An Analytically Solvable Core-Periphery Model*

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Abstract

We develop an analytically solvable version of the central model of "new economic geography", the so called core-periphery model by Krugman (1991). While the modified model reproduces all the appealing features of the original one, it also allows us to obtain additional analytical results that are out of reach in Krugman's set-up. First, we are able assess the exact number of equilibria and their global stability properties. Second, we are able to investigate the implications of exogenous asymmetries between countries. This is achieved by introducing heterogeneity between high-skill mobile and low-skill immobile workers, which may also be an empirically attractive property.

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1 Introduction

The effect of economic integration on industrial location has become an issue of considerable political interest following regional integration agreements such as NAFTA and the EU. Consequently 'geography and trade' or 'new economic geography' has become a mainstream research area of trade theory (Fujita, Krugman and Venables, 1999). In the seminal paper by Krugman (1991), a standard new trade model with monopolistic competition *à la* Dixit and Stiglitz (1977) is modified to analyze industrial location. The outcome is the so called core-periphery (CP) model, which shows how economic integration may lead to a dramatic increase in the geographical concentration of industrial production via a self-reinforcing agglomeration process.

Specifically, the CP model explains agglomeration in terms of *demand linkages*.[1] When a firm moves its production facilities to a new site, local market conditions are affected through two channels. On the one hand, given trade costs, the presence of a new competitor reduces the local price index. This has a negative impact on demand per firm ('market crowding' effect) and a positive impact on consumer surplus ('cost-of-living' effect). On the other, as long as some of the income generated by the new entrant is spent locally, local expenditures grow and this has a positive impact on demand per firm ('market size' effect). This derives from the fact that labour is the only productive factor; firms can employ only local residents and the labour bill absorbs all operating profits. Thus, while the first effect discourages geographical agglomeration, the other two effects foster it by creating a circular causation mechanism among firms' and workers' location decisions.

While yielding valuable insights into the interactions between trade costs, factor mobility, and agglomeration, the standard model is astoundingly difficult to work with, making numerical simulations necessary for most results. In particular, the endogenous variables that are instrumental in determining the location of firms and workers (i.e. wages and operating profits) cannot be expressed as explicit functions of the spatial distribution of economic activities. This implies that all we obtain through numerical investigation is a gallery of possible equilibrium outcomes; each obtained under a particular set of parameter values. A shortcoming of this is that one can not be certain that the gallery is complete.

The aim of this paper is to propose some simple modifications, based on Forslid (1999) and Ottaviano (1996), that make the CP model analytically solvable in the sense that it is possible to derive closed form solutions for the endogenous variables. This allows us to assess analytically the number of equilibria as well as their global stability. To the best of our knowledge this has not been achieved before. The additional usefulness of the simplified model is illustrated by deriving new analytical results also in the realistic cases of exogenous asymmetries between regions in terms of both size and trade barriers. This provides valuable insights in its own right. In particular, we show that, differently from the symmetric model, the asymmetric

[1] An alternative explanation has been put forth by Krugman and Venables (1995) and Venables (1996), who stress the role of input-output linkages between firms.

model exhibits an intermediate range of trade costs in which exogenous asymmetries always dominate and, thus, it does not depend on the initial condition where agglomeration eventually takes place.

Solvability is achieved by introducing skill heterogeneity between workers and by coupling a higher level of skill with higher interregional mobility. This is in line with empirical evidence, which suggests a significantly higher geographical mobility of skilled compared to unskilled workers (Shields and Shields, 1989).

Two analytically solvable CP models have recently been developed. Baldwin (1999) presents a model where factor accumulation, rather than factor migration, causes agglomeration. Ottaviano, Tabuchi and Thisse (1999) develop a model with quadratic utility and linear demands. However, in their quest for analytical tractability both Baldwin (1999) and Ottaviano et al. (2001) have to abandon some of the crucial features of the original CP model. While the former contribution rules out factor mobility, the latter departs from the Dixit-Stiglitz framework. In contrast, the present paper assesses the slightest modifications of the original model which make it analytically tractable.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyses its equilibria and stability properties. Section 4 analyses the cases of asymmetric regional sizes and asymmetric trade costs. Section 5 concludes.

2 The Footloose Entrepreneur Model

The economy consists of two regions, 1 and 2. There are two factors of production, skilled and unskilled labour. Each worker supplies one unit of his type of labour inelastically. Total endowments are H and L for skilled and unskilled labour respectively so that $H_1 + H_2 = H$ and $L_1 + L_2 = L$, where H_i and L_i are the endowments of the two factors in region i. Skilled workers can be thought of as self-employed entrepreneurs who move freely between regions, and we will therefore refer to the model as the 'footloose entrepreneur' (FE) model.

On the demand side, preferences are defined over two final goods, a horizontally differentiated good X ('manufactures') and a homogenous good A ('agriculture').[2] Specifically, the preference ordering of the representative consumer in region $i = \{1, 2\}$ is captured by the utility function:

$$U_i = X_i^{\mu} A_i^{1-\mu}, \tag{1}$$

$$X_i = \left(\int_{s \in N} d_i(s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $\mu \in (0, 1)$ is a constant, X_i is consumption of manufactures, A_i is consumption of agricultural products, $d_i(s)$ is consumption of variety s of good X, N is the total number of varieties,

[2] We maintain Krugman's (1991) labelling of the two sectors for ease of comparison. More generally, sector X should be interpreted as relatively skill-intensive compared to sector A.

and n_i are the varieties produced in region i so that $n_1 + n_2 = N$. Finally, $\sigma > 1$ is both the elasticity of demand of any variety and the elasticity of substitution between any two varieties.

Standard utility maximization of (1) yields CES demand by residents in location i for a variety produced in location j:

$$d_{ji}(s) = \frac{p_{ji}(s)^{-\sigma}}{P_i^{1-\sigma}} \mu Y_i, \quad i, j = \{1, 2\} \tag{3}$$

where p_{ji} is the consumer price of a variety produced in j and sold in i, and P_i is the local CES price index associated with (2):

$$P_i = \left[\int_{s \in n_i} p_{ii}(s)^{1-\sigma} ds + \int_{s \in n_j} p_{ji}(s)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

and Y_i is local income consisting of skilled (w_i) and unskilled wages (w_i^L):

$$Y_i = w_i H_i + w_i^L L_i. \tag{5}$$

Thus, the representative consumer in region i maximizes utility (1) subject to the following budget constraint:

$$\int_{s \in n_i} p_{ii}(s) d_{ii}(s) ds + \int_{s \in n_j} p_{ji}(s) d_{ji}(s) ds + p_i^A A_i = Y_i, \tag{6}$$

where p^A is the price of the agricultural good. Turning to the supply side, firms in sector A produce a homogenous good under perfect competition and constant returns to scale and employ only unskilled labour. Without loss of generality, units are chosen so that one unit of output requires one unit of labour. This implies that the unit production cost for a firm in sector A equals the unskilled wage w_i^L. Then, perfect competition implies marginal cost pricing so that $p_i^A = w_i^L$.

Firms in sector X are monopolistically competitive and employ both skilled and unskilled workers under increasing returns to scale. Product differentiation ensures a one-to-one relation between firms and varieties. Specifically, in order to produce $x(s)$ units of variety s, a firm incurs a fixed input requirement of α units of skilled labour and a marginal input requirement of βx units of unskilled labour. The total cost of production of a firm, in location i, is thus given by:

$$TC_i(s) = w_i \alpha + w_i^L \beta x_i(s). \tag{7a}$$

Given the fixed input requirement α, skilled labour market clearing implies that in equilibrium the number of firms is determined by:

$$n_i = \frac{H_i}{\alpha}, \tag{8}$$

so that the number of active firms in a region is proportional to the number of its skilled residents.

In addition to their employment opportunities, the two types of workers differ also in terms of spatial mobility. Unskilled workers are perfectly mobile between sectors but spatially immobile and assumed to be evenly spread across regions: $L_i = L/2$. Skilled workers, on the contrary, are mobile and free to reside in the region that offers them the higher indirect utility.

The assumption that skilled (geographically mobile) labour is employed in the fixed cost and unskilled (geographically immobile) labour is employed in the variable cost in the production of manufactures is the only difference with respect to Krugman's CP model, where mobile labour is used in both fixed and variable cost. However, it is enough to simplify the analysis without altering the qualitative insights of the original model.[3] However, analitycal convenience is not the only appealing feature of the present set-up. Indeed, the assumption that skilled workers are associated with the fixed cost is quite natural, since this cost often stem from headquarter services, R&D or other high-skill activities.

Also goods differ in terms of their spatial mobility. Good A is freely traded so that its price is the same everywhere. Then, due to marginal cost pricing, interregional price equalization ($p_1^A = p_2^A$) also implies interregional wage equalization ($w_1^L = w_2^L$).[4] This suggests choosing good A as numeraire, so that $p_i^A = w_i^L = 1$.

Trade in X, on the contrary, is inhibited by frictional trade barriers, which are modeled as iceberg costs: for one unit of the differentiated good to reach the other region, $\tau \in [1, +\infty)$ units must be shipped.

Using (7a) a typical manufacturing firm located in region i maximizes profit:

$$\Pi_i(s) = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)] - \alpha w_i, \tag{9}$$

where, due to the choice of numeraire, the unskilled labour wage w_i^L is set equal to 1 and $\tau d_{ij}(s,t)$ represents total supply to the distant location j inclusive of the fraction of product that melts away in transit due to the iceberg costs. The first order condition for maximization gives:

$$p_{ii}(s) = \beta\sigma/(\sigma - 1) \quad \text{and} \quad p_{ij}(s) = \tau\beta\sigma/(\sigma - 1) \tag{10}$$

for every i and j. After using (10), the CES price index (4) simplifies to:

$$P_i = \frac{\beta\sigma}{\sigma - 1} [n_i + \phi n_j]^{\frac{1}{1-\sigma}}, \tag{11}$$

where $\phi \equiv \tau^{1-\sigma} \in (0, 1]$ is the ratio of total demand by domestic residents for each foreign variety to their demand for each domestic variety. It is therefore a measure of the freeness of trade, which increases as τ falls and is equal to one when trade is free ($\tau = 1$). Since by (8) the

[3] To ease comparisons, the crucial equations of the original CP model are reported in the Appendix.

[4] Wage equalization holds as long as the homogeneous good is produced in both regions. That is the case when a single region alone cannot supply economy wide demand, i.e. when good A has a large weight in utility (μ small) and product variety is highly valued by consumers (σ small). The exact condition is $\mu < \sigma/(2\sigma - 1)$ and it is assumed to hold from now on.

total number of X firms is given by $n_i + n_j = H/\alpha$, the price index (11) decreases (increases) with the number of local (distant) firms.

Due to free entry and exit, there are no profits in equilibrium. This implies that a firm's scale of production is such that operating profits exactly match the fixed cost paid in terms of skilled labour. In other words, as in the CP model, the equilibrium wages corresponding to (8) are determined by a bidding process for skilled workers, which ends when no firm can earn a strictly positive profit at the equilibrium market prices. That is, a firm's operating profits are entirely absorbed by the wage bill of its skilled workers:

$$w_i = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)]$$

that is, given (10),

$$w_i = \frac{\beta x_i}{\alpha(\sigma - 1)}, \tag{12}$$

where $x_i = [d_{ii}(s) + \tau d_{ij}(s)]$ is total production by a typical firm in location i. This last expression can be used to determine the output of firms in regions 1 and 2. Using (3), (10) and (11), market clearing for a typical variety produced in region i implies:

$$x_i = \frac{\sigma - 1}{\beta\sigma}\left(\frac{\mu Y_i}{n_i + \phi n_j} + \frac{\phi\mu Y_j}{\phi n_i + n_j}\right). \tag{13}$$

Using (8) and (12), equation (13) can be equivalently written as:

$$w_i = \frac{\mu}{\sigma}\left[\frac{Y_i}{H_i + \phi H_j} + \frac{\phi Y_j}{\phi H_i + H_j}\right], \tag{14}$$

where, by (5) and (8), income equals:

$$Y_i = \frac{L}{2} + w_i H_i. \tag{15}$$

For $i = 1, 2$, the system consisting of equations (8), (10), (12), (13), and (15) determines the endogenous variables n_i, p_i, w_i, x_i, and Y_i for a given allocation of skilled workers H.[5] In particular, plugging (15) into (14) generates a system of two linear equations in w_1 and w_2, that can be solved to obtain the equilibrium skilled wages as explicit functions of the spatial distribution of skilled workers H_i:

$$w_i = \frac{\frac{\mu}{\sigma}}{1 - \frac{\mu}{\sigma}}\frac{L}{2}\frac{2\phi H_i + [1 - \frac{\mu}{\sigma} + (1 + \frac{\mu}{\sigma})\phi^2]H_j}{\phi(H_i^2 + H_j^2) + [1 - \frac{\mu}{\sigma} + (1 + \frac{\mu}{\sigma})\phi^2]H_i H_j}. \tag{16}$$

In the CP model expressions such as (16) are simply not available. As reported in the Appendix, the equations corresponding to (14) are highly non linear in skilled wages and therefore do not admit any explicit solution. Our model, instead, becomes solvable because the *equilibrium prices* (10) *are equalised across regions and independent from the location of firms and workers.* As already discussed, this result holds as long as agriculture is active in both regions, i.e. $\mu < \sigma/(2\sigma - 1)$.

[5] In Krugman (1991), the corresponding system consists of (A4), (A2); (A3), (A5), and (A6).

Defining $h \equiv H_1/H$ as the share of skilled workers that reside in region 1, we have:

$$\frac{w_1}{w_2} = \frac{2\phi h + [1 - \frac{\mu}{\sigma} + (1 + \frac{\mu}{\sigma})\phi^2](1-h)}{2\phi(1-h) + [1 - \frac{\mu}{\sigma} + (1 + \frac{\mu}{\sigma})\phi^2]h}. \tag{17}$$

Differentiating (17) w.r.t. h shows that the region with more skilled workers offers a higher (lower) skilled worker wage whenever ϕ is larger (smaller) than the threshold:

$$\phi_w \equiv \frac{1 - \frac{\mu}{\sigma}}{1 + \frac{\mu}{\sigma}}. \tag{18}$$

with $\phi_w \in (0,1)$. This is the result of a trade-off between two opposing forces. On the one hand, given trade costs, a larger number of skilled workers in a certain region entails a larger number of competing manufacturing firms. For given expenditures on manufactures, that depresses the local price index inducing a fall in local demand per firm ('market crowding' effect). Lower demand leads to lower operating profits and, therefore, lower skilled wages. On the other hand, hosting more firms also implies additional operating profits and thus additional skilled income, a fraction of which is spent on local manufactures. Accordingly, local expenditures are larger, which, for a given price index, increases demand per firm ('market size' effect). When $\phi = \phi_w$ the two effects exactly offset each other, while the former (latter) dominates the latter (former) whenever ϕ is smaller (larger) than ϕ_w. Notice that this is a 'global' analytical result (i.e. true for every h) which is not available in the CP model.

Inspection of (18) shows that the 'market crowding' effect is strong when trade costs (τ) are high because firms in this case sell mainly in the domestic market. It is also strong when the own and cross price elasticity of demand for manufactures (σ) are large because a firm's demand is quite sensitive to the price index. Finally, as intuition would have it, expression (18) also shows that the 'market size' effect is strong when the fraction of income spent on manufactures (μ) is large. Thus, skilled wages are higher in the region with more skilled workers for small τ, large μ, and small σ.

3 Equilibrium and stability

We are now ready to analyse the location decision of skilled workers. Following Krugman (1991) we assume that workers are short sighted and choose location as to maximize their well-being as measured by current indirect utility. Specifically, their migration is regulated by the following simple marshallian adjustment:

$$\dot{h} \equiv dh/dt = \begin{cases} W(h,\phi) & \text{if } 0 < h < 1 \\ \min\{0, W(h,\phi)\} & \text{if } h = 1 \\ \max\{0, W(h,\phi)\} & \text{if } h = 0 \end{cases} \tag{19}$$

where t is time, which is left implicit to simplify notation. $W(h,\phi)$ is the current indirect utility differential associated with (1):

$$W(h,\phi) \equiv \eta \left(\frac{w_1}{P_1^{\mu}} - \frac{w_2}{P_2^{\mu}} \right), \tag{20}$$

where $\eta \equiv \mu^\mu(1-\mu)^{1-\mu}$. Moreover, by (8) and (11), the two price indexes are:

$$P_1 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}[h+\phi(1-h)]^{\frac{1}{1-\sigma}} \tag{21}$$

$$P_2 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}[1-h+\phi h]^{\frac{1}{1-\sigma}} \tag{22}$$

The presence of P_1 and P_2 in (20) adds a new item to the list of location effects. In particular, (21) and (22) show that, for a given wage, the region with more skilled workers, and thus more manufacturing firms, grants higher purchasing power, that is, higher consumer surplus. The reason is its lower price index as the larger number of domestic firms implies that fewer manufacturing varieties are imported and burdened by trade costs ('cost-of-living' effect). Therefore, this additional effect teams up with the market size effect to support the agglomeration of manufactures against the opposition of the market crowding effect.

Substituting (21) and (22) in (20) we obtain:

$$W(h,\phi) = \frac{\Phi}{\phi(h^2+(1-h)^2)+(1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})\phi^2)h(1-h)}\cdot V(h,\phi), \tag{23}$$

where $\Phi \equiv [\eta\mu L\alpha^{\mu/(1-\sigma)}(\sigma-1)^\mu]/[2(\sigma-\mu)H^{(1+\mu-\sigma)/(1-\sigma)}(\sigma\beta)^\mu]$ is a positive bundling parameter and

$$V(h,\phi) \equiv \frac{2\phi h+[1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})\phi^2](1-h)}{[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}}} - \frac{2\phi(1-h)+[1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})\phi^2]h}{[(1-h)+\phi h]^{\frac{\mu}{1-\sigma}}}. \tag{24}$$

Clearly, a spatial equilibrium implies $\dot{h}=0$. If $W(h,\phi)$ is positive, some workers will move from 2 to 1; if it is negative, some will go in the opposite direction. Furthermore, a spatial equilibrium is *stable* for (19) if, for any marginal deviation from the equilibrium, the equation of motion brings the distribution of workers back to the original one. Therefore, a corner configuration ($h=0$ or $h=1$) is always stable when it is an equilibrium, while an interior equilibrium ($0<h<1$) is stable if and only if the slope of $W(h,\phi)$ is non-positive in its neighborhood.

Inspection of (23) reveals that for the determination of equilibria all that matters is $V(h,\phi)$. In particular, all interior equilibria are solutions to $V(h,\phi)=0$ while fully agglomerated configurations $h=0$ and $h=1$ are equilibria if and only if $V(0,\phi)<0$ and $V(1,\phi)>0$ respectively. Since by (24) we have:

$$V(0,\phi) = -V(1,\phi) = \frac{[1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})\phi^2]}{\phi^{\frac{\mu}{1-\sigma}}} - 2\phi$$

full agglomeration in either region is a stable spatial equilibrium whenever trade costs are so small that ϕ is above the threshold value ϕ_s, which is implicitly defined by:

$$1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})(\phi_s)^2 - 2(\phi_s)^{1+\frac{\mu}{1-\sigma}} = 0. \tag{25}$$

where ϕ_s is what Fujita et al (1999) call the 'sustain point'.

Turning to interior equilibria we can prove that $V(h,\phi)=0$ has at most three solutions for $0<h<1$. It is readily verified that one solution exists for any values of parameters. This is the symmetric outcome $h=1/2$, which entails an even geographical distribution of skilled workers and modern firms. This solution is stable whenever $V_h(1/2,\phi)<0$, where the subscript denotes the partial derivative with respect to the corresponding argument. This is the case if and only if trade costs are so large that ϕ is below the threshold value ϕ_b defined as:

$$\phi_b \equiv \phi_w \frac{(1-1/\sigma-\mu/\sigma)}{(1-1/\sigma+\mu/\sigma)}, \tag{26}$$

where, in the terminology of Fujita et al (1999), ϕ_b is the 'break point'. It is seen by inspection that the breakpoint is decreasing in μ and increasing in σ. Moreover, if $\phi_b<0$ the symmetric outcome is never stable independently of parameter values: the market crowding effect is always dominated by market size and cost-of-living effects. We rule out this case by assuming that $\mu<\sigma-1$ (the 'no-black-hole' condition).[6] Note also that the cost-of-living effect always works in favour of the large country. Therefore, at the breakpoint, where real wages are equal, the wage effect must go in favour of the small location so that $\phi_b<\phi_w$.

Apart from $h=1/2$, there exist at most two other interior equilibria that are symmetrically placed around it. This comes from tedious but standard study of the function $W(h,\phi)$, which is symmetric around $h=1/2$ and changes concavity at most twice. In particular, the following local properties can be established in a neighborhood of $h=1/2$:[7]

$$W(1/2,\phi)=0 \;\; \forall\phi \tag{27}$$

$$W_h(1/2,\phi_b)=0, \;\; W_{h\phi}(1/2,\phi_b)>0, \tag{28}$$

$$W_{hh}(1/2,\phi_b)=0, \;\; W_{hhh}(1/2,\phi_b)>0 \tag{29}$$

Property (27) says that $h=1/2$ is always a steady state (persistent steady state). In other words, as ϕ changes, $V(h,\phi)$ rotates around $(h,V)=(1/2,0)$. Properties (28) say that as ϕ increases from 0, the steady state $h=1/2$ turns from stable to unstable as soon as ϕ rises above ϕ_b. Properties (29) say that, as soon as the steady state $h=1/2$ changes stability, two additional steady states appear in its neighbourhood. Due to the symmetry of the model such steady states are symmetric around it. All these properties together say that the differential equation (19) undergoes a (local) 'pitchfork bifurcation' at $\phi=\phi_b$. Since $W_{hhh}(1/2,\phi_b)>0$ the bifurcation is 'subcritical': as ϕ falls below ϕ_b the persistent steady state $h=1/2$ gains stability giving rise to two unstable symmetric steady states in its neighbourhood (Guckenheimer and Holmes, 1990).[8]

[6] This condition is less restrictive than the corresponding one, namely $\mu<(1-1/\sigma)$, in the original CP model (see Fuijita, Krugman, and Venables, 1999).

[7] Using the no-black-hole condition, all signs can be established by inspection.

[8] If $V_{hhh}(1/2,\phi_b)<0$ the bifurcation is 'supercritical': as ϕ rises above ϕ_b the persistent steady state $h=1/2$ loses stability giving rise to two stable symmetric steady states in its neighbourhood.

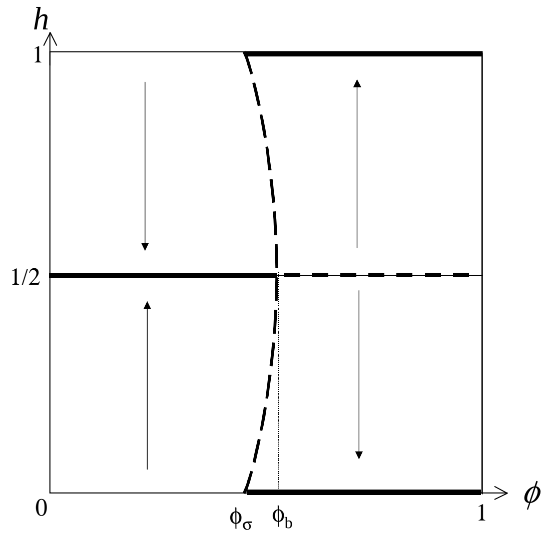

Figure 1: Bifurcation diagram

All these *local* properties pertain also to the CP model. The difference here is that, given the explicit function $W(h,\phi)$, one can claim that they hold also globally. In other words, while in Krugman's model analytical results are only local and numerical methods are required to investigate its global properties, here both local and global properties can be assessed analytically. Therefore, contrary to the original model, we are sure that all equilibria have indeed been captured. In particular, the global extension of $W_{hhh}(1/2,\phi_b) > 0$ implies that $\phi_b > \phi_s$. These results are conveniently summarized by the bifurcation diagram in Figure 1, where, as pointed out by the arrows, heavy lines are stable equilibria while dotted lines are unstable ones.

The fact that trade costs at the break point are lower than at the sustain point implies that, like the CP model, the FE model displays 'hysteresis' in location. Once the core-periphery equilibrium is reached, trade costs have to rise above the breakpoint before the agglomerated equilibrium becomes unstable. Finally, it may be noted that as σ goes towards infinity the manufacturing sector approaches perfect competition, and the break and sustain points converge. Figure 1 shows that the CP and the FE model are qualitatively identical, which is not surprising since the logic of agglomeration they formalize is the same by construction. Specifically, the no-black-hole condition as well as the break and sustain points of the CP model can be readily trasformed into those of the FE model by substituting μ by μ/σ.

4 Exogenous regional differences

The analytical convenience of the FE model can be gauged by considering exogenous asymmetries between regions. Indeed, the availability of analytical solutions is robust to the introduction of such asymmetries.

As a first example, we let regions differ in terms of market size. In particular, we focus on the exogenous components of market sizes: the regional endowments of immobile unskilled workers. The asymmetry is captured by defining a parameter $\varepsilon > 0$ such that $L_2 = \varepsilon L_1$. Thus, when $\varepsilon > (<)\ 1$, region 2 is 'larger' ('smaller') than region 1.

Using the same procedure as before, we find that the equilibria of the asymmetric model and their global stability properties are determined by the zeroes and the slope of the following expression:[9]

$$V(h,\phi) \equiv \frac{(1+\varepsilon)\phi h + [1-\frac{\mu}{\sigma}+(\varepsilon+\frac{\mu}{\sigma})\phi^2](1-h)}{[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}}} - \frac{(1+\varepsilon)\phi(1-h) + [\varepsilon(1-\frac{\mu}{\sigma})+(1+\varepsilon\frac{\mu}{\sigma})\phi^2]h,}{[(1-h)+\phi h]^{\frac{\mu}{1-\sigma}}} \tag{30}$$

This is again an explicit function of h which can be studied analytically. Figure 2 shows the bifurcation diagram for a typical case with $\varepsilon > 1$ so that region 2 is the larger region. Heavy lines are stable equilibria while dotted lines are unstable ones. Compared to Figure 1, a key feature of Figure 2 is the existence of a unique stable asymmetric equilibrium for values of ϕ below ϕ_b with the larger region 2 hosting a larger number of manufacturing firms. For $\phi_{s2} < \phi < \phi_b$ the stable asymmetric equilibrium is accompanied by another stable equilibrium with full agglomeration in region 2, and by another asymmetric equilibrium , which is unstable and has even more firms in region 2. For $\phi_b < \phi < \phi_{s1}$ there is only one equilibrium, which is stable and implies full agglomeration again in region 2. It is only when ϕ is above ϕ_{s1} that full agglomeration in the smaller region 1 becomes a second stable equilibrium.

All this illustrates the market access advantage of the exogenously larger region: larger immobile demand attracts more mobile demand.

Size is, however, not the only determinant of market access. Indeed, a region can provide better market access to manufacturing firms not only because it is larger but also because it is more difficult to reach via exports. An example may highlight why the freeness of trade may be asymmetric. Clearly, driving from a region to the other is as expensive as driving back. However, foreign retail costs may differ due to different non-tariff barriers or freight insurance premia. For identical exogenous market sizes ($L_1 = L_2 = L/2$), this implies that trade costs between regions are asymmetric. To analyse this scenario, we call ϕ the freeness of trade from region 2 to region 1, ϕ^* trade freeness in the other direction. Then, assume that region 2 is more difficult to supply from outside: $\phi^* = \varepsilon\phi$ with $0 < \varepsilon < 1$.

The usual derivation yields:

$$V(h,\phi) \equiv \frac{(1+\varepsilon)\phi h + [1-\frac{\mu}{\alpha\sigma}+\varepsilon(\varepsilon+\frac{\mu}{\alpha\sigma})\phi^2](1-h)}{[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}}} - \frac{(1+\varepsilon)(1-h) + [(1-\frac{\mu}{\alpha\sigma})+(1+\varepsilon\frac{\mu}{\alpha\sigma})\phi^2]h}{[(1-h)+\phi h]^{\frac{\mu}{1-\sigma}}}, \tag{31}$$

[9] Also the expression for Φ changes, remaining nonetheless positive.

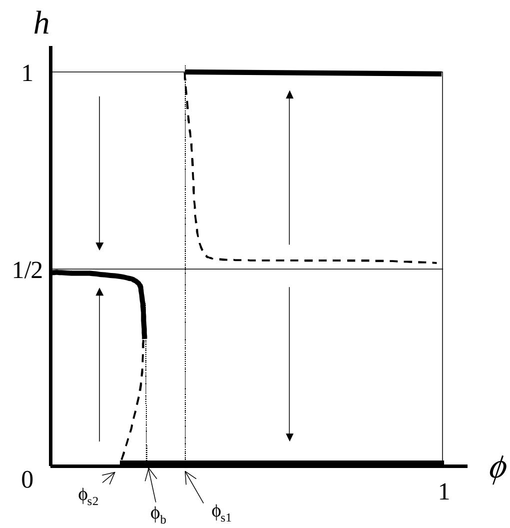

Figure 2: Exogenous regional differences

which is once more an explicit function of h that can be studied analytically: the equilibria correspond to its zeroes while its slope determines their global stability.[10]

In particular, the bifurcation diagram corresponding to (31) is qualitatively identical to Figure 2 drawn in the case of size asymmetry. The reason for this is that a relatively protected region and a larger region both provide better overall access to world markets (i.e., cost reduction in terms of trade barriers).

5 Conclusions

This paper presents an analytically solvable version of the central 'new economic geography' model, the so called core-periphery model by Krugman (1991). The key to solvability is the introduction of skill and mobility heterogeneity among manufacturing workers. In the original model all manufacturing workers are assumed to be homogeneous and sector-specific. On the contrary, in our modified version they are allowed to differ in terms of both skills and mobility. In particular, the unskilled are *intersectorally* mobile while the skilled are *interregionally* mobile. The result is a model of 'footloose entrepreneurs', which seems to better fit the European context where empirical evidence indicates that skilled workers are much more mobile than unskilled ones.

Analytical solvability allows us to assess the exact number of equilibria and their global stability properties, which is not possible in the original core-periphery model. Our modified model is also useful in studying complex problems that are out of reach in Krugman's set-up. For example, we have shown how the introduction of exogenous size asymmetries or asymmetric

[10] Again the value of Φ changes but remains positive.

trade costs between regions can be easily analysed. Solvability is also an advantage when explicitly analyzing dynamics as in Ottaviano (2001) or growth as in Baldwin and Forslid (2000) and in Martin and Ottaviano (2001).

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Appendix: Comparing the FE model to the CP model by Krugman (1991)

The model by Krugman (1991) differs from ours in that manufacturing firms incur both their fixed and marginal trade costs in terms of sector-specific 'skilled' labour:

$$TC_i(s) = w_i[\alpha + \beta x_i(s)], \tag{A1}$$

whereas 'unskilled' labour is used in agriculture only. Thus, prices depend on skilled wages rather than on unskilled ones:

$$p_{ii}(s) = w_i \beta\sigma/(\sigma - 1) \quad \text{and} \quad p_{ij}(s) = w_i \tau\beta\sigma/(\sigma - 1), \tag{A2}$$

Once substituted into profits, these equilibrium prices imply that firm size is independent of wages:

$$x_i = \frac{\alpha(\sigma - 1)}{\beta}. \tag{A3}$$

Plugging (A1) and (A3) into the skilled labour market clearing condition yields:

$$n_i = \frac{H_i}{\alpha\sigma}. \tag{A4}$$

and the manufacturing goods market clearing condition can be written as:

$$1 = \frac{{w_i}^{1-\sigma}\mu Y_i}{{w_i}^{1-\sigma}H_i + \phi {w_j}^{1-\sigma}H_j} + \frac{\phi {w_i}^{1-\sigma}\mu Y_j}{\phi {w_i}^{1-\sigma}H_i + {w_j}^{1-\sigma}H_j}. \tag{A5}$$

This is the equation that corresponds to (14) and, since income is still given by:

$$Y_i = \frac{L}{2} + w_i H_i \tag{A6}$$

it is clearly non-linear in w_i and w_j. This is what prevents any analytical solutions for skilled wages as functions of skilled workers' location.