Stockholm University

Department of Economics

Course name:	**Econometrics 1**
Course code:	**EC7410**
Examiner:	**Björn Tyrefors Hinnerich**
Number of credits:	**7,5 credits**
Date of exam:	**Saturday 11 February 2017**
Examination time:	**3 hours [09:00-12:00]**

Write your identification number on each paper (the number stated in the upper right hand corner on your exam cover).

Use one answer sheet per question. Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked. **No aids are allowed.**

The exam consists of 5 questions. Each question is worth 20 points, 100 points in total. For the grade E 40 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Your results will be made available on your "My Studies" account **(**www.mitt.su.se) on January 31th at the latest.

Good luck!

Question 1.

(a) Prove that the absolute value of the correlation between two random variables (R.V.) X and Y is smaller or equal to one. I.e.: prove

$$|corr(X,Y)| \leq 1$$

(b) Show that $\Pr(Y = y_j) = \sum_{i=1}^{l} \Pr(Y = y_j | X = x_i) \Pr(X = x_i)$

(c) Let $Y_{1,\ldots,} Y_n$ be i.i.d with mean μ_Y and variance $\sigma^2{}_Y$. Show that $cov(\overline{Y}, Y_i) = \sigma^2{}_Y/n$

Question 2.

The density function for a normally distributed R.V. X, with mean μ and variance σ^2 is given by

$$f(y) = \frac{1}{\sqrt{2\pi}\ \ \sigma} e^{-(y-\mu)^2/2\sigma^2}$$

(a) Prove that any normally distributed variable, with mean μ and variance σ^2 can be expressed as standardized normal distribution with mean 0 and variance of 1.

Question 3.

Say that we are interested in the effect of X_{i1} on Y_i. We specify the following equation:

$$Y_i = \beta_0 + \beta_1 X_{i1} + u_i$$

(a) Set up the optimization problem that solves for the OLS-estimators and the first order conditions. In words, please describe the logic of the optimization problem.

(b) Solve for the optimal values

(c) Under which assumptions is the OLS-estimator of β_1 an unbiased and consistent estimator of β_1.

(d) Show that the OLS-estimator of β_1 is an unbiased estimator of β_1.

Question 4.

Social scientists are interested in the efficiency of imprisonment after the criminal served the time. We would like to know if a criminal commit fewer crimes in the future (defined Y_i below) after imprisonment compared to say a criminal that was identical and committed an identical crime but instead only was sentenced to probation.

Assume you observe an indicator $P = 1$ if individual i get prison and $P = 0$ if i get probation. Assume you observe the number of future crimes Y_i (the outcome) for individual i.

(a) If you would run an OLS-regression explaining Y_i with the regressor P_i: ($Y_i = \beta_0 + \beta_1 P_i + u_i$), discuss one potential omitted factor.

We observe a randomized (natural) experiment for a subgroup of the Swedish population, namely drunk drivers. If a drunk driver has a blood alcohol concentrations (BAC) equal to or above the threshold 1.5, he/she is often sentenced to prison while those just below are sentenced to probation. For simplicity, assume the threshold is strictly binding so if a drunk driver has 1.5 or above he/she is sentenced to prison and if below she/he is sentenced to probation. If we compare individuals close enough to the threshold, the treatment, P, will be "as good as" randomly assigned to the drunk drivers. We have in our sample 40 drunk drivers that had 1.501 in BAC and 40 individuals that had 1.491 in BAC and we are given the following:

$$\boldsymbol{P}'\boldsymbol{Y} = \binom{60}{20}$$

(b) Explicitly calculate the $\mathrm{P}'\mathrm{P}$ matrix in numbers

(c) Calculate the determinant of $\mathrm{P}'\mathrm{P}$. What does it mean that it differs from 0?

(d) Calculate the inverse of $\mathrm{P}'\mathrm{P}$

(e) Calculate the OLS-estimates and interpret them.

Question 5.

(a) Discuss the different type of correct standard errors used when using time series data.

(b) Describe the assumptions required for estimating a dynamic causal effect with exogenous regressors using OLS.

(c) Discuss the two major causes of non-stationarity and relevant tests in order to detect these hazards in a standard AR(1) forecasting model.