Macro I, retake exam 2016
August 25, 2016

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Capital-output ratio in the Solow model (15 points)

Consider the following Solow set-up: The resource constraint is

$$K_{t+1} = K_t(1-\delta) + q^t I_t, \tag{1}$$

where $q > 1$ is capital specific technical change and I_t is investment. I_t is equal to savings which is a constant fraction of income

$$I_t = S_t = sK_t^\alpha \left(\gamma^t n^t\right)^{1-\alpha}. \tag{2}$$

(i) **(7 points)** Suppose $\gamma n q^{\frac{1}{1-\alpha}} = 1.03$, $s = 0.15$, and $\delta = 0.02$. Calculate the ratio between nominal wealth and nominal output, i.e., $\frac{q^{-t}K_t}{K_t^\alpha(\gamma^t n^t)^{1-\alpha}}$, in this economy along the balanced growth path.

(ii) **(4 points)** In the data, how large is the measured nominal wealth to output ratio in developed economies like to post-war U.S.? Is the ratio constant over time?

(iii) **(4 points)** What about the ratio of real capital to real output, i.e., $\frac{K_t}{K_t^\alpha(\gamma^t n^t)^{1-\alpha}}$; does it converge according to the model above in the long run? If yes to which level, if no why not?

Problem 2: CRRA, KPR and balanced growth in a small open economy (25 points)

Consider a representative household that maximizes the following preferences

$$\mathcal{U}_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \tag{3}$$

subject to the constraints

$$a_{t+1} = a_t \left[1 + r_t\right] + w_t - c_t, \tag{4}$$

and a standard no-Ponzi game condition

$$\lim_{T \to \infty} \left[a_{T+1} \prod_{s=1}^{T} \frac{1}{1 + r_s} \right] \geq 0, \tag{5}$$

and a given $a_0 = A_0$.

(i) **(4 points)** Solve the household problem and state the consumption Euler equation.

Now consider a "small open economy" set-up where the interest rate is exogenous and constant, i.e., $r_t = \bar{r}$ and the household can borrow or lend as much as she wants at this fixed interest rate. In this case the transversality condition of the problem becomes

$$\lim_{T \to \infty} \left[a_{T+1} \left(\frac{1}{1 + \bar{r}} \right)^T \right] = 0. \tag{6}$$

Further assume that the wage rate is growing at constant gross rate γ, i.e., $w_t = w_0 \gamma^t$.

(ii) **(3 points)** Show that for any given constant interest rate $\bar{r}$ the household side is consistent with a "balanced growth path": Show that the household is willing to have a constant gross consumption growth rate $g_c \equiv \frac{c_{t+1}}{c_t}$, $\forall t$ for *any* constant exogenous interest rate. Calculate g_c as a function of the interest rate and the preference parameters.

(iii) **(6 points)** Calculate the initial consumption level c_0 (as a function of γ, σ $\bar{r}$, β, A_0, and w_0). Note that this initial consumption level is the only one that fulfills the transversality condition. Are there upper bounds for the interest rate, $\bar{r}$, such that the problem is well specified? If yes state these restrictions.

(iv) **(4 points)** How are the dynamics in a_t along this path with a constant wage growth and a constant interest rate? What condition determines whether the gross growth rate of consumption is larger or smaller than the gross growth rate of a_t? Is there an interest rate such that the consumption/wealth ratio is constant?

Now let us additionally consider a set-up with endogenous labor supply. Preferences are given by the following KPR form

$$\mathcal{U}_0 = \sum_{t=0}^{\infty} \beta^t \frac{[c_t \cdot v(h_t)]^{1-\sigma} - 1}{1-\sigma}, \tag{7}$$

where $v(\cdot)$ is an arbitrary (differentiable, monotonically decreasing) function and h_t denotes hours worked. The budget constraints are now given by

$$a_{t+1} = a_t \left[1 + r_t\right] + w_t h_t - c_t, \forall t. \tag{8}$$

Again consider a constant interest rate $r_t = \bar{r}$ and a constant wage growth, i.e., $w_t = w_0 \gamma^t$.

(v) **(4 points)** Solve for the intratemporal consumption-hours first-order condition as well as for the (new) Euler equation. Will households be willing to supply constant hours h to the labor market? If yes why, if no is there there a special interest rate for which this is fulfilled?

Finally suppose the preferences are instead given by

$$\mathcal{U}_0 = \begin{cases} \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}-1}{1-\sigma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right) & \text{if } \sigma \neq 1, \\ \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right) & \text{if } \sigma = 1. \end{cases} \tag{9}$$

We have $\sigma \geq 0$, $\theta > 0$, and $\psi > 0$. The budget constraint is still given by (8).

(vi) **(4 points)** Is there a balanced growth path in the sense that for *any* exogenous interest rate and any constant rate wage growth labor supply is changing at a constant gross rate g_h? If yes calculate the growth rate g_h.

Problem 3: Convergence, period (10 points)

What do we understand under the term "convergence"? Is there empirical evidence for absolute/conditional convergence? What is the evidence over the last 50 years as well as over the last 200 years? Can the neoclassical growth theory replicate the data qualitatively and quantitatively? Are simple endogenous growth models like the AK model doing better in terms of explaining the data?

[I don't expect you to write more than 1/2-3/4 page.]

Problem 4: Overlapping generations (50 points)

Consider an economy with overlapping generations of a constant population of two-period-lived agents. At each date $t \geq 1$ there are born N individuals of type 1 and N individuals of type 2. An agent of type i born at time t is endowed with $w_t^{it} \geq 0$ units of a single consumption good when young and $w_{t+1}^{it} \geq 0$ units of the good when old. The consumption good is not storable. A young agent of type i born at time t ranks utility streams according to $u^i(c_t^{it}, c_{t+1}^{it})$ where c_j^{it} is his consumption of the time j good;

$$u^1(c_t^{1t}, c_{t+1}^{1t}) = \ln(c_t^{1t}) + \ln(c_{t+1}^{1t}) \quad \text{and} \quad u^2(c_t^{2t}, c_{t+1}^{2t}) = c_t^{2t} + c_{t+1}^{2t}.$$

In addition, at time 1, there are N old people of type 1 alive and N old people of type 2 alive. An old agent of type i at time 1 is endowed with w_1^{i0} units of the good and his utility is increasing in his consumption c_1^{i0} of the time 1 good.

All intertemporal trade takes place in a market for private credit. Let R_t denote the gross interest rate between dates t and $t+1$. An agent of type i born at time t chooses how much to save b_t^{it} between dates t and $t+1$. (If b_t^{it} is negative, the agent is borrowing.) There are no initial credit contracts between old people alive in period 1.

(a) [**3 points**] Define an equilibrium for this economy.

(b) [**7 points**] Find the saving function of a young agent of type i born at time t.

From hereon, we make the following assumptions about endowments: $w_t^{1t} = w_y^1 > 0$, $w_t^{2t-1} = w_o^2 > 0$, and $w_t^{1t-1} = w_t^{2t} = 0$, for all $t \geq 1$.

(c) [**15 points**] Compute an equilibrium, i.e., express the interest rate and consumption allocation in terms of primitives. (Be sure to characterize equilibrium outcomes for all possible values of our two primitives, $w_y^1 > 0$, and $w_o^2 > 0$.)

Suppose a government institutes a pay-as-you-go social security system in period 1. Specifically, in every period $t \geq 1$, a lump-sum tax $\tau > 0$ is levied on each young agent, and a lump-sum transfer $\epsilon > 0$ is given to each old agent. Since there are equal numbers of young and old agents, the government balances its budget by setting $\tau = \epsilon$.

(d) [**15 points**] Compute an equilibrium.

(e) [**10 points**] Compare the interest rate, consumption allocation and welfare in the economy with social security in question **d** to the outcomes without social security in question **c**. Explain your findings.