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Exam for EC7112, Game theory, 25 March 2017

1. (35 points) Consider the following game which models an international conflict where countries **X** and **Y** choose between standing firm (s) and backing down (b), and payoffs are represented in the following matrix:

Payoff matrix N

		Country **Y** s	Country **Y** b
Country **X**	s	$1,1$	$4,2$
	b	$2,4$	$3,3$

a) Derive the best response functions of both countries. Illustrate the best response functions in a figure. Use the figure to identify all Nash equilibria of this game.

In the above game both countries' leaders are normal in the sense that they regard the outcome where no one backs down as the worst. If we instead assume that country **Y**'s leader is insane, such that he/she always prefers standing firm to backing down, the interaction between the two countries is represented by the following payoff matrix (country **X**'s payoffs are the same as above):

Payoff matrix I

		Country **Y** s	Country **Y** b
Country **X**	s	$1,3$	$4,1$
	b	$2,4$	$3,2$

b) Identify the unique Nash equilibrium of this game. (Note: you are not required to illustrate the best response functions in a figure as above.)

Now assume that country **X** is uncertain regarding the the type of country **Y**'s leader, i.e. whether he/she is normal (state N) or insane (state I). The leader of country **X** assigns probability $\alpha = \frac{1}{3}$ to country **Y**'s leader being normal. Thus, from country **X**'s point of view, there is a probability of $\frac{1}{3}$ that the interaction with country **Y** is represented by payoff matrix N, and a probability of $\frac{2}{3}$ that the interaction with country **Y** is represented by payoff matrix I. The leader of country **Y** knows for sure whether he/she is normal or insane (really!).

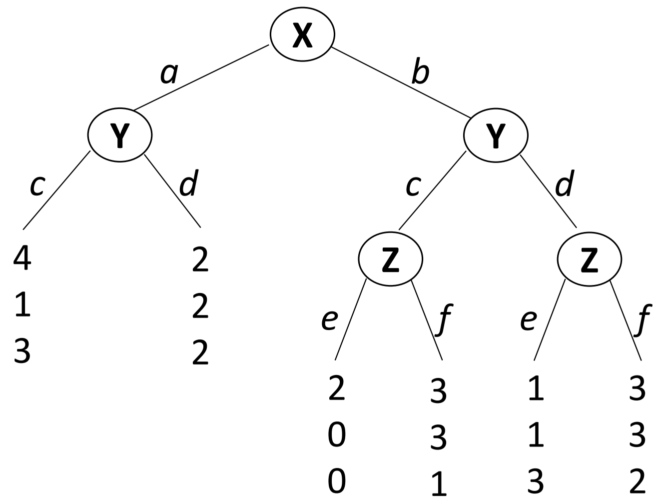
c) Calculate the expected payoffs of country **X** for all possible combinations of actions taken by the two types of country **Y** (i.e. consider country **Y** as two different players, **YN** in case its leader is normal, and **YI** in case its leader is insane).

d) Represent the interaction under imperfect information as a three-player normal form game. (That is, illustrate the interaction in a payoff matrix.)

e) Apply IDSDS to identify the unique Bayesian Nash equilibrium. (Note: you have to clarify the order in which you eliminate strictly dominated strategies.) Provide an intuitive explanation for why there exists only one Bayesian Nash equilibrium.

f) Use your results in a) and b) to explain in words (no calculations!) that there exists a second pure strategy Bayesian Nash equilibrium if α is sufficiently high.

2. (30 points) Consider the following extensive form game between players **X**, **Y** and **Z**, where payoffs are presented in the following order: u_X, u_Y, u_Z.



a) Define the strategy sets, the player function and the set of terminal histories of this game. Identify all subgames of the game.

b) Apply backward induction to identify the unique subgame perfect Nash equilibrium strategy profile.

c) Is the subgame perfect Nash equilibrium outcome Pareto efficient?

3. (35 points) Consider the interaction between two competing firms that simultaneously have to decide between complying with environmental legislation (strategy C) and violating the law (strategy V). Being compliant is associated with higher marginal costs and hence, also affects the strategic interaction between the two firms (i.e. by unilaterally breaching the law a firm gets a competitive advantage). More specifically, if both firms comply with legislation, both make a profit of 1; if both firms violate the law, both make a profit of 2; and if one firm unilaterally violates legislation, this firm makes a profit of 4, while its compliant competitor's profit is 0.

a) Represent this interaction on normal form and identify the Nash equilibrium/equilibria.

Since violations of the law are harmful for the environment, there exists an agency that carries out inspections and punishes firms that do not comply with legislation. Let p be the probability that a firm is inspected, and let F be the fine that a violating firm has to pay if it is inspected. Hence, a violating firm's profit is reduced by the expected penalty $F^e = pF$. Compliant firms do not incur any costs from being inspected.

b) Represent the interaction between the two firms on normal form, taking into account the impact of inspection activities on firms' expected payoffs.

c) Determine the pure strategy Nash equilibria of this game for different values of F^e. (Hint: there are five different cases to consider, i.e. threshold values of F^e have to be treated as separate cases.)

d) Assume that $F = 4$. Use the results in c) to determine the equilibrium compliance rates for (i) $p = \frac{2}{5}$; (ii) $p = \frac{3}{5}$; and (iii) $p = \frac{4}{5}$. (That is, what is the share of compliant firms under the pure strategy Nash equilibria for these different p-values?)

e) Obviously a higher inspection frequency is associated with higher costs for the agency. Use the results in d) to explain the trade-off that a decision-maker faces when determining the level of funding for the inspection agency. (No formulas, just words!)