transition matrix is attractive for my initial analysis as it ensures that households have identical income risk profiles. Thus households differ only on the amount of resources available to them in the endowment period. I leave the assessment of persistent income states for sensitivity analysis. Figure 2 reveals that there is little difference between households preferences for carbon taxation.

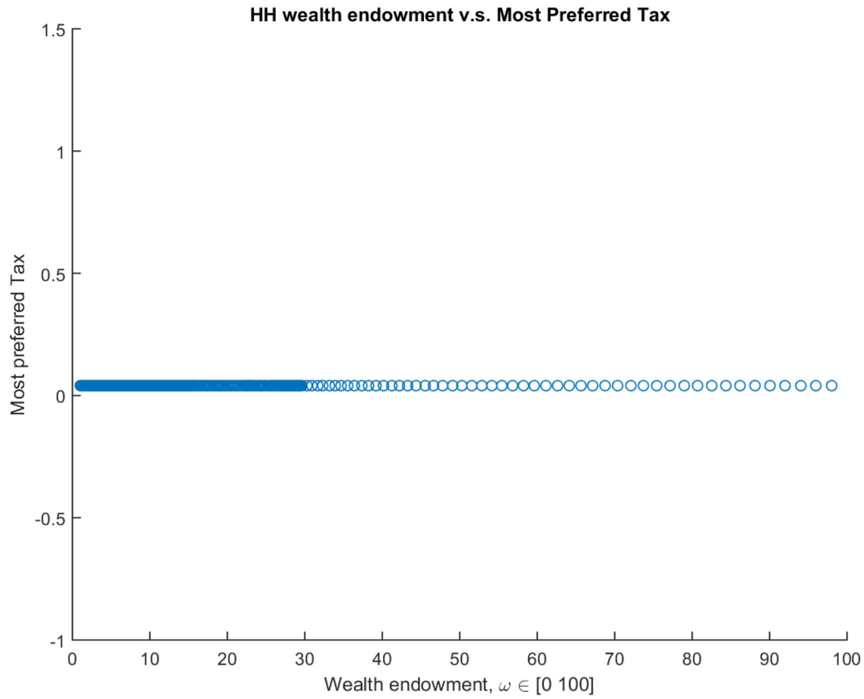


Figure 2: HH Wealth Endowment vs Most Preferred Carbon Tax - no rebate

The reason for this result comes from the symmetry in how the sources of household earnings are impacted by climate change damage in the model. With full depreciation and Cobb-Douglas production, the proportional change in the factor earnings are quantitatively very similar in this case.[4] This symmetry means that households who receive their earnings entirely from labour or entirely from capital benefit from the carbon tax similarly.

### High damage, no rebate, correlated risk

If, on the other hand, households idiosyncratic risk is positively correlated with the aggregate climate state, this symmetry between earnings sources breaks down. Following the notion that climate impacts will be unequally distributed across the population, I explore the implications of adverse shocks hitting a subset of the population. The idea being that low productivity households will be impacted more by climate change than high productivity households. Some examples of how this might occur is from the evidence of temperature on labour productivity. Dell et al. (2012) discuss the existing empirical evidence noting that sectors which involve outdoor labour, such as agriculture, mining, construction, forestry,

[4] In addition, if utility was logarithmic there would be no savings adjustment at all by households, even in the case below where I assume idiosyncratic risk is positively correlated with aggregate climate risk.

etc. see drops in productivity during high temperature weather. Agriculture is arguably the most susceptible to climate change impacts and the global agricultural labour force is largely concentrated in low income countries and amongst low earners.

From the equity premium literature Mankiw (1986) shows that when asset markets are incomplete the concentration of ex post adverse shocks can increase the ex ante value of existing market assets. This logic translates into the climate change framework when idiosyncratic productivity is correlated with the aggregate climate state. To explore this, I introduce a mean-preserving spread to the idiosyncratic productivity states when the aggregate climate state is bad. Thus, the volatility of labour productivity increases when climate damage is most severe. Since all households are equally likely of being subject to these shocks in the future, it does not change their expected labour income, only its volatility.

$$\begin{pmatrix} h_1 - \mu & h_2 - \mu & h_3 & h_4 & h_5 + 2\mu \end{pmatrix}$$

In this case, the uncertainty of labour income increases, putting additional pressure on poor constrained households who rely completely on their future productivity realization for period 2 welfare. I choose a $\mu$ equal to 0.01, which under my income state calibration results in a roughly 70% loss in productivity for the lowest quintile should the climate realization be the high damage state. From an aggregate perspective this may seem like a high number, however recent studies on disaggregated impacts, such as Krusell and Smith Jr (2015), find damage impacts similar to these magnitudes even in scenarios which correspond to aggregate global damages that are in line with the low damage aggregate state. Figure 3 reveals a relationship between a households most preferred tax and their wealth endowment.

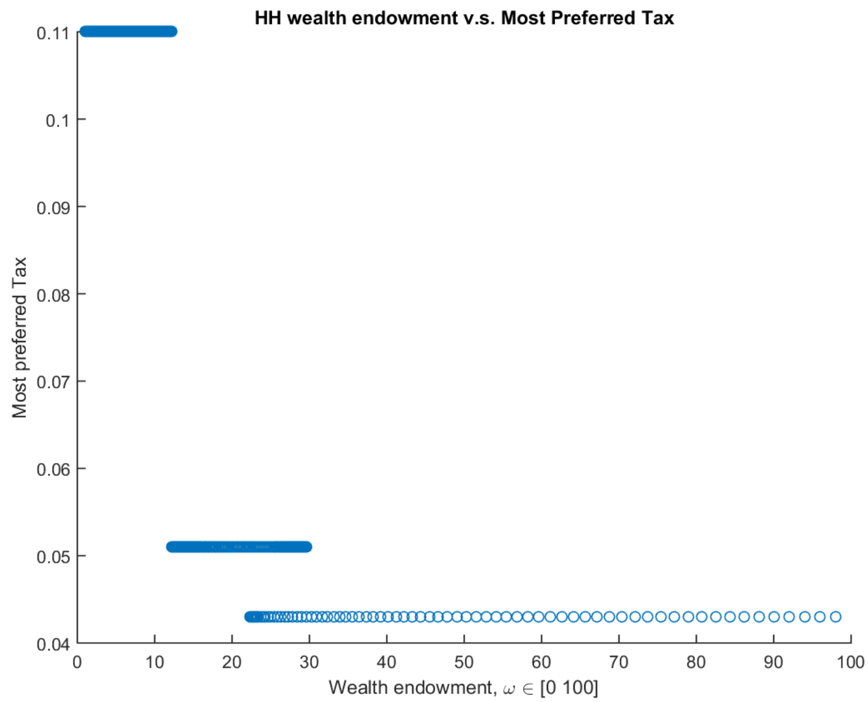


Figure 3: HH Wealth Endowment vs Most Preferred Carbon Tax - no rebate, correlated risk

Poor constrained households prefer a tax more than twice as large as their wealthier unconstrained counterparts. This result arises from constrained households greater exposure to climate risk. Agents who rely entirely upon labour income benefit from stronger action on climate change since cutting emissions reduces losses associated with the worst potential outcomes. Agents with private savings will not be hurt as badly in these realizations since they will have additional resources on hand for adaptation regardless of their labour productivity state. Amongst unconstrained households, the relative composition of earnings determines their most preferred tax, with the fourth income quintile still receiving a large enough proportion of their earnings from labour to prefer a higher carbon tax than the wealthier quintile above them.

There are however, additional general equilibrium considerations for wealthy households' tax preference. Figure 4 charts household tax preference against their wealth endowment, when only considering the portion of their earnings from capital. This reveals a reversal of the pattern under total earnings. Here wealthier households prefer a *higher* tax. The reason for this is that carbon taxation reduces savings more quickly amongst poorer households. This reduction in savings benefits wealthier households through higher returns on their own savings, which they are not inclined to adjust as quickly in the face of increased carbon taxation.

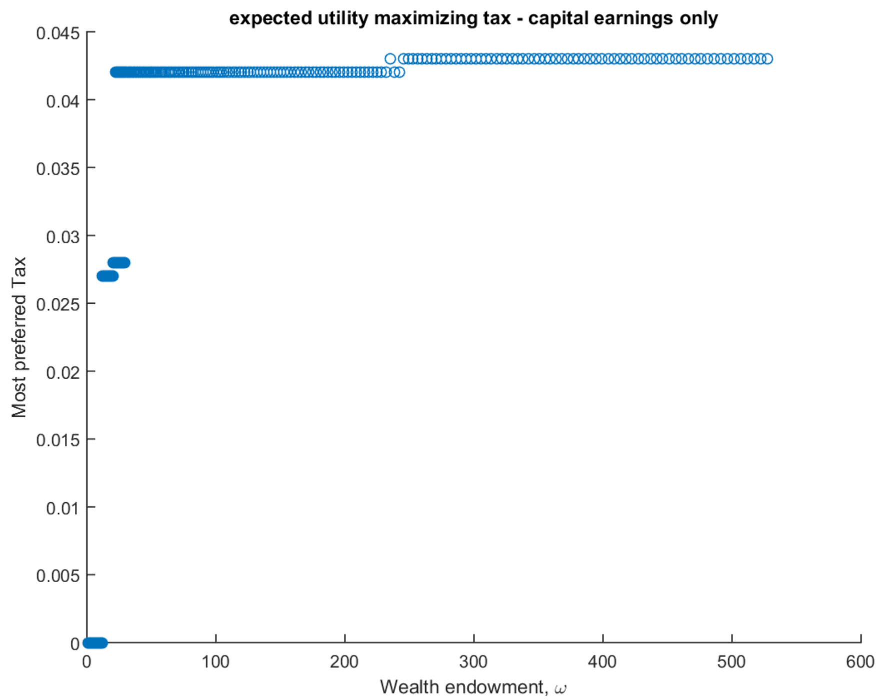


Figure 4: HH Wealth Endowment vs Most Preferred Carbon Tax - capital earnings only

This general equilibrium effect only slightly mitigates the income composition effect, and in this scenario carbon tax preference is decreasing in wealth.

### *6.1 Neutral rebate experiment*

Clearly, discarding tax revenue is a suboptimal policy. There exists a substantial opportunity to improve welfare by rebating the revenue to households, and given the inequality in the world, there is a potential for the climate tax to redistribute resources in the future. Returning the carbon revenue in a way that, in expectation, redistributes future resources in a neutral fashion is difficult in the face of both idiosyncratic and aggregate uncertainty. As a means of illustrating the impact of climate risk explicitly on carbon tax preference, I construct an experiment that compares households under a situation where the revenue rebate rule has been parametrized to be non-redistributive. To accomplish this I eliminate the influence of the carbon stock on economic activity. Specifically, I set the damage elasticity, $\theta_k$, to zero, which implies that there is no climate change externality from fossil fuel use. In this situation, a tax on fuel use is distortionary in the sense that it pushes the firm away from its socially optimal level of use.

In a representative agent framework with complete markets and no externalities, a fuel tax would not be welfare improving. However, under incomplete markets with heterogeneous households, there are two channels through which welfare can be improved for at least some agents. The first is redistribution, which replaces the loss of welfare from decreased factor earnings with lump sum rebates of the fuel tax revenue. The other channel arises from the incomplete markets setting specifically. Previous literature has shown that taxes which are distortionary and inefficient in a complete markets setting, can be Pareto improving when markets are incomplete due to their ability to reduce uncertainty in the income stream.[5] Here, taxing fuel and returning a predictable rebate to households allows them to reduce their precautionary savings and improve their overall welfare position, despite the expected fall in factor earnings from the fuel tax.

The goal then is to find a tax level and redistribution rule such that all agents are worse off in the absence of the climate change externality. Finding a starting point for the revenue recycling rule reveals how strong the insurance motive of the rebate scheme is in this incomplete markets setting. Due to the large degree of idiosyncratic risk that households face, there is substantial welfare to be gained through taxing fuel and returning a predictable lump sum. I settle on a tax level of 150 % of the fuel price, and rebate the revenue to households as a function of their period 1 endowment. Households who are wealthy in the initial period will receive a larger share than their poorer counterparts, regardless of what their future income realization is.

This rebate rule becomes the starting point for the analysis in this experiment, where I reintroduce the climate externality at the normal damage parameterization from the calibration section and solve for each household's most preferred tax. I then repeat this most-preferred tax exercise, but at the higher damage parameterization and compare the *change* between tax preferences after the damage parametrization increase. To do this, I take the ratio of a household's most preferred tax under both high and low damage. The idea being to control for the idiosyncrasies of the household risk profile and rebate scheme by comparing the same household under two climate damage regimes. Thus I compare a household's tax preference in response to the intensity of climate damage only. The pattern which emerges in Figure 5 is consistent with relatively poor agents being more vulnerable to climate risk. Agents with low period 1 endowments prefer a stronger increase in taxation in response to higher climate sensitivity. The poorest households prefer a tax around 1.8 times higher when the climate is more sensitive, while the wealthiest prefer a tax between 1.5 and 1.6 times

[5] For example Krusell and Smith (2006) provide an example where a tax on investment can improve household welfare by lowering the equilibrium capital stock, which decreases wage risk and boosts capital returns

higher. However, in terms of magnitude there is not a large difference over the distribution of tax preference. A rebate designed to be non-redistributive allows for a small variation in tax preference across the distribution of households, however this preference variation is swamped by the impact of transfers.

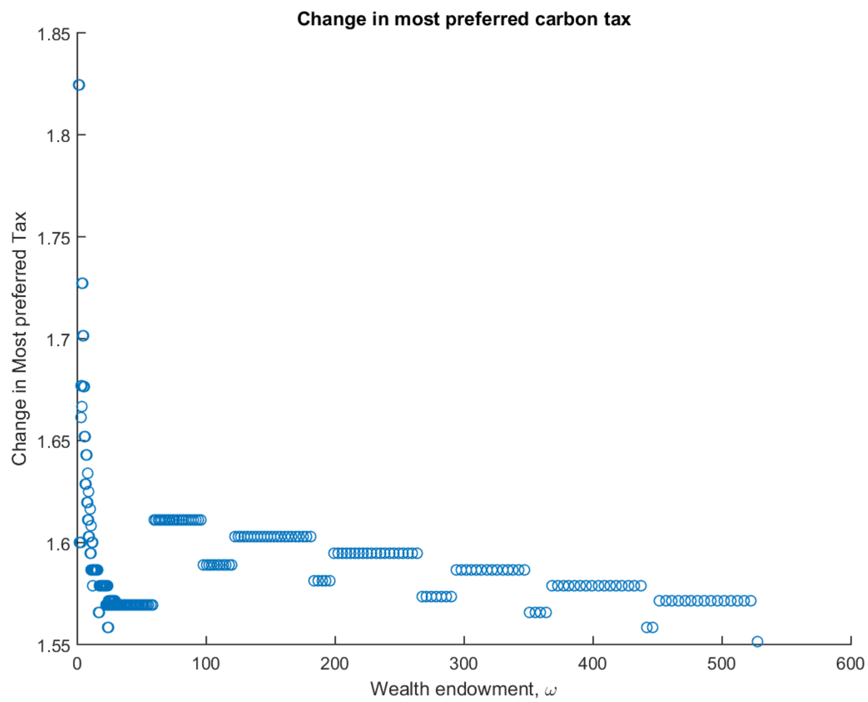


Figure 5: Change in HH most preferred tax by endowment position

# 7 Conclusion and discussion

Currently, models of climate and the economy answer normative questions about optimal carbon taxation under assumptions of complete markets and representative agents. Relaxing these two assumptions allows a better understanding of how implicit vulnerability of poor households and distributional impacts can shape the optimal policy problem. Modifying a simple integrated assessment model with a standard incomplete markets framework is a first step in incorporating concerns of global household inequality in a familiar policy evaluation framework for addressing climate change.

A common theme in the literature on incomplete markets is the role of public policy as implicit insurance. In the absence of comprehensive risk markets, policy makers can improve welfare by implementing various policies, e.g. progressive income taxation and public pension plans. This study shows that carbon taxation can fill a simliar role in an integrated assessment model setting. A carbon policy fulfils this role in several ways. First, the tax reduces the use of fossil fuel and thus mitigates the severity of climate damage, especially in extreme realizations of aggregate risk. In the model there are two sources of risk for which agents self-insure through precautionary savings, idiosyncratic productivity and aggregate climate risk. Lower wealth agents make a relatively costly trade-off by reducing current consumption to insure against both lower labour earnings and a bad realization for the climate state. Since emissions negatively impact aggregate prices, the carbon policy can both improve tomorrow's expected earnings, as well as, reduce the volatility of tomorrow's consumption. Furthermore, if idiosyncratic productivity outcomes are correlated with the aggregate climate state, such that household productivity is more volatile in a bad climate state, then the tax becomes more important for all households - and poor households in particular. Secondly, the carbon policy presents an opportunity to increase the gross resources available by internalizing the climate change externality. The way these additional resources are distributed amongst the population can have substantial welfare implications, particularly for constrained households who are unable to self-insure.

The most interesting result from this experiment is the heterogeneity in tax preference in the absence of redistribution. Household earnings arise from two sources, capital and labour. Since households differ in their endowment, the relative importance of the source of their earnings can introduce asymmetries in their tax preference. When idiosyncratic productivity shocks are correlated with the aggregate climate state, households face asymmetric risk to the two components of their earnings. Households which rely more heavily on their labour earnings, i.e. the poor, will prefer a tax that improves both the expected value and volatility of labour earnings. In addition, agents which have substantial private savings will face less volatility in their utility, implying that risk *per se* is more harmful to poor households. For these reasons, we observe a result where tax level preference is decreasing in wealth.

The magnitude of differences in carbon tax preferences across the distribution of households is largely dominated by concerns over the direction of transfers. Even under consideration of high climate damage a distribution neutral rebate rule yields only marginally higher tax preference from the poorer households. The degree of economic vulnerability that arises in the standard incomplete market setting places a large emphasis on how the transfers are directed. The role that the carbon policy plays as a form of public insurance will be a significant part in determining the identity a global tax agreement in a setting of incomplete markets and substantial household inequality.

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