

Department of Economics

Course name:	Econometrics 1
Course code:	EC7410
Examiner:	Björn Tyrefors Hinnerich
Number of credits:	7,5 credits
Date of exam:	Tuesday 10 January 2017
Examination time:	3 hours [09:00-12:00]

Write your identification number on each paper (the number stated in the upper right hand corner on your exam cover).

**Use one answer sheet per question.** Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked. **No aids are allowed.** 

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The exam consists of 5 questions. Each question is worth 20 points, 100 points in total. For the grade E 40 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

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Your results will be made available on your "My Studies" account (<u>www.mitt.su.se</u>) on January 31th at the latest.

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Good luck!

## Question 1.

(a) Prove that the absolute value of the covariance between two random variables (R.V.)X and Y, is smaller or equal to the square root of the product of their variances. I.e.:prove

$$|\sigma_{XY}| \leq \sqrt{\sigma_X^2 \sigma_Y^2}$$

- (b) State the Law of Iterated Expectations. Also describe what it means using an example.
- (c) Prove the Law of Iterated Expectations.

## **Question 2.**

- (a) Define a p-value, null hypotheses, the t-statistics and a confidence interval.
- (b) State the central limit theorem
- (c) Prove that that a standard t-statistic used to test for a null hypothesis of the form H<sub>0</sub>:  $E(Y) = \mu_{Y,0}$  is approximately normally distributed.

#### **Question 3.**

Say that we are interested in the effect of *X*<sub>*i*1</sub>on *Y*<sub>*i*</sub>. We specify the following equation:

$$Y_i = \beta_0 + \beta_1 X_{i1} + u_i$$

- (a) Set up the optimization problem that solves for the OLS-estimators and the first order conditions. You need <u>not</u> to simplify further. In words, please describe the logic of the optimization problem.
- (b) Under which assumptions is the OLS-estimator of  $\beta_1$  an unbiased and consistent estimator of  $\beta_1$
- (c) Derive the omitted variable bias (*X*<sub>*i*2</sub> omitted). Give two cases when there is no bias?

# **Question 4.**

Social scientists are interested in the efficiency of imprisonment after the criminal served the time, so called specific deterrence. In particular, we would like to know if a criminal commit fewer crimes in the future (defined  $Y_i$  below) after imprisonment compared to say a criminal that was identical and committed an identical crime but instead only was sentenced to probation.

Assume you observe an indicator P = 1 if individual *i* is being sentenced to prison and P = 0 if the individual *i* is being sentenced to probation. Assume further you observe the number of future crimes  $Y_i$  (the outcome) for individual *i*.

(a) If you run an OLS-regression explaining  $Y_i$  with the regressor  $P_i$ : ( $Y_i = \beta_0 + \beta_1 P_i + u_i$ ), discuss one potential omitted factor.

We are lucky to observe a randomized (natural) experiment for a subgroup of the Swedish population, namely the drunk drivers. If a drunk driver has a blood alcohol concentrations (BAC) equal to or above the threshold 1.5, he/she is often sentenced to prison while those just below are sentenced to probation. For simplicity, assume the threshold is strictly binding so if a drunk driver has 1.5 or above he/she is sentenced to prison and if below she/he is sentenced to probation. If we compare individuals close enough to the threshold, the treatment, *P*, will be "as good as" randomly assigned to the drunk drivers. We have in our sample 20 drunk drivers that had 1.50000001 in BAC and 20 individuals that had 1.499999991 in BAC and we are given the following information

$$\mathbf{P}'\mathbf{Y} = \begin{pmatrix} 60\\20 \end{pmatrix}$$

(a) Explicitly state the **P**'**P** matrix in numbers

(b) Calculate the determinant of P'P. What does it mean that a determinant is different from zero?

- (c) Calculate the inverse of **P'P**
- (d) Calculate the OLS-estimates and interpret them.

#### **Question 5.**

- (a) Discuss the notion of Granger Causality and the appropriate test.
- (b) Describe the assumptions required for estimating a dynamic causal effect with exogenous regressors using OLS.
- (c) Discuss the two major causes of non-stationarity and relevant tests in order to detect these hazards in a standard AR(1) forecasting model.