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Exam for EC 2110, Uncertainty and asymmetric information, 25 May 2016

1. Short questions:

a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
(i) $v(c) = e^{\ln(1+c)}$
(ii) $v(c) = c^{\frac{3}{2}}$
(iii) $v(c) = c^{\frac{2}{3}}$
(iv) $v(c) = e^{c} + e^{-c}$

b) Consider a market with a monopsonist employer. There are two types of workers. Type 0 has marginal product $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = 1$. Type 1 has marginal product $\Theta_1 = 3$ and an outside opportunity wage of $w_0(\Theta_1) = 2$. The share of type 1 workers is given by $\frac{2}{3}$. Workers know their own type but the employer cannot tell the high from the low productivity workers. In the absence of any educational screening, will there be adverse selection in the market?

c) Explain briefly how deductibles can be used to deal with different types of asymmetric information in the insurance market.

d) The owner of a farm hires a worker to grow crops. The crop yield is random (depending on e.g. weather conditions), either high or low. However, the probability of the crop yield being high π_e also depends on the effort $e \in \{0, 1\}$ that the worker exerts, such that $\pi_0 = \frac{1}{4}$ and $\pi_1 = \frac{3}{4}$. The cost that the worker incurs from exerting effort e is $\Psi(e) = e$. The farm owner, who is the only employer, offers a contract $(\underline{t}, \bar{t})$ that induces the worker to exert high effort. The worker's expected utility is given by $EU_e = \pi_e \bar{t} + (1 - \pi_e)\underline{t} - \Psi(e)$. Unfortunately it is not possible for the employer to observe how much effort has been exerted. Moreover, the worker's liability is limited such that losses from transfers cannot exceed 1. State all the constraints that need to be satisfied for a contract $(\underline{t}, \bar{t})$ offered by the farm owner. Illustrate these constraints graphically in a figure, with $\underline{t}$ on the x-axis and $\bar{t}$ on the y-axis.

2. Ann has the following elementary utility function: $v(c) = \ln(1 + c)$, where c is consumption. Assume that there are only two possible states of the world, 1 and 2, where the probability of state 1 being realized is π. Consumption in state 1 is denoted by c_1, and consumption in state 2 is denoted by c_2.

   a) State Ann's von Neumann-Morgenstern utlility function.

   b) Derive a mathematical expression for Ann's marginal rate of substitution (MRS) between consumption in the two possible states of the world. What does the MRS measure? (Hint: $v'(c) = \frac{1}{1+c}$.)

   Ann's endowment of state claims is given by $\bar{c}_1^A = 2$ and $\bar{c}_2^A = 8$, and $\pi = \frac{2}{3}$. It is possible to trade in state claims at prices $p_1 = 1$ and $p_2 = 1$.

   c) Which two conditions need to be satisfied to find Ann's optimal amounts of state claim 1 and state claim 2?

   d) What are Ann's optimal amounts of state claim 1 and state claim 2?

   e) Are market prices actuarily fair? Use your answer in d) to provide an intuitive explanation.

   f) Determine the actuarily fair market price ratio.

   g) Now assume that trading takes place in an economy which only consists of Ann and another person called Barbara, i.e. the market equilibrium is determined by the interaction between Ann and Barbara. Barbara has the same utility function as Ann, and Barbara's endowment is given by $\bar{c}_1^B = 8$ and $\bar{c}_2^B = 2$ (both attribute the same probability $\pi = \frac{2}{3}$ to state 1 occurring). Explain in words why the market equilibrium price ratio will be actuarily fair - relate your answer to Ann's and Barbara's attitude to risk and to the total amounts of each state claim. (Note: you are not required to derive the solution analytically - you are supposed to provide an intuitive explanation.)

3. Consider a farmer (the agent) who needs to borrow money to grow crops. The farmer turns to a bank (the principal) which provides a loan of size k (at cost k). The repayment of the farmer is given by t. The bank's profit is thus given by $V = t - k$. The value of the farmer's output P is determined by the size of the loan and his/her type: $P(k, \Theta) = 2\Theta k^{\frac{1}{2}}$, where $\Theta = \underline{\Theta} = 1$ if the farmer is inefficient and $\Theta = \overline{\Theta} = 2$ if the farmer is efficient. (Note: A higher Θ implies higher efficiency!) The farmer's profit is given by $U_\Theta = P(k, \Theta) - t$.

   a) What is the socially optimal loan size for each type of farmer?

   b) Show that the socially optimal loans generate a social surplus for both types of farmer.

   c) Which repayments will be paid by farmers if the bank is the only money lender? Which is the first-best menu of contracts?

   d) Calculate the information rent that an efficient farmer can extract by mimicking an inefficient farmer.

   Assume now that information regarding farmers' types is hidden to the bank. Let the share of type $\overline{\Theta}$ farmers be given by $\nu = \frac{1}{3}$.

   e) State the bank's optimization problem and all constraints that need to be satisfied. Which constraints are relevant? Explain why the other constraints are not relevant.

   f) Simplify the optimization problem by taking into consideration that the bank is a monopolist. Solve the optimization problem to determine the second-best menu of contracts.

   g) Explain in words who gains and who looses when second-best contracts instead of first-best contracts are implemented. (Note: You are not supposed to calculate gains and losses.)