Macro I, Final Exam
March 24, 2017

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Solow model and development accounting (12 points)

Consider two countries $i = B, D$ that are both characterized by the following Solow setups: The law of motion of the capital stock is in country i

$$K_{i,t+1} = K_{i,t}(1-\delta) + I_{i,t}. \tag{1}$$

The saving/investment behavior is

$$I_{i,t} = s_i F\left[K_{i,t}, A_{i,t} h_i L_t\right], \tag{2}$$

and the resource constraint is

$$F\left[K_{i,t}, A_{i,t} h_i L_t\right] = C_{i,t} + I_{i,t}. \tag{3}$$

$F\left[K_{i,t}, A_{i,t} h_i L_t\right]$ is a neoclassical production function that fulfills the standard assumptions. We have $A_{i,t} = A_{i,0}\gamma^t$ and $L_t = \eta^t$. $1 > s_i > 0$, $\gamma > 1$ and $\eta > 1$ are some constants. The variable $h_i > 0$ denotes the (fixed) level of human capital. The two countries may differ in the saving rate, s_i, the level of human capital, h_i, as well as in the (initial) technology level, $A_{i,0}$. The rate of technical change, γ, the depreciation rate, δ, population, L_t, and the production function, $F[\cdot]$, is identical across countries.

(a) **(6 points)** Let us focus on the balanced growth path along which $k_{i,t} \equiv \frac{K_{i,t}}{A_{i,t}L_t}$ is constant. Express the capital-output ratio, $\frac{K_{i,t}}{F[K_{i,t}, A_{i,t}h_i L_t]}$, in country i in this stationary point in terms of exogenous model parameters. How does the long-run capital-output ratio depend on s_i and h_i? Give an intuition.

In the following we again assume that the two countries $i = B, D$ are along their long-run balanced growth path. Country B has an output per-capita, $\frac{F[K_{i,t}, A_{i,t}h_i L_t]}{L_t}$, of 32,000 international \$, a capital-labor ratio, $\frac{K_{i,t}}{L_t}$, of 128,000 international \$, and a level of human capital, h_i, that is normalized to 1. In contrast country D is much poorer with an output per-capita of 1,000 international \$, a capital-labor ratio of only 1,000 international \$, and a level of human capital of 1/2.

(b) **(2 points)** Suppose the saving rate in country B, s_B, is 0.20. How large is then the saving rate in country D? (Note that both countries are on their balanced growth path and (potential) only differ in the following fundamental parameters values: s_i, h_i, and $A_{i,0}$.)

(c) **(4 points)** The production function takes the following form:

$$Y_{i,t} = F\left[K_{i,t}, A_{i,t}h_i L_t\right] = K_{i,t}^{\alpha}\left[A_{i,t}h_i L_t\right]^{1-\alpha}. \tag{4}$$

Furthermore we have $\alpha = 1/3$. Do a "development accounting exercise" that is consistent with the framework above and a long-run perspective, i.e., countries being on their balanced growth path. Output per capita differs between the two countries by a factor of 32. By what factor does the level of technology differ between the two countries? What fraction of the income difference is attributed to technological differences?

[Hint: You might find it useful to write the production function as

$$\frac{Y_{i,t}}{L_t} = A_{i,t}h_i\left(\frac{K_{i,t}}{Y_{i,t}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{5}$$

where $Y_{i,t}$ is total output.]

Problem 2: Feasibility of balanced growth (14 points)

Consider the Solow model with labor-augmenting technical change, changing hours worked, investment specific technical change, and population growth. The production function is Cobb-Douglas. Then, the resource constraint can be written as

$$K_{t+1} = Q_t s K_t^\alpha \left(A_t \ell_t L_t\right)^{1-\alpha} + (1-\delta) K_t, \tag{6}$$

where K denotes capital, $s > 0$ the saving rate, and $0 < \delta < 1$ the depreciation rate. Q_t is the investment specific technology term that grows at gross rate $q > 1$, i.e., $Q_t = q^t$. A_t is the labor augmenting technology term that grows at gross rate γ. The number of worker (=population), L_t, grows at gross rate $n > 1$. Finally, hours worked per worker, ℓ_t, changes at exogenous gross rate $\phi < 1$, i.e., $\ell_t = \phi^t$. We assume $L_0 = A_0 = Q_0 = \ell_0 = 1$. Aggregate consumption is given by

$$C_t = (1-s) K_t^\alpha \left(A_t \ell_t L_t\right)^{1-\alpha}. \tag{7}$$

(a) **(6 points)** Show that there is a balanced growth path by defining a detrended capital stock k (capital detrended by an appropriate growth rate) and show that there is a stationary point in k. Calculate the stationary point $k^\star$.

(b) **(2 points)** Show that the point $k^\star$ is globally stable by drawing a corresponding diagram.

(c) **(6 points)** Along the balanced growth path: At what rate does aggregate capital grow? At what rate does aggregate consumption grow? Assume that the wage rate is equal to the marginal product of an hour worked of one worker in terms of consumption. At what rate does then the real wage rate grow along the balanced growth path (i.e., the wage per hour expressed in consumption goods)? Are the Kaldor facts satisfied?

Problem 3: Lucas model (14 points)

Consider the following simplified version of a Lucas growth model: Preferences are given by

$$\mathcal{U}(0) = \sum_{t=0}^{\infty} \beta^t \log(c_t), \tag{8}$$

where $0 < \beta < 1$. There is no population growth. Output, Y_t is produced according to the following technology

$$Y_t = AK_t^{\alpha} \left[N_t H_t\right]^{1-\alpha}, \tag{9}$$

where $A > 0$ and $0 < \alpha < 1$ are some constants. K and H denotes physical and human capital which both can be accumulated. N_t is the share of labor used in production as opposed to human capital accumulation. The resource constraint is standard and given by:

$$Y_t = c_t + I_t, \tag{10}$$

where I denotes investment. The depreciation rate on physical capital is 100% such that the law of motion of K becomes

$$K_{t+1} = I_t. \tag{11}$$

Finally, a share $1 - N_t$ of labor is used in human capital accumulation and the law of motion of H is given by

$$H_{t+1} = B\left[1 - N_t\right] H_t, \tag{12}$$

where $B > 0$ is a constant.

(a) **(8 points)** Let us focus on the social planner's problem in this framework. For a given level of K_0 and H_0 the social planner maximizes welfare subject to the resource constraint and law of motion of human and physical capital with respect to $\{c_t, N_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}$. State the planner's problem and solve for the first-order conditions.

(b) **(6 points)** Assume that the planner's solution is indeed given by an interior solution characterized by the first-order conditions derived in (a): Guess and verify that the solution to the planner's problem takes the following simple functional form: $c_t = \mu_1 Y_t$ and $N_t = \mu_2$, where μ_1 and μ_2 are some constants. Determine μ_1 and μ_2.

Problem 4: Return on capital (10 points)

How does the average return on capital change in the long run in the data and what is roughly its level? How would you measure the long-run average return to capital? Suppose the economy is characterized by a neoclassical growth model and suppose the population growth rate will fall in future persistently to a lower level (whereas the rate of technical change, technology and preferences remain unchanged). Would you predict that this decrease in the population growth rate increases or decreases the return on capital in the long-run (or is is unaffected)? Why?

Problem 5: Overlapping generations and (in)efficiency (50 points)

Consider an economy that consists of overlapping generations of two-period-lived agents. At each date $t \geq 1$ there is born a constant number N of young people with preferences given by

$$\log c_t^t + \log c_{t+1}^t ,$$

where c_s^t is time s consumption of an agent born in period t. For all dates $t \geq 1$, young people are endowed with one unit of labor when they are young and zero units when they are old. The economy has a constant-returns-to-scale production technology, and output in period t is equal to $K_t^\alpha L_t^{1-\alpha}$, where $\alpha \in (0,1)$, and K_t and L_t are the aggregate stock of capital in period t and the number of young people working in period t, respectively. Capital depreciates fully after production. The economy's resource constraint in period t becomes

$$N c_t^t + N c_t^{t-1} + K_{t+1} = K_t^\alpha L_t^{1-\alpha}. \tag{13}$$

The inital old generation is endowed with a capital stock K_1, and their utility is strictly increasing in c_1^0.

In each period, there are markets in labor and capital services. In period t, young agents supply their labor at the market wage rate w_t, and old agents sell capital services at the market rental rate r_t.

[In your analysis, please feel free to normalize $N = 1$, i.e., you have a representative agent in each generation and all quantities are expressed in per capita terms.]

a. [5 points] Define a competitive equilibrium.

b. [5 points] Derive the optimal decision rules of an agent born in period t. That is, given a wage rate w_t and a gross return r_{t+1} on savings between periods t and $t+1$, compute the agent's optimal labor supply, savings and consumption.

c. [2 points] Find equilibrium expressions for the wage rate w_t and the rental rate r_t in terms of the aggregate capital stock in period t.

d. [3 points] Impose market clearing and derive an equilibrium law of motion for the capital stock.

e. [5 points] Compute the steady state of the economy, i.e., find expressions for the allocation and prices in terms of primitives.

f. [10 points] Discuss how the steady state can fail to be Pareto optimal and if that is so, describe what sort of feasible changes to the allocation could constitute Pareto improvements.

Based on your discussion and on your derivation of the steady state in question e, try to come up with a restriction on parameter values under which the steady state fails to be Pareto optimal. Interpret any such parameter restriction in terms of the economic forces at work under such parameter values.

We now introduce a government that levies a tax rate τ_t^n on labor income in period t and a tax rate τ_t^k on capital income in period t. (If a tax rate is negative, it is a subsidy.) In each period, the government runs a balanced budget, i.e.,

$$\tau_t^n w_t L_t + \tau_t^k r_t K_t = 0. \tag{14}$$

g. [5 points] Suppose that a given tax policy $\{\tau_t^n, \tau_t^k\}_{t=1}^{\infty}$ is government budget feasible. Tell how to compute a competitive equilibrium.

h. [5 points] Find an equilibrium expression for the ratio τ_t^n/τ_t^k in terms of primitives. That is, find an expression for τ_t^n/τ_t^k in terms of preference, technology and endowment parameters (rather than in terms of endogenous prices and quantities). Provide an economic interpretation of that expression.

i. [10 points] Go as far as you can to explore the existence of Pareto-improving tax policies. Is it possible that tax/subsidy policies can improve upon all generations' welfare?

[Hint: You may first want to characterize how future generations' welfare is affected in a steady state by time-invariant tax rates (τ^n, τ^k). Next, you can ask if earlier generations would agree to that kind of tax/subsidy policies.]