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Exam for EC 2110, Uncertainty and asymmetric information, 21 February 2015

1. Short questions:

a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
(i) $v(c) = \ln(e^{c+3})$
(ii) $v(c) = c + c^2$
(iii) $v(c) = c + \sqrt{c}$
(iv) $v(c) = 1 - e^{-c}$

b) Tim has the following elementary utility function: $v(c) = \sqrt{c}$, where c is the value of Tim's fortune which consists of a bike worth 100. The likelihood of the bike being stolen is 20%. Tim has to take a decision on whether to buy bike theft insurance. The insurance contract on offer specifies a premium $P = 19$ and a deductible $D = 32$. Will Tim buy insurance?

c) Consider a market with a monopsonist employer. There are two types of workers. Type 0 has marginal product $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = 1$. Type 1 has marginal product $\Theta_1 = 3$ and an outside opportunity wage of $w_0(\Theta_1) = 2$. Workers know their own type but the employer cannot tell the high from the low productivity workers. When educational screening is possible, what contract will be offered to type 1 workers? (Assume that, if two contracts yield the same level of utility, a type 0 worker prefers the one which requires less education.)

d) The owner of a farm hires a worker to grow crops. The crop yield is random (depending on e.g. weather conditions), either high or low. However, the probability of the crop yield being high π_e also depends on the effort $e \in \{0, 1\}$ that the worker exerts, such that $\pi_0 = \frac{1}{3}$ and $\pi_1 = \frac{2}{3}$. The cost that the worker incurs from exerting effort e is $\Psi(e) = e$. The farm owner, who is the only employer, offers a contract $(\underline{t}, \bar{t})$ that induces the worker to exert high effort. The worker's expected utility is given by $EU_e = \pi_e \bar{t} + (1 - \pi_e)\underline{t} - \Psi(e)$. Unfortunately it is not possible for the farm owner to observe how much effort has been exerted. Moreover, the worker's liability is limited such that losses from transfers cannot exceed 1. State all the constraints that need to be satisfied for a contract $(\underline{t}, \bar{t})$ offered by the farm owner. Illustrate these constraints graphically in a figure, with $\underline{t}$ on the x-axis and $\bar{t}$ on the y-axis.

2. There are two states of the world, state 1 and state 2. The probability for state 1 occurring is $\pi = \frac{1}{4}$ (and the probability for state 2 occurring is $1 - \pi = \frac{3}{4}$). It is not possible to directly trade in state claims. However, there exists a complete asset market, where two assets, asset A_1 and asset A_2, can be traded.

a) State the conditions that have to be satisfied for a complete asset market.

The price of A_1 is given by $P_1^A = 1$, and the price of A_2 is given by $P_2^A = 1$. The following yield matrix indicates how much each asset yields in each state (e.g. A_1 yields $z_{11} = 4$ in state 1):

	State 1	State 2
Asset A_1	$z_{11} = 4$	$z_{12} = 1$
Asset A_2	$z_{21} = 2$	$z_{22} = 2$

The individual's preference-scaling function is given by $v(c) = \ln c$, and the individual is endowed with $\overline{q_1} = 2$ units of A_1 and $\overline{q_2} = 2$ units of A_2.

b) What are the implicit prices of state claim 1 (P_1) and state claim 2 (P_2)? (Hint: $P_1^A = z_{11}P_1 + z_{12}P_2$ and $P_2^A = z_{21}P_1 + z_{22}P_2$.)

c) What will the individual's portfolio of assets (i.e. the endowment $\overline{q_1}$ and $\overline{q_2}$) yield in the two different states? (That is, what is the individual's implicit endowment of state claims $\overline{c_1}$ and $\overline{c_2}$?)

d) Set up the von-Neumann-Morgenstern expected utility function.

e) To obtain the optimal amounts of implicit state claims, two conditions need to be satisfied. State these two conditions. Then calculate the optimal amounts of state claims.

f) Given the optimal amounts of state claims, what are the optimal amounts of assets A_1 and A_2? (Note that it is possible to go short, i.e. either q_1^* or q_2^* can be negative!)

3. Consider an entrepreneur (the agent) who needs to borrow money to set up a production facility. A bank (the principal) provides a loan of size k (at cost k). The repayment of the entrepreneur is given by t. The bank's profit is thus given by $V = t - k$. The value of the entrepreneur's output P is determined by the size of the loan and his/her type: $P(k, \Theta) = 2\Theta\sqrt{k}$, where $\Theta = \underline{\Theta} = \frac{1}{2}$ if the entrepreneur is inefficient and $\Theta = \overline{\Theta} = 1$ if the entrepreneur is efficient (note: a higher Θ implies higher efficiency). The entrepreneur's profit is given by $U_\Theta = P(k, \Theta) - t$.

a) What is the socially optimal loan size for each type of entrepreneur?

b) What are the socially optimal output values?

c) Which repayments will be paid by entrepreneurs if the bank is the only money lender? Which is the first-best menu of contracts?

Assume now that information regarding entrepreneurs' types is hidden to the bank. However, the bank knows that the share of efficient entrepreneurs is $v = \frac{1}{3}$.

d) State the bank's optimization problem and all constraints that need to be satisfied. Which constraints are relevant? Explain why the other constraints are not relevant.

e) Simplify the optimization problem by taking into consideration that the bank is a monopolist. Solve the optimization problem to determine the second-best menu of contracts.

f) Show that it is not optimal for the bank to shut down inefficient entrepreneurs. Provide a brief intuitive explanation for why the bank chooses not to shut down inefficient entrepreneurs.