Department of Economics, Stockholm University
Mathias Herzing
Exam for EC 2110, Uncertainty and asymmetric information, 5 June 2014

1. Short questions:

a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
(i) $v(c) = \ln(c+1)$
(ii) $v(c) = \ln(c^2)$
(iii) $v(c) = e^{5c-1}$
(iv) $v(c) = 5c - 1$

b) What is a "complete asset market"? Which conditions need to be satisfied for a "complete asset market"?

c) Consider an economy consisting of two individuals (A and B), who will end up in state 1 with probability π and in state 2 with probability $1 - \pi$. Individual A is endowed with state claims $(\overline{c}_1^A, \overline{c}_2^A)$, and individual B is endowed with state claims $(\overline{c}_1^B, \overline{c}_2^B)$. Both individuals are risk averse. Their elementary utility functions are $v_A(c)$ and $v_B(c)$. It is possible for A and B to trade state claims. Let (c_1^{A*}, c_2^{A*}) and (c_1^{B*}, c_2^{B*}) denote the market equilibrium amounts of state claims. State all six conditions that need to be satisfied to obtain the market equilibrium under uncertainty. (Note: You are not supposed to calculate the equilibrium outcome. Just state the conditions that have to be satisfied.)

d) Consider a market with a monopsonist employer. There are two types of workers. Type 0 has marginal product $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = 1$. Type 1 has marginal product $\Theta_1 = 3$ and an outside opportunity wage of $w_0(\Theta_1) = \frac{3}{2}$. The share of type 1 workers is given by $\frac{1}{3}$. Workers know their own type but the employer cannot tell the high from the low productivity workers. In the absence of any educational screening, will there be adverse selection in the market?

2. Consider a factory owner who needs to hire staff. The value of output produced is given by $S(q) = 3q^{\frac{1}{3}}$, where q is the amount of effort exerted by an employee. The factory owner pays a wage t to employees. Profit per employee is thus given by $\pi = S(q) - t$.

   There are two types of employees who differ with respect to their cost of effort. Employees of type $\underline{\Theta} = 1$ incur cost $C(q, \underline{\Theta}) = q$, while employees of type $\overline{\Theta} = 4$ incur cost $C(q, \overline{\Theta}) = 4q$ when exerting effort q. Employees' utilities are given by $u_\Theta(q, t) = t - C(q, \Theta)$. The share of type $\underline{\Theta}$ agents is given by $\nu = \frac{5}{8}$.

   a) What are the socially optimal effort levels of each employee type?

   b) Will the socially optimal effort levels generate a social surplus?

   c) Which wages will be paid to employees if the factory owner has all bargaining power? Which is the first-best menu of contracts?

   d) Calculate the information rent that efficient employees can extract by mimicking inefficient.

   Assume now that information regarding employees' types is hidden to the factory owner.

   e) State the factory owner's optimization problem and all constraints that need to be satisfied. Which constraints are relevant? Explain why the other constraints are not relevant.

   f) Simplify the optimization problem by taking into consideration that the factory owner has all bargaining power. Solve the optimization problem to determine the second-best menu of contracts.

   g) Explain in words who gains and who loses when second-best contracts instead of first-best contracts are implemented? (Note: You are not supposed to calculate gains and losses!)

3. The owner of a farm hires a worker to grow crops. The crop yield is random (depending on e.g. weather conditions), either high ($\overline{S} = 11$) or low ($\underline{S} = 2$). However, the probability of the crop yield being high π_e also depends on the effort $e \in \{0, 1\}$ that the worker exerts, such that $\pi_0 = \frac{1}{3}$ and $\pi_1 = \frac{2}{3}$. The cost that the worker incurs from exerting effort e is $\Psi(e) = 2e$. The farm owner, who is the only employer, offers a contract $(\underline{t}, \overline{t})$ that specifies the transfers to be paid to the worker, depending on the crop yield. The farm owner's expected profit is given by $EV_e = \pi_e(\overline{S} - \overline{t}) + (1 - \pi_e)(\underline{S} - \underline{t})$, and the worker's expected utility is given by $EU_e = \pi_e \overline{t} + (1 - \pi_e)\underline{t} - \Psi(e)$.

   a) State the participation constraint of the worker for contracts that specify effort $e = 1$.

   b) State the farm owner's optimization problem under complete information.

   c) What high-effort-inducing contract(s) will be offered?

   Assume now that information regarding the worker's actions is hidden to the farm owner.

   d) Which constraints need to be satisfied?

   e) What high-effort-inducing contract(s) will be offered?

   f) If the worker's liability is limited such that losses from transfers cannot exceed 1, which constraints need to be satisfied? What high-effort-inducing contract(s) will be offered?

   g) Provide an intuitive explanation for why hidden action and limited liability leads to a decrease in the farm owner's profits in this model. (No formulas, just words!)

   h) Illustrate your answers in c), e) and f) in a figure, with $\underline{t}$ on the x-axis and $\overline{t}$ on the y-axis.