STOCKHOLM UNIVERSITY
Department of Economics

Course name:	**The Macroeconomy in the Long Run**
Course code:	**EC7215**
Examiner:	**Johan Söderberg**
Number of credits:	**7.5 credits**
Date of exam:	**6 December 2015**
Examination time:	**3 hours (09.00-12.00)**

Write your identification number on each paper and cover sheet (the number stated in the upper right hand corner on your exam cover).

Do not write answers to more than one question in the same cover sheet. Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked. No aids are allowed.

The exam consists of 4 questions. Each question is worth 25 points, 100 points in total. For the grade E 45 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Only students who have NOT received credits from the seminar series should answer question 4.

Results will be posted on mitt.su.se three weeks after the exam, at the latest

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Good luck!

Question 1 (25 p)

Consider an economy with an infinitely lived representative household where population grows at constant rate n. The household head solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \widetilde{\beta}^t \log\left(\frac{C_t}{L_t}\right), \tag{1}$$

where $\widetilde{\beta} = \beta(1+n) < 1$ is the effective discount factor, subject to the budget constraint

$$I_t + (1+\tau_t) C_t = w_t L_t + R_t K_t + T_t, \tag{2}$$

where τ_t is a (time-varying) consumption tax, T_t denotes transfers, and the evolution of the capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{3}$$

Firms in the economy are price-takers, operating on a perfectly competitive market. Production in the economy is determined by the production function

$$Y_t = AK_t, \tag{4}$$

where $A > 0$ is a technological constant. The representative firm's problem is to choose K_t to maximize its profit function

$$\Pi_t = AK_t - w_t L_t - R_t K_t. \tag{5}$$

a) Derive the household's and the firm's first-order conditions.

b) Assume that the proceeds from the consumption tax are transfered back to the households lump sum. Derive the model's equilibrium conditions.

c) Let lower case variables denote per-capita variables. Rewrite the equilibrium conditions in per-capita terms. Suppose that the economy is in a steady state where the consumption tax is constant. Solve for k_{t+1} and c_t in terms of k_t. What is the growth rate of output? Hint: Guess that the solution of the model is of the form:

$$k_{t+1} = F_k k_t, \tag{6}$$
$$c_t = F_c k_t. \tag{7}$$

d) Assume that the consumption tax evolves according to

$$\tau_{t+1} = \rho \tau_t, \tag{8}$$

where $0 < \rho < 1$. Consider two economies (A and B) with the same values of the deep parameters and the same initial capital stocks. In economy A, $\tau_0 = 0$; in economy B, $\tau_0 > 0$. Which economy will have the highest growth rate of consumption per capita during the transition to the steady state with constant taxes. Explain intuitively.

Question 2 (25 p)

Consider an economy without population growth where the household head solves

$$\max_{\{C_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log\left(\frac{C_t}{L_t}\right), \tag{9}$$

subject to the budget constraint

$$(1-\tau) I_t + \left(1-\tau^H\right) I_t^H + C_t = R_t K_t + w_t H_t + T, \tag{10}$$

where τ is a subsidy on investments in the physical capital stock, τ^H is a subsidy on investments in the human capital stock, and T_t denotes transfers. The evolution of the physical capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{11}$$

and the evolution of the human capital stock by

$$H_{t+1} = (1-\delta) H_t + I_t^H. \tag{12}$$

Production in the economy is determined by the production function

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \tag{13}$$

where $0 < \alpha < 1$. The representative firm's problem is to choose K_t and H_t to maximize its profit function

$$\Pi_t = K_t^{\alpha} H_t^{1-\alpha} - R_t K_t - w_t H_t. \tag{14}$$

a) Derive the household's and the firm's first-order conditions.

b) Solve for the equilibrium ratio of physical to human capital.

c) Show that the economy can grow forever. Explain why.

d) Solve for the growth rate of output when the economy is on a balanced growth path where output, capital, and consumption grow at the same rates. Explain how the growth rate depends on δ, τ, and τ^H.

Question 3 (25 p)

Consider an economy without population growth where a perfectly competitive final good firm combines intermediate goods into a final good, using the production function

$$Y_t = \left(\sum_{j=1}^{N_t} X_{jt}^{\alpha} \right) L_t^{1-\alpha}, \tag{15}$$

where X_{jt} is the input of intermediate variety j and N_t is the number of varieties available. The final good firm's problem is to choose $\{X_{jt}\}_{j=1}^{N_t}$ and L_t to maximize its profit function

$$\left(\sum_{j=1}^{N_t} X_{jt}^{\alpha} \right) L_t^{1-\alpha} - w_t L_t - \sum_{j=1}^{N_t} P_{jt} X_{jt}, \tag{16}$$

where P_{jt} is the price of intermediate variety j.

The intermediate goods are sold to the final good firms by research firms, each producing a specific variety of the intermediate good. The cost of producing one unit of the intermediate good is one unit of the final good. The profit function of the research firm producing intermediate variety j is thus

$$\Pi_{jt} = [P_{jt} - 1] X_{jt}. \tag{17}$$

The representative household invests in research firms; the cost of investing in a firm is η. The household retains ownership of the newly started firms, and the profits that the firms make is distributed back to the households at a period basis. The household head solves

$$\max_{\{C_t, N_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log \left(\frac{C_t}{L_t} \right), \tag{18}$$

subject to the budget constraint

$$I_t^R + C_t = w_t L_t + \sum_{j=1}^{N_t} \Pi_{jt}, \tag{19}$$

where I_t^R denotes the household's investment in research firms.

a) Derive the final good firm's demand for good j.

b) The optimal price charged by research firm j is $P_{jt} = 1/\alpha$. Find an expression for profits at research firm j in terms of α and L.

c) If intermediate varieties never become obsolete, the household's investment in research is given by

$$I_t^R = \eta \left(N_{t+1} - N_t \right). \tag{20}$$

Suppose instead that, in each period, a random fraction γ of the intermediate varieties become obsolete and cannot be used in production of the final good. Write down the expression for I_t^R under this assumption.

d) Show that the household's Euler equation is given by

$$\frac{1}{C_t} = \beta\left(1 + r_t\right)\frac{1}{C_{t+1}}, \tag{21}$$

where r_t is the interest rate (the return from investing in research firms). Determine r_t and explain how it is affected by η and γ?

Question 4 (25p)

Consider an economy where the social planner solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t, \tag{22}$$

subject to the aggregate resource constraint

$$K_{t+1} + C_t = K_t^{\alpha}. \tag{23}$$

a) Formulate the Bellman equation for the social planner's problem. Let K denote this period's capital stock and K' next period's capital stock.

b) Assume that the capital stock can take on all non-negative values. Guess that the initial value function $V^0\left(K\right)$ is 0 for all K. Use value function iteration to update the guess twice, i.e., calculate $V^2\left(K\right)$.

c) Would you consider $V^2\left(K\right)$ having converged to the true value function? Motivate your answer!

d) If you, regardless your answer in (c), consider $V^2\left(K\right)$ to be the true value function, what is the policy function $K'\left(K\right)$?