# EC7104 The Climate & the Economy
## Spring 2015
August 2015

**Instructions**. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. Consider the following simple climate model:

$$\begin{aligned} T_t &= T_{t-1} + \sigma_1 \left( F_{t-1} - \kappa T_{t-1} - \sigma_2 \left( T_{t-1} - T_{t-1}^L \right) \right) \\ T_t^L &= T_{t-1}^L + \sigma_3 \left( T_{t-1} - T_{t-1}^L \right) \end{aligned} \tag{1}$$

where $T_t$ and $T_t^L$, respectively, denote the atmospheric and ocean temperatures in period $t$, both measured relative to a pre-industrial steady state. $F_t$ is forcing in period $t$ (change in energy balance relative to pre-industrial steady state) and $\kappa$ and the $\sigma$s are positive parameters.

(a) Explain the term $\kappa T_{t-1}$: what it is capturing?

(b) Suppose forcing is constant a some level $F$. Compute the steady state value of the two temperatures, denoted $T$ and $T^L$. (Hint: start with second equation and show the implication of $T_t^L = T_{t-1}^L$ for the relation between $T$ and $T^L$.)

(c) In our calibrations, we have used a value for $\sigma_3$ much smaller than for $\sigma_1$ and $\sigma_2$. Explain why this is reasonable.

(d) Suppose we shut down the influence of the ocean temperature on the atmospheric temperature in the model by setting $\sigma_2 = 0$. Draw a stylized graph with time on the $x$ axis and $T_t$ on the $y$ axis showing the temperature dynamics in the two cases $\sigma_2 > 0$ and $\sigma_2 = 0$.

(e) Let us now introduce a positive feedback mechanism, e.g., the change in albedo when higher temperatures reduce the arctic ice covers. We specify this as a contribution to the effective forcing proportional to the temperature, i.e., $xT_t$ where $x$ is a positive number. Introduce the term $xT_t$ in the climate model by showing where and in which equation it belongs. Make sure you get the right sign. Now calculate $T$ and $T^L$ as functions of $F$.

(f) It is reasonable to assume that $\kappa > x$. Why?

2. Consider an economy with two sectors, one for the production of final output and one for the production of fossil fuel. A firm producing final output has the production function

$$Y \equiv F\left(A, H_1, E, \overline{E}\right) = AH_1^{1-\alpha} E^{\alpha} \overline{E}^{\phi},$$

where $H_1$ denotes labor used to produce final output, $E$ is fossil fuel and $\overline{E}$ is an externality, which is taken as exogenous by the representative firm. Fossil energy is produced by firms using labor according to the following technology

$$E = BH_2,$$

where $H_2$ is the amount of labor used to produce fossil energy. Labor is supplied inelastically and the total supply of labor is normalized to 1, which implies that in equilibrium,

$$H_1 + H_2 = 1.$$

The price of fossil fuel is denoted by $p$, and the wage rate is denoted by $w$. The price of final output can be normalized to one. There are no savings, so all final output has to be consumed in equilibrium, i.e.,

$$C = AH_1^{1-\alpha} E^{\alpha} \overline{E}^{\phi}.$$

(a) Set up the profit maximization problem for the representative firm that produces final output.

(b) Derive the necessary first-order conditions.

(c) Set up the maximization problem for the firm that produces fossil energy.

(d) Derive the necessary first-order condition.

(e) Combine the three first-order conditions, the production function for fossil fuel, and the equation $L_1 + L_2 = 1$ into one equation that pins down the equilibrium value of $L_1$. Solve for $L_1$ in terms of the parameters. How does the value of $\phi$ influence $L_1$? Interpret.

3. A Solow model with no technological progress but with population growth at net rate $g$ can be summarized by the following equations:

$$c_t + i_t = y_t$$
$$y_t = Ak_t^{\alpha} \left((1+g)^t n\right)^{1-\alpha}$$
$$k_{t+1} = i_t + (1-\delta)k_t$$
$$i_t = sy_t.$$

(a) Derive the law of motion for capital, i.e., how $k_{t+1}$ must depend on $k_t$ (and on no other endogenous variables).

(b) What is the growth rate of output per capita in the long run in this model?

(c) In this model, $k_t/y_t$ will converge in the long run. To which value will it converge?

(d) Consider two countries whose macroeconomic variables both satisfy the equations above, though the two countries have different primitive parameters. Assuming that we are comparing long-run outcomes for two countries and that one of the countries has high output per capita whereas the other one has low output per capita, answer the following questions:

i. Suppose that $A$, $\alpha$, $n$, and $\delta$ are the same across countries so that only $g$ and $s$ differ between them. If $g$ is higher in the rich country, what does this tell you—if anything—about the relative values of $s$ in the two countries? Defend your answer.

ii. Suppose now instead that all the parameters may be different across countries. Based on the empirical literature, which one or two parameters appear to be the most important ones in explaining the distribution of relative output per capita between rich and poor countries?

4. Consider the following static IAM model: the representative consumer has preferences $u(c) = \log c - \gamma S$, the production of the consumption good occurs according to

$$c = Dk^{\alpha} n_c^{1-\alpha-\nu} E^{\nu},$$

(coal) energy is produced according to

$$E = \chi n_e,$$

there is a constraint $n_e + n_c = 1$ for the labor input, the carbon cycle satisfies $S = \varphi E$, and TFP does not involve any damages (climate damages only appear in utility, and no firms take this damage into account into account in any decisions).

(a) Derive an equation determining market coal use.

(b) Derive the first-order condition for a social planner maximizing consumer welfare.

(c) Find an expression for the optimal tax on carbon in this economy.

(d) Given any outcome for coal production, $E$, what is the outcome for temperature (relative to its preindustrial level)?

(e) Verbally describe one concrete *tipping point* and show, in concrete terms, how the model could be changed to accommodate it.

B. Short questions

5. Explain the difference between positive and negative climate feedback effects. Could they theoretically cancel? If so, is this realistic?

6. Matthews et al. (2009) introduced the concept of a constant Climate-Carbon Response ($CCR$). Assume the value of CCR is 1.5 Celsius/1000GtC. Suppose we want to limit global warming to 3 Celsius and have already emitted 500 GtC. How much more can we emit?

7. In the public debate you sometimes hear the claim that we are consuming the earth's resources at too fast a pace. Critically discuss this claim. Provide one argument supporting this claim and one argument that speaks against it. No formal analysis is required but you should base your reasoning on economic theory.

8. Explain the concept of *discounting* and its relevance in the climate-economy debate. Provide one argument in favor of using the market-determined real interest rate as a discount rate in climate calculations. Then provide one argument against using the market-determined real interest rate for this purpose.

9. Some economists claim that it does not matter whether we handle the climate problem with a carbon tax or with cap-and-trade. How might one defend this position?