Macro II, Spring 2016

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Instructions. The exam consists of two parts, part one covering John's part of the course and part two covering Karl's.

Exam score. Part one contains four exam questions with a total maximum of 100 points. Part two contains three exam questions with a total maximum of 100 points. The exam score is the sum of the scores on part one and two divided by two.

Course score The course score is a weighted average of the exam score and the score on the problem sets. Weights are 4/5 on the exam and 1/5 on the problem sets. The threshold for pass is 50 and for pass with distinction 75.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

Part I

1. **Monopolistic competition(30 points)**

   Suppose individuals have utility given by

$$E_t \sum_{s=0} \beta^s U\left(C_{t+s}, L_{t+s}\right)$$

where C_t aggregates a continuum of different varieties of goods $C(i), i \in [0,1]$,

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}}, \tag{1}$$

The problem of minimizing the cost of getting C_t, can be written

$$\min_{\{C_t(i)\}_{i=0}^1} \int_0^1 P_t\left(i\right) C_t\left(i\right) di - \lambda_t \left(\left(\int_0^1 C_t\left(i\right)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}} - C_t\right).$$

(a) (7) We interpreted λ_t as a price index which we denote P_t. Explain intuitively why we can do that.

(b) (7) Derive the set of symmetric first-order conditions for the cost minimization. Use $\left(\int_0^1 C_t\left(i\right)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}-1} = C_t^{\frac{1}{\varepsilon}}$ to derive an expression for relative demand $\frac{C_t(i)}{C_t}$ as a function of the relative price $\frac{P_t(i)}{P_t}$.

(c) (7) Use the expression for $\frac{C_t(i)}{C_t}$ in the definition of aggregate expenditure $P_t C_t = \int_0^1 P_t\left(i\right) C_t\left(i\right) di$ to show that $P_t = \left(\int_0^1 P_t\left(i\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

(d) (9) Now extend the analysis and assume that there are two categories of goods – 1 and 2. In each category, a continuum of varieties are aggregated like in (1), i.e., $C_{1,t} \equiv \left(\int_0^1 C_{1,t}\left(i\right)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}}$ and $C_{1,t} \equiv \left(\int_0^1 C_{2,t}\left(i\right)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}}$. The two categories are then aggregated in a CES function so that (1) is replaced by

$$C_t \equiv \left(\left(1-\omega\right) C_{1,t}^{\frac{\sigma-1}{\sigma}} + \omega C_{2,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

For each category of goods, the exact price index is $P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}, j \in \{1,2\}$. Derive the exact price index for C_t. Proceed as above using the exact price indices for $P_{j,t}$. This means that the objective function to be minimized is $P_{1,t}C_{1,t} + P_{2,t}C_{2,t}$.

2. **Balanced Growth (30 points)**
   Consider the planning problem

$$\max_{\{C_t, K_{t+1}, L_t, N_t\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$
$$\text{s.t. } C_t + K_{t+1} = Z_t F(K_t, N_t) + (1-\delta) K_t \forall t \geq 0$$
$$N_t = 1 - L_t$$
$$K_0 \text{ given.}$$

   (a) (10) Derive the optimality conditions for the planner by using the first order conditions for C_t, K_{t+1} and L_t. Express your results as
      1. an Euler condition expressing the intertemporal optimality condition involving a trade-off between C_t and C_{t+1} and
      2. an intratemporal optimality condition involving a trade-off between labor and leisure. Show the steps in your derivations!

   (b) (10) Suppose utility is $\ln(C_t) - \frac{1}{1+\nu}\phi(1-L_t)^{1+\nu}$ with $\nu, \phi > 0$. Does this utility function admit balanced growth (constant common growth rates in consumption and wages with constant interest rates and labor supply)? Prove your result.

   (c) (10) Generalize utility to $\ln(C_t) - \frac{1}{1+\nu}\phi(1-L_t)^{1+\nu}C^{\frac{\varphi}{1-\varphi}}$. Show that for some values of φ we can have labor N constantly falling in a growth path with constant common growth rates of consumption and wages.

3. **Wedge accounting (20 points)**. Chari, Kehoe and McGrattan (2007) devised a method for calculating how the perfect market model predictions deviate from observed data. They defined wedges as deviations from the equilibrium conditions.

   (a) (10) Suppose individuals have a period utility function $U(C_t, L_t) = \ln C_t + \frac{\nu}{\nu-1}\phi_t L_t^{\frac{\nu-1}{\nu}}$. Let w_t denote the wage and normalize the price of the consumption good C_t to unity. Define the labor-leisure wedge (make sure it is zero if the perfect market equilibrium is exactly satisfied).

   (b) (10) In US data, how does the wedge you just defined vary over the business cycle? (Make sure your statement corresponds to the way you defined the wedge above).

4. **HP-filter (20 points)**. Describe briefly what the Hodrick-Prescott filter does and how it's parameter λ determine its properties.

Part II

5. **Short questions. (25 points)** Please answer briefly.

   (a) (5 points) What is the main exercise performed in the paper by Jones and Klenow? For a given level of aggregate output, why is consumption inequality bad for welfare in their analysis?

   (b) (6 points) Characterize a good measure of inequality in terms of goals and properties. Mention two specific inequality measures used in the literature and their pros and cons.

   (c) (10 points) The magazine 'The Economist' recently stated that the real median income of households headed by 45- to 54-year-olds in the U.S. has decreased by 7% from 1989 to 2014, in spite of substantial growth in aggregate output over the same time period. How does this relate to the data on inequality in income reported by Piketty? What is the overall shape of the time series for the last 100 years of the share of the top income decile in total income for the US? In what way is this time series different for Europe?

   (d) (4 points) What are the cyclical characteristics of labor force participation?

5. **The Aiyagari model (32 points)**

   (a) (5 points) Define the stationary equilibrium in the Aiyagari model.

   (b) (15 points) For this model, please draw the demand and supply of capital curves (capital K on the x-axis, interest rate r on the y-axis) as well as the borrowing limit. Mark the equilibrium interest rate and capital stock. Finally, mark the interest rate that would obtain if markets where complete.

   (c) (7 points) Quantify the precautionary savings using the figure from b). What are the two reasons for precautionary savings in this model?

   (d) (5 points) How does the equilibrium interest rate and capital stock change, in steady state, if we assume that idiosyncratic income risk is higher? Draw changes in the figure.

6. **The Diamond-Mortensen-Pissarides model (43 points)**
   Consider a setup similar to the standard DMP model discussed in class. In particular, we have: risk-neutrality, homogenous workers, homogenous firms, a worker-firm match produces output y_t. Wages, w_t, are determined by Nash bargaining, the cost of creating a vacancy is c, free entry of firms, unemployment benefits, b, exogenous break-up rate, δ, constant labor force participation normalized to unity, no intensive margin (choice of hours/worker). The discount factor is β. Denote the job finding rate with f_t and the vacancy filling rate with q_t.
   Please define clearly any notation that you add.

   (a) (7 points) Please write down and explain the value, W, to a worker of being employed, compared to being unemployed. Assume that a worker that loses his job doesn't have the ability to look for a new job until the next period.

   (b) (5 points) Assume a matching function $M_t = \gamma_t V_t^{1-\lambda} U_t^{\lambda}$ where V denotes vacancies and U unemployment. Derive expressions for the job finding rate and the vacancy filling rate in terms of market tightness, $\theta_t \equiv V_t/U_t$.

   (c) (7 points) State the job creation condition implied by free entry, assuming that a vacancy that is filled today becomes a productive match the next period.

   (d) (7 points) Describe, using the relevant equations, how a temporary, one-period, increase in γ_t affect the equilibrium outcomes. Mention which variables are affected and for how long.

(e) (4 points) Describe a mechanism related to the job creation condition that stabilizes unemployment over the business cycle.

(f) (7 points) Using equations for flows into and out of unemployment in steady state, derive a steady state expression for the job finding rate f as a function of the job separation rate and unemployment.

(g) (6 points) Explain the appropriation externality and the congestion externality that affect the vacancy posting level in the DMP model defined in class, including in what direction each of them go.