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Exam for EC7112, Game theory, 24 April 2017

1. (35 points) Consider two players (1 and 2) that participate in a first-price sealed-bid auction. The object on sale is valued at $v_1 = 3$ and $v_2 = 2$ by the two players. Let b_1 and b_2 be the bids that are submitted by players 1 and 2, respectively. Bids are made in discrete numbers, and the highest bid that either person can submit is 4, i.e. the set of bids is given by $B = \{0, 1, 2, 3, 4\}$ for both players. The person who has submitted the highest bid wins and pays his/her bid. If there is a tie, a coin is tossed such that there is a probability of $\frac{1}{2}$ of winning for each player. The expected utility of player i is $EU_i = v_i - b_i$ if $b_i > b_{-i}$, $EU_i = \frac{1}{2}(v_i - b_i)$ if $b_i = b_{-i}$, and $EU_i = 0$ if $b_i < b_{-i}$.

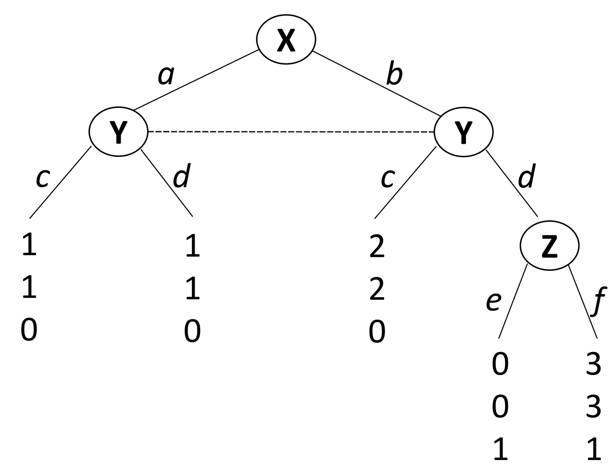
   a) Calculate the expected utilities of player 1 for all combinations of b_1 and b_2. State the best response function of player 1.

   b) Calculate the expected utilities of player 2 for all combinations of b_1 and b_2. State the best response function of player 2.

   c) Illustrate the best response functions in a figure, with $b_1 \in B$.on the horizontal axis and $b_2 \in B$.on the vertical axis.

   d) Use the figure in c) to identify all pure strategy Nash equilibria.

2. (30 points) Consider the following extensive form game between players **X**, **Y** and **Z**, where payoffs are presented in the following order: u_X, u_Y, u_Z.



   a) Define the strategy sets, the player function and the set of terminal histories of this game. Identify all subgames of the game.

   b) Identify the pure strategy subgame perfect Nash equilibria of this game.

   c) Determine the subgame perfect Nash equilibria outcomes. Is any of these outcomes Pareto optimal?

3. (35 points) Consider the following Prisoner's Dilemma game where two thieves choose between the strategies *Quiet* (Q) and *Fink* (F):

	Quiet	*Fink*
Quiet	$-1, -1$	$-3, 0$
Fink	$0, -3$	$-2, -2$

Now assume that finking while the other thief remains quiet is associated with a retribution such that the payoff in this case becomes $u_i(F, Q) = -R$. All other payoffs are unaltered; in particular, there is no retribution if both thieves fink.

a) Represent this modified Prisoner's Dilemma game in a payoff matrix.

b) Show that there exists a threshold value for R, below which both thieves finking is the unique Nash equilibrium of the game.

c) Consider the case when R is at least as large as the threshold level derived in b). Determine the best response functions for both thieves. Illustrate the best response functions in a figure (for some R-value of your choice). Determine all (pure and mixed strategy) Nash equilibria when R equals the threshold level and when R is strictly larger than the threshold level.

d) Provide an intuitive explanation for why retribution for finking is important among criminals. Describe how an increase in R affects the Nash equilibria. How does a higher R impact on the stability of the Nash equilibria?

e) Let R become infinitely large. Determine the Nash equilibria in this case.