Macro I, Spring 2016, Final Exam
March 18, 2016

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Solow with investment specific technical change (12 points)

Consider the following Solow set-up: The resource constraint is

$$K_{t+1} = K_t(1-\delta) + q^t I_t, \tag{1}$$

where $q > 1$ is capital specific technical change and I_t is investment. I_t is equal to savings which is a constant fraction of income

$$I_t = S_t = sK_t^{\alpha}\left(\gamma^t n^t\right)^{1-\alpha}. \tag{2}$$

(a) **(4 points)** Show that there exists a balanced growth path along which K_t grows at a constant rate. Calculate the gross growth rate of K_t along the balanced growth path.

(b) **(8 points)** Approximate the speed of convergence $\frac{\partial \log(k_{t+1}/k_t)}{\partial \log k_t}$ around the balanced growth path and express it terms of the exogenous variables δ, α, n, γ, q and s. Here k_t is the "detrended" capital (i.e., K_t detrended by its long-run growth rate).

Problem 2: Balanced growth with endogenous labor supply and changing hours (28 points)

Consider the following neoclassical set-up: the is a representative household with the following preferences over per-capita consumption and per-capita hours worked

$$\mathcal{U}_0 = \begin{cases} \sum_{t=0}^{\infty} (n\beta)^t \left(\frac{c_t^{1-\sigma}-1}{1-\sigma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right) & \text{if } \sigma \neq 1, \\ \sum_{t=0}^{\infty} (n\beta)^t \left(\log(c_t) - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right) & \text{if } \sigma = 1. \end{cases} \tag{3}$$

We have $\sigma \geq 1$, $\theta > 0$, $1 > n\beta > 0$ and $\psi > 0$. The preferences (3) are as in MaCurdy (1981). There is exogenous population growth at gross rate $n > 0$, i.e., $L_t = n^t$. The resource constraint can be written as

$$K_{t+1} = F(K_t, A_t h_t L_t) + (1-\delta)K_t - L_t c_t, \tag{4}$$

where K is aggregate capital and $F(\cdot)$ is a neoclassical production function that fulfills the standard assumptions. The Harrod-neutral technical change takes place at gross rate $\gamma \geq 1$, i.e., $A_t = \gamma^t$. In the following we are going to analyze the planner's solution of this economy. The time endowment per capita is normalized to one, i.e., $0 \leq h \leq 1$.

(a) **(6 points)** The planner's problem is sometimes written in "detrended variables" in anticipation of a balanced growth path along which these detrended variables turn out to be constant. In the textbook version of the neoclassical growth model c is growing at gross rate γ, K is growing at gross rate γn and h is constant along the balanced growth path. Set up the planner's problem in terms of $\tilde{c}_t \equiv \frac{c_t}{\gamma^t}$, $k_t \equiv \frac{K_t}{n^t \gamma^t}$ and h_t as well as $\frac{F(K_t, A_t h_t L_t)}{\gamma^t n^t} \equiv f(k_t, h_t)$ and solve for the first-order conditions.

(b) **(4 points)** Is there a balanced growth path where $\tilde{c}$, k and h are constant? If not, can you put additional restrictions on preference and/or technology parameters such that a balanced growth path with constant hours exists?

(c) **(4 points)** How is the finding in (b) related to the class of King-Plosser-Rebelo (1988) preferences and to the relative size of income and substitution effects on labor supply? What is needed to get a balanced growth path with constant hours? (Maybe you find it useful to draw a picture.)

(d) **(6 points)** Without additional restrictions on technology and preference parameters: Is there a balanced growth path along which hours worked h change at constant gross rate $\gamma^{-\nu}$ and c and K is growing at constant gross rate $\gamma^{1-\nu}$ and $n\gamma^{1-\nu}$, respectively? Here $\nu \in [0, 1)$ is some constant. If yes, solve for the constant ν in terms of exogenous (preference and technology) parameters by guessing and verifying. If no, why not?

(e) **(4 points)** Empirically, how do average hours worked behave in major advanced economies (like the U.S., Germany, or Japan) over the last 6 decades?

(f) **(4 points)** Assume a Cobb-Douglas production function, i.e., $F\left(K_t, A_t h_t L_t\right) = K_t^{\alpha}\left(\gamma^t h_t n^t\right)^{1-\alpha}$, and solve for the steady state capital stock $\hat{k}^{\star} \equiv \frac{K_t}{\gamma^{(1-\nu)t} n^t}$.

Problem 3: Growth and development accounting (10 points)

Are observed differences in real purchasing power parity adjusted GDP per capita to some extent explained by observed differences in the level of physical and human capital? Can observed differences in physical and human capital fully account for the observed income differences? What is the order of magnitude of observed PPP adjusted per-capita income differences between rich countries like the U.S. and poor countries like Burkina Faso? What about changes in income over time: Can the accumulation of physical and human capital fully account for the observed growth in output in the U.S.? If not, what is a potential interpretation of the unexplained part?

[I don't expect you to write more than 1/2-3/4 page.]

Problem 4: Complete and incomplete markets (50 points)

Consider an infinite-horizon pure endowment economy. In each period $t \geq 0$, there is a realization of a stochastic event $s_t \in \{1, 2, 3\}$. The history of events up and until time t is denoted $s^t = [s_0, s_1, \ldots, s_t]$. The unconditional probability of observing a particular sequence of events s^t is given by a probability measure $\pi_t(s^t)$. We assume the following stochastic process. The economy starts with $s_0 = 2$, i.e., $\pi_0(2) = 1$. Next period, the stochastic event is either $s_1 = 1$ with probability π, or $s_1 = 3$ with probability $1 - \pi$, i.e., $\pi_1(2, 1) = \pi$ and $\pi_1(2, 3) = 1 - \pi$, where $\pi \in [0, 1]$. Thereafter, the realization of the stochastic event in period 1 persists forever, $s_t = s_1$ for all $t > 1$.

There are equal numbers of two types of consumers, $i = A, B$. Consumers of type i order consumption streams of the one good according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \left\{ \alpha c_t^i(s^t) - \frac{[c_t^i(s^t)]^2}{2} \right\}, \qquad \beta \in (0, 1), \;\; \alpha > 2,$$

where $c_t^i(s^t) \geq 0$ is the consumption of a type i consumer after history s^t. The good is tradable but nonstorable. The endowment process of a consumer of type A is given by

$$y_t^A(s^t) = \begin{cases} 1 - x, & \text{if } s_t = 1; \\ 1, & \text{if } s_t = 2; \\ 1 + x, & \text{if } s_t = 3; \end{cases}$$

and the endowment of a consumer of type B is given by $y_t^B(s^t) = 2 - y_t^A(s^t)$, where

$$x \in \left[0, \; \frac{1 + \beta}{2 + \beta}\right]. \tag{5}$$

a. [10 points] Solve the social planner problem with Pareto weights λ_A and λ_B for consumers of type A and B, respectively. Find the range of relative Pareto weights, λ_A / λ_B, that fully maps out the Pareto frontier.

b. [5 points] Define a competitive equilibrium with time 0 trading.

c. [25 points] Compute a time 0 trading equilibrium, i.e., find an allocation $\{c_t^i(s^t);\ \forall i, \forall t, \forall s^t\}$ and prices $\{q_t^0(s^t); \forall t, \forall s^t\}$.

d. [5 points] Define a competitive equilibrium with sequential trading.

e. [25 points] Compute a sequential trading equilibrium Besides an allocation, find prices $\{Q_t(s_{t+1}|s^t); \forall t, \forall s_{t+1}, \forall s^t\}$ and asset holdings $\{a_{t+1}^i(s_{t+1}, s^t); \forall i, \forall t, \forall s_{t+1}, \forall s^t\}$.

f. [20 points] Consider a two-period version of this economy, i.e., the economy ends after period 1. Also, markets are now assumed to be incomplete in a sequential trading equilibrium. Specifically, there are no markets for state-contingent claims but just a market for risk-free bonds. Compute such an incomplete-market equilibrium, i.e., find an allocation, asset holdings and the gross interest rate R between periods 0 and 1. Compare outcomes to a complete-market equilibrium for different values of the parameter $\pi \in [0, 1]$.

g. [10 points] Explain where parameter restriction (5) is needed in your calculations and how your answers would change with the alternative restriction $x \in [0, 1)$.