Macro I, Spring 2015, Final Exam
March 19, 2015

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Solow model (10 points)

Consider the following Solow setup: The law of motion of the capital stock is

$$K(t+1) = K(t)(1-\delta) + qI(t) \tag{1}$$

The saving behavior is

$$I(t) = sF\left[K(t), A(t)L(t)\right] \tag{2}$$

and the resource constraint is

$$F\left[K(t), A(t)L(t)\right] = C(t) + I(t). \tag{3}$$

$F\left[K(t), A(t)L(t)\right]$ is a neoclassical production function that fulfills the standard assumptions. We have $A(t) = \gamma^t$ and $L(t) = n^t$. $q > 0, 1 > s > 0$, $\gamma > 1$ and $n > 1$ are some parameters.

(a) **(6 points)** Let us focus on the steady state where $k(t) \equiv \frac{K(t)}{A(t)L(t)}$ is constant. Calculate the capital-output ratio, $\frac{K(t)}{F[K(t),A(t)L(t)]}$, in this stationary point. Is this capital-output ratio increasing or decreasing in q? Give an intuition.

(b) **(4 points)** The production function is:

$$F\left[K(t), A(t)L(t)\right] = K(t)^{\alpha}\left[A(t)L(t)\right]^{1-\alpha}. \tag{4}$$

Calculate the golden rule capital stock per efficiency units, $k^{\star}_{gold}$, for this economy.

Problem 2: Neoclassical growth in discrete time (30 points)

Consider a version of the neoclassical growth model with Greenwood, Hercowitz, and Huffman (AER, 1988) preferences, no population growth and 100% physical capital depreciation. Households have the following preferences:

$$\mathcal{U}(0) = \sum_{t=0}^{\infty} \beta^t \log\left(c(t) - \psi h(t)^{\theta}\right), \quad \theta > 1, \quad \psi > 0,$$

where $c(t)$ is consumption and $h(t)$ are hours worked. We have $0 < \beta < 1$. The resource constraint is:

$$\begin{aligned} K(t+1) &= Y(t) - c(t) \\ Y(t) &= AK(t)^{\alpha} h(t)^{1-\alpha}, \quad 0 < \alpha < 1, \end{aligned}$$

where $K(0) > 0$ is given. $K(t)$ is aggregate physical capital, $Y(t)$ is aggregate production, A is total factor productivity in production, α is the aggregate income share from physical capital, θ relates to the Frisch elasticity of labor supply, and ψ is a measure for the weight of leisure relative to consumption in the utility function. $h(t) \leq 1$ are hours worked, where the total time endowment is normalized to one. The transversality condition for this economy can be expressed as:

$$\lim_{T\to\infty} \beta^T \frac{\partial \log\left[c(T) - \psi h(T)^{\theta}\right]}{\partial c(T)} K(T+1) = 0.$$

(a) **(6 points)** Greenwood, Hercowitz, and Huffman preferences are special because they imply no wealth effects on the labor supply. Assume that there are perfect competitive markets. In a perfect competitive equilibrium households maximize utility subject to the budget constraint $a(t+1) = a(t)\left[1 + r(t)\right] + w(t)h(t) - c(t)$ and a no-Ponzi game condition. Solve for the optimal labor supply and show that it can be written just as a function of the wage rate (and exogenous parameters).

[You don't need to solve for the entire competitive equilibrium path. Stating the household problem in a competitive environment and solving for the labor supply (and showing that it only depends on exogenous parameters and the wage rate) is enough.]

(b) **(6 points)** State the **social planner's problem** of this economy and derive the first-order conditions.

(c) **(4 points)** Derive the consumption Euler equation from the social planner's problem.

[Hint: The Euler equation takes the following functional form $\frac{c(t+1) - z_1 h(t+1)^{z_2}}{c(t) - z_1 h(t)^{z_2}} = z_3 \left[\frac{K(t+1)}{h(t+1)}\right]^{z_4}$ *where* z_1, z_2, z_3 *and* z_4 *are some constants.]*

(d) **(6 points)** Is there a steady state? If yes solve for capital, hours worked and consumption in this stationary point and show that the transversality condition is fulfilled.

[Hint: Note that there is no technical progress.]

(e) **(4 points)** Now let us look at the transition: Guess that consumption is proportional to output, $c(t) = \mu Y(t)$ (where μ is a constant that you will have to determine) and show that this is indeed the case. Determine μ.

(f) **(4 points)** Introduce exogenous technical progress into the model. Then, the new resource constraint becomes

$$K(t+1) = AK(t)^{\alpha} \left[X(t)h(t)\right]^{1-\alpha} - c(t),$$
$$X(t+1) = \gamma X(t), \quad X(0) > 0, \, \gamma > 1.$$

Is there a balanced growth path for this economy where $Y(t)$ and $K(t)$ grow both at gross rate γ?

[Hint: Check whether there is a balanced growth path where labor supply is constant.]

Problem 3: Labor income share (10 points)

Describe (or draw) the dynamics of the labor income share in a developed country like the U.S.? What is roughly the level? How did it change over the last 80 years? How do the dynamics look for the last two decades? Discuss the following questions: Are there any challenges to measure the labor income share? How is it measured in practice? What are potential sources for changes in the labor income share?

[I don't expect you to write more than 1/2-3/4 page.]

Problem 4 (50 points)

Consider an infinite-horizon pure endowment economy. In each period $t \geq 0$, there is a realization of a stochastic even $s_t \in \mathbf{S}$. The history of events up and until time t is denoted $s^t = [s_0, s_1, \ldots, s_t]$. The unconditional probability of observing a particular sequence of events s^t is given by a probability measure $\pi_t(s^t)$. We assume a nondegenerate probability distribution over the initial stochastic event s_0, i.e., $\pi_0(s) \in [0,1)$ for all $s \in \mathbf{S}$.

The economy is populated by large and equal numbers of two types of agents named $i = 1, 2$. Each agent of type i evaluates streams of a single nonstorable consumption good according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \log\Big(c_t^i(s^t)\Big), \qquad \beta \in (0,1),$$

where $c_t^i(s^t)$ is the agent's consumption at time t after history s^t. A feasible allocation satisfies $\sum_i c_t^i(s^t) \leq \sum_i y_t^i(s^t) \equiv Y_t(s^t)$ for all $t \geq 0$ and for all s^t, where $y_t^i(s^t)$ is the endowment of agent i.

(a) **(7 points)** Formulate and solve a Pareto problem for this economy. Characterize how the aggregate endowment is allocated among agents of type 1 and 2 in a Pareto optimal allocation.

(b) **(4 points)** Define a competitive equilibrium with time 0 trading. Trading is assumed to take place before the realization of the stochastic event s_0. Let $q_t(s^t)$ denote the Arrow-Debreu prices.

(c) **(10 points)** Characterize a competitive equilibrium with time 0 trading. Go as far as you can to find equilibrium expressions in terms of primitives for the allocation and for relative prices $q_t(s^t)/q_k(\tilde{s}^k)$.

From hereon we assume a two-state time-invariant Markov chain, $s \in \{1, 2\}$, as defined by transition probabilities

$$\text{Prob}\Big(s_{t+1} = s' | s_t = s\Big) \;\equiv\; \pi(s'|s) = \begin{cases} 1, & \text{if } s' = s; \\ 0, & \text{if } s' \neq s; \end{cases}$$

and initial probabilities $\pi_0(1) = \bar{\pi}_I > 0$ and $\pi_0(2) = \bar{\pi}_{II} > 0$, where $\bar{\pi}_I + \bar{\pi}_{II} = 1$. The endowments of the two agents are

$$\begin{aligned} y_t^1(s^t) &= 2 - s_t\,, \\ y_t^2(s^t) &= s_t - 1\,. \end{aligned}$$

(d) Deduce $\pi_t(s^t)$ from the described Markov chain.

(e) **(11 points)** Compute a competitive equilibrium with time 0 trading. In terms of primitives, find equilibrium expressions for the allocation, and for the Arrow-Debreu prices $\hat{q}_t(s^t)$ when the good in period 0 and history $s^0 = [\,1\,]$ is the numeraire. Provide an economic explanation to your findings.

From hereon we assume that the agents have heterogenous beliefs about the Markov process. Specifically, while they agree about the transition probabilities $\pi(s'|s)$ as stated above, they disagree about the probability distribution over s_0. Let $\bar{\pi}_I^i > 0$ and $\bar{\pi}_{II}^i > 0$ denote agent i's probabilities over $s_0 = 1$ and $s_0 = 2$, respectively, where $\bar{\pi}_I^i + \bar{\pi}_{II}^i = 1$. Thus, the diverse beliefs are $\bar{\pi}_I^1 \neq \bar{\pi}_I^2$ and $\bar{\pi}_{II}^1 \neq \bar{\pi}_{II}^2$.

(f) Deduce the subjective probability over history s^t for agent i, $\pi_t^i(s^t)$.

(g) **(10 points)** Compute a competitive equilibrium with time 0 trading. In terms of primitives, find equilibrium expressions for the allocation, and for the Arrow-Debreu prices $\hat{q}_t(s^t)$ when the good in period 0 and history $s^0 = [\,1\,]$ is the numeraire.

(h) **(3 points)** Formulate and solve a Pareto problem. Go as far as you can to characterize the set of Pareto optimal allocations. Is the competitive equilibrium allocation Pareto optimal?

(i) **(5 points)** Suppose that the true probability distribution over s_0 is symmetric, $\bar{\pi}_I = 0.5$. Moreover, we assume that agent 1 knows the truth, $\bar{\pi}_I^1 = \bar{\pi}_I$, but that agent 2 does not, $\bar{\pi}_I^2 \neq \bar{\pi}_I$. In terms of realized utilities, which agent is better off in a competitive equilibrium? Provide an economic explanation to your finding.