STOCKHOLM UNIVERSITY
Department of Economics

Course name:	**The Macroeconomy in the Long Run**
Course code:	**EC7215**
Examiner:	**Johan Söderberg**
Number of credits:	**7.5 credits**
Date of exam:	**30 October 2016**
Examination time:	**3 hours (16.00-19.00)**

Write your identification number on each answer sheet. Use the printed answer sheets for all your answers.

Do not write answers to more than one question in the same cover sheet. Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked. No aids are allowed.

The exam consists of 4 questions. Each question is worth 25 points, 100 points in total. For the grade E 45 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Only students who have NOT received credits from the seminar series should answer question 4.

Results will be posted on mitt.su.se three weeks after the exam, at the latest

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Good luck!

Question 1 (25 p)

Consider an economy with an infinitely lived representative household where population grows at constant rate n and technology at constant rate g. The household head solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \widetilde{\beta}^t \log\left(\frac{C_t}{L_t}\right), \tag{1}$$

where $\widetilde{\beta} = \beta(1+n) < 1$ is the effective discount factor, subject to the budget constraint

$$C_t + I_t = w_t L_t + R_t K_t - T_t, \tag{2}$$

where T_t is a lump-sum tax, and the evolution of the capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{3}$$

Firms in the economy are price-takers, operating on a perfectly competitive market. Production in the economy is determined by the production function

$$Y_t = F(K_t, A_t L_t), \tag{4}$$

which is assumed to exhibit constant returns to scale in K_t and L_t. The representative firm's problem is to choose K_t and L_t to maximize its profit function

$$\Pi_t = F(K_t, A_t L_t) - w_t L_t - R_t K_t. \tag{5}$$

a) Derive the household's and the firm's first-order conditions.

b) The government's budget constraint is

$$G_t = T_t, \tag{6}$$

where G_t is government expenditures (which are not used for anything productive). The government sets G_t such that the fraction of government expenditures in GDP is constant at S_G, i.e,

$$\frac{G_t}{Y_t} = S_G. \tag{7}$$

Derive the model's equilibrium conditions under this assumption.

c) Assume that the economy is in a steady state, where variables normalized by the number of effective workers, $A_t L_t$, are constant. Rewrite the equilibrium conditions in intensive form (in terms of normalized variables). Solve for the steady state values of the capital stock per effective worker and consumption per effective worker. How are they affected by S_G. Explain intuitively!

Question 2 (25 p)

Assume an OLG economy where individuals live for two periods with certainty. The size of the generation born in period t is L_t. Assume that an individual supplies 1 unit of labor and earns w_t in the first period of his life and is retired and not working in the second period. An individual born at time t has life-time utility

$$\log C_{1t} + \beta \log C_{2t+1}, \tag{8}$$

where C_{1t} is the individual's consumption in the first period of his life and C_{2t+1} his consumption in the second period. The individual is born without assets and does not wish to leave a bequest. Savings are invested in the aggregate capital stock, which yields a return of $r_t = R_{t+1} - \delta$.

Firms are price-takers, operating on a perfectly competitive market. The representative firm's problem is to choose inputs of capital and labor to maximize its profit function

$$\Pi_t = F(K_t, L_t) - w_t L_t - R_t K_t. \tag{9}$$

a) Assume that the young individuals' savings, s_t, are taxed at rate τ and that the proceeds from the tax are transferred back to the individuals of the same generation through a lump-sum transfer T_t per individual. Write down the first and second period budget constraints for an individual born in period t.

b) Derive an expression for s_t as a function of w_t. Explain intuitively how savings in the economy are affected by the tax rate τ.
Hint: After deriving the first-order conditions, impose the condition that the total sum of transfers must equal total tax income.

c) One equilibrium condition in the model is the resource constraint, given by

$$K_{t+1} + C_t = (1-\delta) K_t + F(K_t, L_t). \tag{10}$$

Derive the other equilibrium condition in the model.

d) Assume that the capital stock in the economy is larger than the golden rule capital stock if $\tau = 0$. Suppose that the government sets $\tau > 0$ to reduce the capital stock and increase consumption. Are there any drawbacks with this approach compared to the alternative of imposing an unfunded social security system discussed in class? No formal calculations are required, but discuss intuitively.

Question 3 (25 p)

Consider an economy without population growth where the household head solves

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t \tag{11}$$

subject to the budget constraint

$$C_t + I_t = w_t L_t + R_t K_t, \tag{12}$$

where the evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta) K_t + I_t. \tag{13}$$

In the economy, there are a large number of profit-maximizing firms, indexed by i and measured on the unit interval. The production function of firm i is given by

$$Y_{it} = K_{it}^{\alpha} (A_{it} L_{it})^{1-\alpha}, \tag{14}$$

where K_{it} and L_{it} are the inputs of capital and labor used by the firm, and $0 < \alpha < 1$. Technology, which is firm-specific, evolves according to

$$A_{it} = \lambda K_{it}^{\phi} K_t^{1-\phi}, \tag{15}$$

where K_t is the aggregate capital stock, λ a positive constant, and $0 \leq \phi \leq 1$. The firms are price-takers, operating on a perfectly competitive market.

a) Find an expression for aggregate output (the aggregate production function). Can the economy grow forever?

b) Solve for the growth rate of output per capita along a balanced growth path where output, consumption and the capital stock grow at the same constant rate.

c) How does the growth rate of the economy depend on ϕ? Explain intuitively!

Question 4 (25p)

Consider a household that solves

$$\max_{\{C_t, B_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t, \tag{16}$$

where B_t is the household's assets at the beginning of period t, subject to its budget constraint

$$B_{t+1} + C_t = (1 + r) B_t + Y, \tag{17}$$

where the constant r is the interest rate and the constant Y the household's income.

a) Formulate the Bellman equation for the household's problem. Let B denote this period's assets and B' next period's assets.

b) Assume that the $r = 0.1$ and $Y = 10$ and that assets only can take on the values $\{10, 20\}$. Guess that the initial value function $V^0(B)$ is 0 for all B. Use value function iteration to update the guess twice, i.e., calculate $V^2(B)$. You can use the table below to lookup approximate values of the *log* function. Assume that consumption cannot be negative.

x	$\log(x)$	$\beta \log(x)$
1	0	0
11	2.4	2.0
12	2.5	2.1
22	3.1	2.6

c) Would you consider $V^2(B)$ having converged to the true value function? Motivate your answer!

d) If you, regardless your answer in (c), consider $V^2(B)$ to be the true value function, what is the policy function $B'(B)$?