Macro I, retake exam
August 25, 2015

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Capital-output ratio in the Solow model (13 points)

Consider the following standard Solow setup: The law of motion of the capital stock is given by

$$K(t+1) = K(t)(1-\delta) + I(t). \tag{1}$$

The saving behavior is

$$I(t) = sF\left[K(t), A(t)L(t)\right], \tag{2}$$

and the resource constraint reads

$$F\left[K(t), A(t)L(t)\right] = C(t) + I(t). \tag{3}$$

The neoclassical production function, $F\left[K(t), A(t)L(t)\right]$, takes the following functional form:

$$F\left[K(t), A(t)L(t)\right] = \left[\omega K(t)^{\frac{\sigma-1}{\sigma}} + (1-\omega)\left(A(t)L(t)\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

with $\sigma \geq 0$ and $\omega \in (0,1)$. We have $A(t) = \gamma^t$, $L(t) = n^t$ with $\gamma > 1$, $n > 1$. Let us denote the capital per efficiency units as $k(t) \equiv \frac{K(t)}{A(t)L(t)}$.

(a) **(6 points)** Derive the golden rule capital stock per efficiency units, $k^\star_{gold}$, for this economy.

(b) **(7 points)** Assume that the production factors are rewarded according to their marginal products (due to perfect competition on the factor markets). Given the production function (4): Does the labor income share $\alpha_L(t) \equiv \frac{w(t)L(t)}{F[K(t),A(t)L(t)]}$ increase or decrease in $k(t)$? (Here, we normalized the price of the output good to one in the definition of $\alpha_L(t)$.) Which parameter does your answer depend on? Give an intuition.

Problem 2: Rebelo model (27 points)

Consider an economy with a representative household and exogenous gross population growth rate of n. Labor is inelastically supplied to the labor market and we have

$$L(t) = n^t L(0). \tag{5}$$

The economy consists of <u>two sectors</u>: a consumption good sector, C, and an investment sector, I. The investment good is produced according to an "AK" technology

$$I(t) = AK_I(t), \tag{6}$$

where $K_I(t)$ is capital used in the investment sector. The technology for the consumption sector is

$$C(t) = \gamma_c^t \left[\frac{K_C(t)}{\alpha}\right]^{\alpha} \left[\frac{L(t)}{1-\alpha}\right]^{1-\alpha}, \tag{7}$$

where $K_C(t)$ is capital used in the consumption good sector. $\gamma_c > 0$ is an exogenous rate of technical change in the consumption sector. $C(t)$ denotes aggregate consumption. Capital is fully mobile across the two sectors. Consequently, in equilibrium, capital has to earn the same rental rate $R(t) = r(t) + \delta$ across both sectors.[1] Clearing of the capital market requires:

$$K(t) = K_I(t) + K_C(t), \tag{8}$$

where $K(t)$ is aggregate capital in the economy. The preferences of the representative household are:

$$\mathcal{U}(0) = \sum_{t=0}^{\infty} (\beta n)^t \frac{c(t)^{1-\sigma} - 1}{1-\sigma},$$

where $c(t) \equiv \frac{C(t)}{L(t)}$ is per-capita consumption. We have $\beta n < 1$. In the following we consider a decentralized market equilibrium in which all the markets are perfectly competitive. We choose the investment good as a numéraire, i.e.,

$$P_I(t) = 1, \forall t. \tag{9}$$

$P_C(t)$, which denotes the price of the consumption good, can then be interpreted as the <u>relative price</u>. (This relative price is important because it will change over time due to the differences in the production technologies.) Under these assumptions, the resource constraint of the representative household can be written as:

$$K(t+1) = K(t) + w(t)L(t) + r(t)K(t) - P_C(t)L(t)c(t), \tag{10}$$

where $1 \geq \delta \geq 0$ is the depreciation rate. $K(0)$ is exogenously given.

[1] To be precise, what we consider here is an interior solution where both sectors produce strictly positive quantities.

(a) **(5 points)** The representative household chooses a sequence of $\{c(t), K(t+1)\}_{t=0}^{\infty}$ to maximize utility subject to the budget constraint and a standard transversality condition (which you don't need to state). The household takes $\{P_C(t), w(t), r(t)\}_{t=0}^{\infty}$ as given. State the intertemporal problem of the household and solve for the Euler equation. Show that it takes the form

$$z_1(1 + r(t+1)) = \left[\frac{c(t+1)}{c(t)}\right]^{z_2} \left[\frac{P_C(t+1)}{P_C(t)}\right]^{z_3}, \tag{11}$$

where z_1, z_2 and z_3 are some constants you have to determine.

(b) **(5 points)** In equilibrium, the representative firm in the consumption sector solves

$$\max_{K_C(t), L(t)} P_C(t)\gamma_c^t \left[\frac{K_C(t)}{\alpha}\right]^{\alpha} \left[\frac{L(t)}{1-\alpha}\right]^{1-\alpha} - R(t)K_C(t) - w(t)L(t). \tag{12}$$

Show that this firm's problem leads to the optimality conditions:

$$\frac{\alpha L(t)}{(1-\alpha)K_C(t)} = \frac{R(t)}{w(t)} \tag{13}$$

and

$$P_C(t) = \Xi_1 \gamma_c^{-t} R(t)^{\Xi_2} w(t)^{\Xi_3}, \tag{14}$$

where Ξ_1, Ξ_2, and Ξ_3 are to be determined.

(c) **(4 points)** Set up the firm's problem in the investment sector and solve it. Show that it implies—given the choice of numéraire—that $R(t)$ is constant over time.

(d) **(5 points)** Is this framework consistent with a balanced growth path? Look at the budget constraint, the Euler equation, equations (13) and (14), the result in (c), the production functions etc. to illustrate your statement. Can $C(t)$ and $K(t)$ grow at identical constant rates?

[Hint: Remember, we defined a balanced growth path as "a path along which all variables grow at constant rates (that could vary between one variable and the other and can even be zero)".]

(e) **(4 points)** Calculate the equilibrium gross growth rate of capital $\frac{K(t+1)}{K(t)}$ in terms of exogenous parameters.

(f) **(4 points)** With $\gamma_c \neq 1$ the technology changes exogenously. Now suppose, $\gamma_c \to 1$. Is in this case strictly positive, sustained growth in per-capita consumption possible?

Problem 3: Kaldor facts of growth (10 points)

What are the so-called Kaldor facts? Are the facts a good approximation of post-war data of developed countries? In which sense can the neoclassical growth model be viewed as a theory constructed around those facts? What are the required functional forms for technologies and preferences to match the Kaldor facts in a standard one sector neoclassical growth model?

[I don't expect you to write more than 1/2-3/4 page.]

Problem 4 (50 points)

Consider an economy that consists of overlapping generations of two-period-lived agents. At each date $t \geq 1$ there is born a constant number N of young people with preferences given by

$$\alpha c_t^t - \frac{[c_t^t]^2}{2} + \beta\left(\alpha c_{t+1}^t - \frac{[c_{t+1}^t]^2}{2}\right), \qquad \beta > 0,\ \alpha > 1,$$

where c_j^t is time j consumption of an agent born in period t. For all dates $t \geq 1$, young people are endowed with $w_y \in (0,1)$ goods when they are young and $w_o \in (0,1)$ units when they are old.

The initial old generation is of size N and its members are also endowed with w_o goods, and their utility is strictly increasing in c_1^0.

In each period $t \geq 1$, agents who are alive can trade in one-period risk-free bonds. Let R_t be the gross interest rate on a bond issued in period t, i.e., the inverse of the price of a sure claim to one unit of the good in period $t+1$.

(a) **(4 points)** Define a competitive equilibrium for this economy.

(b) **(10 points)** Compute the competitive equilibrium. Provide economic interpretations of how the equilibrium interest rate depends on various parameters of the model.

Suppose that the old-age endowment is subject to independent and identically distributed aggregate shocks. Specifically, with probability 0.5, the endowment of every old agent in a period is equal to $(1-\epsilon)w_o$, and with probability 0.5, it is equal to $(1+\epsilon)w_o$, where $\epsilon \in [0,1)$ and $(1+\epsilon)w_o < 1$. Agents maximize expected utility, and the market structure is the same as before, i.e., there are only markets in one-period risk-free bonds.

(c) **(10 points)** For this stochastic version of the model, compute the competitive equilibrium. Explain how and why the equilibrium interest rate depends on the uncertainty in the economy, as indexed by the parameter ϵ. Discuss the welfare implications of uncertainty.

(d) **(10 points)** In addition to risk-free bonds, suppose now that in each period $t \geq 1$, agents who are alive can also trade in one-step ahead Arrow securities. Compute the competitive equilibrium. Provide economic interpretations of how the equilibrium asset prices and the interest rate depend on various parameters of the model. Discuss how and why an agent's welfare is affected by the assumption of complete markets.

We now return to a deterministic old-age endowment w_o, and introduce fiat money. There is a time-invariant stock of money M that initially belongs to the old agents in period 1, i.e., each initial old agent owns M/N units of fiat money. In each period $t \geq 1$, agents who are alive can trade in fiat money as well as in one-period risk-free bonds. Let p_t be the price level in period t.

(e) **(4 points)** Define a competitive equilibrium with valued fiat money.

(f) **(12 points)** Derive a restriction on parameters for the existence of a stationary equilibrium with valued fiat money, in which the price level stays constant over time. Compute such a stationary monetary equilibrium. Please include expressions for the consumption allocation, the risk-free interest rate and the price level in terms of primitives.