Stockholm University

Department of Economics

Course name:	**Climate and the Economy**
Course code:	**EC7104**
Semester:	**Spring 2017**
Type of Exam:	**Main**
Examiner:	**Per Krusell**
Number of credits:	**7,5 credits (hp)**
Date of exam:	**Monday, May 29, 2017**
Examination time:	**3 hours (13:00-17:00)**

Write your identification number on each paper and cover sheet (the number stated in the upper right hand corner on your exam cover).

Use one cover sheet per question. Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked.

The exam consists of 9 questions. Each of the first 4 questions is worth 15 points; the remaining 5 questions are worth 8 points each. The total exam score is 100 points. Problem sets are given weight and factored into the final grade as described on the next sheet, for a total course score between 0 and 100 points.

For the grade E 45 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Your results will be made available on your "My Studies" account (www.mitt.su.se), on Monday 19 June 2017 at the latest.

Good luck!

EC7104 The Climate & the Economy
Spring 2017
May 2017

Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. **A climate model**

Consider the following simple climate model:

$$\begin{aligned} T_{t+1} - T_t &= \sigma_1 \left(F_t - (\kappa - x)\, T_t - \sigma_2 \left(T_t - T_t^L\right)\right) \\ T_{t+1}^L - T_t^L &= \sigma_3 \left(T_t - T_t^L\right) \end{aligned}$$

where T_t and T_t^L, respectively, denote the atmospheric and ocean temperatures in period t, both measured relative to a pre-industrial steady state. F_t is forcing in period t (change in energy balance relative to pre-industrial steady state) and κ, x, and the σs are positive parameters.

(a) We sometimes call the parts inside parenthesis on the RHS energy budgets.
$F_t - (\kappa - x)\, T_t - \sigma_2 \left(T_t - T_t^L\right)$ is the energy budget for the atmosphere representing energy flows to and from the atmosphere. F_t is forcing, representing changes to the energy budget due to the greenhouse effect. Provide a verbal explanation of the other two terms. Use one sentence per term.

(b) Use the two equations to provide an expression for the relation between steady-state temperature and a constant forcing. (Hint: if forcing is constant at F, what is the long-run value of the atmospheric temperature T?)

(c) Suppose $F_0 = 4$ and $T_0 = T_0^L = 0$ and that $\sigma_1 = 0.25$. Can we calculate T_1 and T_1^L? If so, what are their values?

(d) In our model, we used a much higher value for σ_1 than for σ_3, 0.23 vs. 0.02. That σ_1 is much larger than σ_3 implies that dynamics is much faster for the first equation than for the second. Thus, the first equation reaches a state relatively fast, where $F_t - (\kappa - x)\, T_t - \sigma_2 \left(T_t - T_t^L\right) = 0$. This fact has been used to find an empirical estimate of $\kappa - x$.[1] Let us try to follow this procedure.

Suppose we observe $F_t = 2W/m^2$, $T_t = 0.8$ and $\sigma_2 \left(T_t - T_t^L\right) = 0.8W/m^2$.
i) What is $\kappa - x$?
ii) If a doubling of the CO_2 concentration leads to a forcing $F_t = 3.7W/m^2$, what is the equilibrium climate sensitivity?

2. **Accounting for income differences**

In all of the subquestions below, consider the populations in each country to be constant over time.

[1] E.g., Lewis and Curry, (2014), *Climate Dynamics*.

(a) As a result of recent reports that the GDP per capita of the country of Buranda Rwundi is only one 50th of that in the United States, the World Bank initiates a fact collection mission: they send a crew of economists to the country to painstakingly go around and measure the entire capital stock so as to obtain an estimate for the market value of Buranda Rwundi's capital stock. The economists are also asked to count up all the people in the country. The results in hand, the economists then compute a value for the per-capita capital stock. They estimate this value to be one tenth of that in the United States.

The World Bank economists also assess the rate at which people save in Buranda Rwundi. They find that the saving rate is 30% and, actually, identical to that in the United States.

Based on Solow's growth model and on the assumption that the United States and Buranda Rwundi have been growing at the same rate for a long time (i.e., they are both on a balanced growth path where output and capital grow at a net rate g), can you figure out what could account for the gap in GDP between the countries? (Hint: what is the balanced-growth capital-output ratio, as a function of model parameters?)

(b) A third country, Lethoso, has just experienced a massive earthquake, destroying half of its capital. Fortunately, however, the population is unharmed. Lethoso used to grow in a balanced way, and early post-earthquake indications are that people still save at the same rate as before. Now the World Bank is asked to make a prediction for how its ratio of capital to output will develop over in Lethoso over the next ten years or so. Can you help—should this ratio be expected to rise, stay constant, or fall? And why? You are expected to use Solow's growth model again, and more specifically use a Cobb-Douglas production function with a capital share $\alpha < 1$.

(c) As a result of global warming, it turns out that Lethoso is hit hard by damages, unlike the United States where the damages are estimated to be nil. In terms of the Nordhaus-style damage function, the damages elasticity is significant in Lethoso: for each increase in the atmospheric carbon concentration by one Gton, the resulting damages are $\gamma = 0.02$ percent of Lethoso's GDP, which is around ten times the global average value. (Output can be described by $y_t = Ae^{-\gamma S_t} k_t{}^{\alpha} \left(l(1+g)^t\right)^{1-\alpha}$, where A is a nation-specific constant, S is the global carbon concentration measured in Gton of carbon, k is the nation's physical capital, l is its labor input, and $(1+g)^t$ describes global technical change.) Approximating the value of γ in the United States to be zero and supposing that global human emissions lead to a long-run (permanent) change in S of 1,000 Gton of carbon, what will be the long-run relative change—due to global warming—in the levels of TFP between the United States and Lethoso? What will be the long-run relative change in output across the countries (here, if you want to use a numerical value, use $\alpha = 1/3$)? Is this value large or small compared to the current ratio of outputs between the richest and the poorest countries in the world?

3. Natural-resource economics

Assume that an economy exists for two periods. This economy is inhabited by a representative consumer with preferences given by

$$U\left(C_0, C_1\right) = \log\left(C_0\right) + \beta \log\left(C_1\right).$$

The consumer owns a stock of oil, R_0, that is sold to a representative firm in the two periods. This implies the following constraint

$$E_1 = R_0 - E_0,$$

where E denotes oil use.

The consumer's budget constraints in period 0 and 1, respectively, are given by

$$C_0 + K_1 = r_0 K_0 + p_0 E_0$$

and

$$C_1 = r_1 K_1 + p_1 E_1.$$

In each period, the total income thus comes from capital (rK) and oil (pE).

A representative firm rents capital and buys oil from the consumer to produce a final good.

(a) Write down the consumer's maximization problem and derive the relevant first-order conditions.

(b) Write down the firm's profit-maximization problems and derive the relevant first-order conditions.

(c) Provide intuition for the Euler equation and the Hotelling equation.

(d) Show how K_1 depends on current income $K_0^{\alpha} E_0^{1-\alpha}$.

4. **A static IAM**

Consider the static IAM with coal considered in class, but where labor is not used in the sector for producing the consumption good: it is only used in the coal sector. I.e., the following equations describe the physical environment:

- Preferences: $\log c$.
- Production of consumption: $c = Dk^{1-\nu}e^{\nu}$.
- Production of energy (coal): $e = \chi n$.
- Constraint for the labor input: $n \leq 1$ (there is one unit of labor used only for coal; not all labor necessarily has to be used up).
- Carbon cycle: $S = \varphi e$.
- TFP net of damages: $D = Ae^{-\gamma S}$.

(a) Solve for a competitive equilibrium without taxes (you need to find all the quantities and all the prices).

(b) State the planning problem and solve it. Here pay attention to the condition (if any) under which all the labor will be used.

(c) What is the optimal carbon tax as a fraction of (optimal) output?

(d) Suppose one has solved the model for a given tax rate on coal, τ_1, with a resulting equilibrium carbon concentration S_1. Suppose one then increases the carbon tax to $\tau_2 > \tau_1$ and computes the new equilibrium carbon concentration to be $S_2 < S_1$. How much will global temperature fall as a result? You do not need to compute a value but explain what kind of an equation (or equations) you would use.

B. Short questions

1. Explain briefly the differences between the two general ways to estimate climate damages, namely the bottom-up and reduced-form approaches.

2. Carbon dioxide is a greenhouse gas. Give two examples of other greenhouse gases.

3. Consider a world with three countries: one oil-producing country and two oil-consuming countries that use oil to produce a final output. Explain what "carbon leakage" is in this world. Under what conditions can we expect a high degree of carbon leakage between the two oil-consuming countries? Under what conditions can we expect a low degree of carbon leakage?

4. Suppose discounting is geometric, i.e., that the discount factor for utils in year t is β^t. Nordhaus considers $\beta = 0.985$ and Stern $\beta = 0.999$. Explain how these numbers translate into "half-life generational statements" of the following kind: *I attach the weight 0.5 on utils x years from now (relative to the weight 1 I place on utils today).* I.e., find Nordhaus's x and Stern's x. Very briefly describe the arguments Nordhaus and Stern use to motivate their different positions on what x ought to be.

5. Suppose the optimal tax on carbon is USD500 per ton of carbon and that a "carbon capture and storage" (CCR) technology is available that allows us to remove carbon from the atmosphere and store it indefinitely elsewhere. Suppose that the CCR technique amounts to a cost of USDx per ton of carbon. Can we decide whether or not to use the CCR technique based on how x relates to 500? Explain.