STOCKHOLM UNIVERSITY
Department of Economics

Course name:	**The Macroeconomy in the Long Run**
Course code:	**EC7215**
Examiner:	**Johan Söderberg**
Number of credits:	**7.5 credits**
Date of exam:	**10 December 2016**
Examination time:	**3 hours (09.00-12.00)**

Write your identification number on each answer sheet. Use the printed answer sheets for all your answers.

Do not write answers to more than one question in the same cover sheet. Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked. No aids are allowed.

The exam consists of 4 questions. Each question is worth 25 points, 100 points in total. For the grade E 45 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Only students who have NOT received credits from the seminar series should answer question 4.

Results will be posted on mitt.su.se three weeks after the exam, at the latest

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Good luck!

Question 1 (25 p)

Consider an economy with an infinitely lived representative household where population grows at constant rate n and technology at constant rate g. The household's disposable income in period t is given by

$$D_t^Y = W_t L_t + (1-\tau) R_t K_t + T_t \tag{1}$$

where τ is a capital income tax (a subsidy if negative) that is distributed back to the household via the lump-sum transfer T_t.

The household head solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \widetilde{\beta}^t \log\left(\frac{C_t}{L_t}\right), \tag{2}$$

where $\widetilde{\beta} = \beta(1+n) < 1$ is the effective discount factor, subject to the budget constraint

$$C_t + I_t = D_t^Y, \tag{3}$$

where the evolution of the capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{4}$$

Firms are price-takers, operating on a perfectly competitive market. Production is determined by the production function

$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}. \tag{5}$$

The representative firm's problem is to choose K_t and L_t to maximize its profit function

$$\Pi_t = K_t^{\alpha} (A_t L_t)^{1-\alpha} - w_t L_t - R_t K_t. \tag{6}$$

a) Assume that the government runs a balanced budget every period. Derive the model's equilibrium conditions under this assumption.

b) Assume that the economy is in a steady state, where variables normalized by the number of effective workers, $A_t L_t$, are constant. Rewrite the equilibrium conditions in intensive form (in terms of normalized variables). How is the steady state value of capital per effective worker in the economy affected by τ and β? Explain intuitively!

c) Suppose that household instead of maximizing utility saves a fraction $s \in [0,1)$ of their disposable income every period (and thus consumes a fraction $1-s$). Derive the equilibrium conditions in intensive form under this assumption. How is the steady state value of capital per effective worker affected by τ and s? Explain intuitively!

Question 2 (25 p)

Assume an OLG economy without population growth where individuals live for two periods with certainty. The size of the generation born in period t is L_t. Assume that an individual supplies 1 unit of labor and earns w_t in the first period of his life and is retired and not working in the second period. An individual born at time t has life-time utility

$$\log C_{1t} + \beta \log C_{2t+1}, \tag{7}$$

where C_{1t} is the individual's consumption in the first period of his life and C_{2t+1} his consumption in the second period. The individual is born without assets. Savings are invested in the aggregate capital stock, which yields a return of $r_t = R_{t+1} - \delta$.

Firms are price-takers, operating on a perfectly competitive market. The representative firm's problem is to choose inputs of capital and labor to maximize its profit function

$$\Pi_t = F\left(K_t, L_t\right) - w_t L_t - R_t K_t. \tag{8}$$

a) Assume that each old individual is required by law to leave a bequest of size κ to an inheritance fund. In any period t, the inheritance fund invests the bequested amount from the current period old in the aggregate capital stock and distributes (in period $t+1$) the bequested amount with interest evenly among the individuals that are old in period $t+1$. Write down the first and second period budget constraints for an individual born in period t.

b) Derive an expression for the individual's savings between the first and second period of his life as a function of w_t and κ. Explain intuitively how the individual's savings are affected by κ.

c) One equilibrium condition in the model is the resource constraint, given by

$$K_{t+1} + C_t = (1-\delta) K_t + F\left(K_t, L_t\right). \tag{9}$$

Derive the other equilibrium condition in the model.

d) How is the size of the aggregate capital stock affected by κ? Explain intuitively!

Question 3 (25 p)

Consider an economy without population growth where the household head solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log\left(\frac{C_t}{L_t}\right), \tag{10}$$

subject to the budget constraint

$$C_t + I_t = w_t L_t + R_t K_t, \tag{11}$$

where the evolution of the capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{12}$$

In the economy, there are a large number of profit-maximizing firms, indexed by i and measured on the unit interval. The production function of firm i is given by

$$Y_{it} = K_{it}^{\alpha} \left(A_t L_{it}\right)^{1-\alpha}, \tag{13}$$

where K_{it} and L_{it} are the inputs of capital and labor used by the firm, and $0 < \alpha < 1$. Technology, which is taken as given by the individual firm, evolves according to

$$A_t = \phi K_t^{\lambda} \widetilde{K}_t^{1-\lambda}, \tag{14}$$

where K_t is the aggregate capital stock, ϕ a positive constant, $0 \leq \lambda \leq 1$, and $\widetilde{K}_t = K_t / L_t$. The firms are price-takers, operating on a perfectly competitive market.

a) Derive the household's and the firm's first-order conditions.

b) Derive the model's equilibrium conditions.

c) Guess that the solution to the model is of the form

$$\begin{aligned} K_{t+1} &= F_k K_t, \qquad (15)\\ C_t &= F_c K_t \qquad (16) \end{aligned}$$

Determine the coefficients F_k and F_c.

d) Solve for the growth rate of output. For which values of λ is the growth rate independent of the size of the population. Explain intuitively!

Question 4 (25p)

Consider an economy where the social planner solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t, \tag{17}$$

subject to the aggregate resource constraint

$$K_{t+1} + C_t = K_t^{\alpha}. \tag{18}$$

a) Formulate the Bellman equation for the social planner's problem. Let K denote this period's capital stock and K' next period's capital stock.

b) Assume that the capital stock can take on all non-negative values. Guess that the initial value function is

$$V^0(K) = \frac{\log(1-\alpha\beta)}{1-\beta} + \frac{\alpha\beta\log\alpha\beta}{(1-\beta)(1-\alpha\beta)} + \frac{\alpha}{1-\alpha\beta}\log K \tag{19}$$

for all K. Calculate the policy function $K'(K)$ for the guessed value function.

c) Use value function iteration to update the guess one time, i.e., calculate $V^1(K)$.

d) Would you consider $V^1(K)$ having converged to the true value function? Motivate your answer!