EC7104 The Climate & the Economy
Spring 2016

Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. **A climate model**
   Consider the following simple climate model:

$$\begin{aligned} F_t &= \eta \frac{\ln\left(\frac{S_t}{\bar{S}}\right)}{\ln(2)} \\ T_t - T_{t-1} &= \sigma_1 \left(F_{t-1} - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T^L_{t-1}\right)\right), \\ T^L_t - T^L_{t-1} &= \sigma_3 \left(T_{t-1} - T^L_{t-1}\right) \end{aligned} \tag{1}$$

   where S_t measures the stock of carbon in the atmosphere at t, $\bar{S}$, pre-industrial carbon, F_t is forcing at t, T_t is the global mean atmospheric temperature, T^L_t the mean temperature in the ocean (both measured in excess of their pre-industrial levels) and η, σ_1, σ_2, σ_3, and κ are all constant parameters.

   (a) Calculate the steady-state temperatures in the atmosphere and in the ocean, respectively, if the level of atmospheric carbon (S_t) permanently stabilizes at $2\times\bar{S}$.

   (b) We have also considered a simpler climate system without an ocean energy budget. In this, the second line of (1) is replaced by

$$T_t - T_{t-1} = \sigma_1 \left(F_{t-1} - \kappa T_{t-1}\right)$$

   and the third line is dropped. Assume that σ_1 and κ are the same in both cases. Compare this system to the one with the ocean energy budget. Specifically, compare their

   i. *Equilibrium Climate Sensitivity,* and

   ii. speed of adjustment. To do this, calculate T_2 and T_1 in both systems under the assumption that $T_0 = T^L_0 = 0$ and $F_0 = F_1 = 1$. Compare T_2 for the two cases and provide an explanation for your result.

   (c) Let us now add a simple carbon cycle, using the idea of a carbon depreciation function. Specifically, let

$$d\left(s\right) = \varphi_L + \left(1 - \varphi_L\right)\varphi_0\left(1 - \varphi\right)^s$$

   defining how much remains in the atmosphere of a unit of carbon s periods after it was emitted. Assume as in class that $\varphi_L = 0.2$, $\varphi = 0.023$, and $\varphi_0 = 0.38$, for s measured in decades.

   Until know, mankind has emitted around 500 GtC since we started burning fossil fuel. Consider that we burn 500 GtC more before completely phasing out fossil fuel. Use the carbon cycle and the climate model (1) to calculate the steady-state temperature. Assume $\bar{S} = 600$ GtC, $\eta = 4$ and $\kappa = 4/3$. If you don't have a calculator, provide a mathematical expression for the answer. (Hint: use the carbon depreciation function and determine how much carbon remains in the atmosphere of everything that has and will be emitted as s approaches infinity. Use this as the steady-state value of S_t.)

2. **Intertemporal energy supply without climate damages**

   Assume that an economy exists for two periods and that final output is produced with capital and energy. In the first period, only fossil fuel (E) is available as an energy source. However, in the second period there is also an alternative energy source B_1. Specifically, production in the two periods are given by

$$Y_0 = A_0 K_0^{\alpha} E_0^{1-\alpha} \quad \text{and} \quad Y_1 = A_1 K_1^{\alpha} \left(R_0 - E_0 + B_1\right)^{1-\alpha},$$

   where K is capital A_0 and A_1 are the exogenously given technology levels in period 0 and 1, respectively. Capital is assumed to depreciate fully between the periods. There is a fixed supply of fossil energy so the resource constraint for fossil fuel is given by

$$E_0 + E_1 = R_0.$$

   The production of final output can be used for consumption or savings which implies two more aggregate resource constraints:

$$C_0 = A_0 K_0^{\alpha} E_0^{1-\alpha} - K_1 \quad \text{and} \quad C_1 = A_1 K_1^{\alpha} \left(E_1 + B_1\right)^{1-\alpha}.$$

   The utility function is logarithmic in consumption and households maximize the discounted present value of utility:

$$U\left(C_0, C_1\right) = \log\left(C_0\right) + \beta \log\left(C_1\right),$$

   where β is the discount factor.

   (a) Set up the social planning problem.

   (b) Derive the Euler equation for capital as well as the Hotelling equation.

   (c) Solve explicitly for K_1, E_0, and E_1.

   (d) Show how the intertemporal allocation of fossil fuel (i.e., the relation between E_0 and E_1) is affected by A_0 and A_1.

   (e) How does the alternative energy source B_1 affect the allocation of fossil fuel between the periods?

3. **Secular stagnation**

   Consider an economy where technology has grown steadily, and for a long period of time, through a continuing rise in total-factor productivity ("TFP"), with an accompanying growth in the capital stock and output. In particular, this growth path has been *balanced*, with both output and capital growing at rate $g = 0.02$, under a constant saving rate $s = 0.3$. Two citizens of this economy, however, the economist oracles Bob Summers and Larry Gordon, all of a sudden claim that TFP has stopped growing, something they dub "secular stagnation". Everybody is baffled and start asking themselves a number of questions. Your task is now to try to answer these questions.

   In answering the questions, use Solow's growth model and suppose that nothing will change except TFP growth, which will now be 0 forever. The economy's current capital stock (before the hypothetical secular stagnation would set in) is 3 times as large as current annual output.

   (a) What will happen in the long run—will capital and output not be growing any more? Answer by showing, in a graph, the capital-stock dynamics of the Solow growth model without technological change. Can the capital stock be decreasing over time in this model?

(b) Denote the rate of depreciation of the capital stock δ. Try to use the information given in order to figure out what the value of δ must be.

(c) What will the long-run value of the capital-output ratio be? Based on your answer (whether it will be higher than, equal to, or smaller than its current value 3), will the capital stock in the short run go up, down, or remain constant?

(d) Someone raises an additional worry that, since the economy's resources are finite and we will be using them up over time in producing goods and services, we are facing an even more dystopic future. Suppose this point is correct and that this factor amounts to a lower and lower use of resources that is equivalent to a fall in labor input by 2 percent per year. What will this imply for the long-run level and growth rate of output and for the long-run capital-output ratio?

4. **An IAM with a variety of damages**

Consider a static IAM where, in the absence of climate change, output is given by $y = k^{\alpha}E^{1-\alpha}$, where k is capital and E energy used in production. Energy comes from oil and oil is available for free, but there is a total amount of it, R, that cannot be exceeded: we think of energy in oil units and hence the economy's energy resource restriction is $E \leq R$. Capital is also given. All of output is consumed and the consumer's utility function is $\log c$. Labor is not used as an input in this economy so we can abstract from it.

(a) Define a market equilibrium for this economy where the consumer owns and sells the oil and owns and sells capital and consumes the resulting income, and where firms buy oil and capital from consumers and produce the consumption good, all under perfect competition. What is the equilibrium use of oil? Compute the equilibrium prices of capital and oil as a function of the primitives (k, R, and α).

Now suppose that climate change sets in and that temperature goes up as a function of emissions. The mechanism is that emissions increase atmospheric carbon concentration, S (which is measured as a departure from the pre-industrial level), which in turn increases temperature via Arrhenius's greenhouse law. In particular, $S = \varphi E$, where $\varphi > 0$ is the fraction of emissions that end up in the atmosphere. Temperature increases, moreover, cause a variety of damages. These damages are of three kinds: directly lower utility (due to effects on health and consumer happiness more generally), destruction of the economy's capital stock, and lower TFP. The former makes utility now be $\log c - \gamma_1 S$, where S is the stock of carbon concentration. The latter two appear in production, $c = D(uk)^{\alpha}E^{1-\alpha}$, with $D = e^{\gamma_2 S}$ and $u = e^{\gamma_3 S}$. So D is the usual Nordhaus-like damage formulation, whereas u implies a less than full use of the economy's capital; both D and u are less than one to the extent that $S > 0$. The parameters γ_1, γ_2, and γ_3 are all finite, positive numbers.

(a) Based on this model, would it be optimal, from a social planner's perspective, to completely prohibit the use of oil, if it were possible?

(b) Show that the marginal damages from emitting a unit of oil in the laissez-faire equilibrium are $\gamma_1 + \alpha\gamma_2 + \gamma_3$ measured in "utils" and $(\gamma_1 + \alpha\gamma_2 + \gamma_3)c$ measured in consumption units, where c is laissez-faire consumption. (Hint: to do this, formulate consumer utility as a function of E and take the appropriate derivative. Also, note that one util is worth the same as 1/MU units of consumption, where MU is the marginal utility of consumption.)

(c) Define Γ to equal $\gamma_1 + \alpha\gamma_2 + \gamma_3$. Show that the optimal use of oil is R if $\Gamma < \frac{1-\alpha}{R}$, or otherwise $e < R$, where e is given by $e = \frac{1-\alpha}{\Gamma}$.

(d) How could the optimal allocation be attained with a policy? Be explicit about the policy instrument and how to set it.

B. Short questions

5. Water vapor is a greenhouse gas. This creates an important positive feedback mechanism. Explain how this works in words.

6. Suppose that the damage function is very convex so that marginal damages from climate change increase much faster than assumed in class. Then the result that the optimal tax is independent of future emissions becomes invalid. How? In what way would assumptions of future emission paths affect the optimal tax today?

7. Consider a world with three countries: one oil-producing country and two oil-consuming countries that use oil to produce a final output. Explain what "carbon leakage" is in this world. Under what conditions can we expect a high degree of carbon leakage between the two oil-consuming countries? Under what conditions can we expect a low degree of carbon leakage?

8. An investment of 1 unit of output today gives a return of $1 + r$ units in 100 years, using a long-run government bond. This value came about by the government selling bonds in the marketplace to finance road and bridge constructions. Give a short argument for, and one against, using r as a basis for choosing the discount factor in a dynamic IAM.

9. Are the following statements correct? Defend your answers very briefly.

   (a) Consumption smoothing is a result of decreasing marginal utility of consumption.

   (b) The real interest rate goes down, ceteris paribus, with consumption growth.