Macro I, retake exam
August 22, 2017

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Capital-output ratio in the Solow model (12 points)

Consider the following standard Solow setup: The law of motion of the capital stock is given by

$$K(t+1) = K(t)(1-\delta) + I(t). \tag{1}$$

The saving behavior is

$$I(t) = sF\left[K(t), A(t)L(t)\right], \tag{2}$$

and the resource constraint reads

$$F\left[K(t), A(t)L(t)\right] = C(t) + I(t). \tag{3}$$

The neoclassical production function, $F\left[K(t), A(t)L(t)\right]$, takes the following functional form:

$$F\left[K(t), A(t)L(t)\right] = \left[\omega K(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)\left(A(t)L(t)\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

with $\epsilon \geq 0$ and $\omega \in (0,1)$. We have $A(t) = \gamma^t$, $L(t) = n^t$ with $\gamma > 1$, $n > 1$. Let us denote the capital per efficiency units as $k(t) \equiv \frac{K(t)}{A(t)L(t)}$.

(a) **(5 points)** Derive the golden rule capital stock per efficiency units, $k^\star_{gold}$, for this economy.

(b) **(7 points)** Assume that the production factors are rewarded according to their marginal products (due to perfect competition on the factor markets). Given the production function (4): Does the labor income share $\alpha_L(t) \equiv \frac{w(t)L(t)}{F[K(t),A(t)L(t)]}$ increase or decrease in $k(t)$? (Here, we normalized the price of the output good to one in the definition of $\alpha_L(t)$.) Which parameter does your answer depend on? Give an intuition.

Problem 2: Neoclassical growth (28 points)

Consider the following neoclassical set-up: the is a representative household with the following preferences over per-capita consumption and per-capita hours worked

$$\mathcal{U}_0 = \sum_{t=0}^{\infty} (n\beta)^t \left[\log(c_t) + \psi \log(1 - h_t)\right], \tag{5}$$

We have $\psi > 0$, and $1 > n\beta > 0$. There is exogenous population growth at gross rate $n > 0$, i.e., $L_t = n^t$. The resource constraint can be written as

$$K_{t+1} = K_t^{\alpha} \left(A_t h_t L_t\right)^{1-\alpha} + (1 - \delta)K_t - L_t c_t, \tag{6}$$

where K is aggregate capital and we have $\alpha \in (0, 1)$, and $\delta \in (0, 1]$. The Harrod-neutral technical change takes place at gross rate $\gamma \geq 1$, i.e., $A_t = \gamma^t$. In the following we are going to analyze the planner's solution of this economy. The time endowment per capita is normalized to one, i.e., $0 \leq h \leq 1$ (but in the following we will consider an interior equilibrium in which this constraint will not bind).

(a) **(6 points)** State the planner's problem and solve for the first-order conditions.

Now let us assume $\delta = 1$.

(b) **(4 points)** Is there a balanced growth path? If yes at what gross rate g_k, will the capital stock grow in the long run? At what rate will per-capita consumption, c, grow in the long run? Define a detrended capital stock as $k_t \equiv \frac{K_t}{g_k^t n^t}$ and and a detrended per-capita consumption level, $\tilde{c}_t \equiv \frac{c_t}{g_c^t}$ calculate its steady state value in terms of exogenous model parameters. Also solve for h_t along the balanced growth path.

(c) **(6 points)** Show that there is a closed form solution for the *transitional dynamics*. Solve for consumption and hours worked as a function of the initial capital stock. (Hint: This is usually done by guessing and verifying.)

(d) **(6 points)** Calculate the speed of convergence $d\log(y_{t+1}/y_t)/d\log y_t$ in this economy (where y_t denotes detrended output). Give an intuition how this speed of convergence depends on the model parameters.

(e) **(6 points)** How would you calibrate the model parameters α, γ, ψ, β, and n. Describe what data you would use for each parameter as a target. What would this calibration imply for the speed of convergence? How does this implied speed of convergence compare to the "conditional rate of convergence" found in typical growth regressions?

Problem 3: Kaldor facts of growth (10 points)

What are the so-called Kaldor facts? Are the facts a good approximation of post-war data of developed countries? In which sense can the neoclassical growth model be viewed as a theory constructed around those facts? What are the required functional forms for technologies and preferences to match the Kaldor facts in a standard one sector neoclassical growth model?

[I don't expect you to write more than 1/2-3/4 page.]

Problem 4: Government finance in an OLG model (50 points)

Consider an endowment economy that is populated by overlapping generations of two-period-lived agents. At each date $t \geq 1$ there is born a constant number N of young people with preferences given by

$$\log(c_y^t) + \log(c_o^t),$$

where c_y^t and c_o^t are consumption in young and old age, respectively, of an agent born in period t. Every agent is endowed with $w_y > 0$ goods as young and $w_o \geq 0$ goods as old. The initial old generation at date $t = 1$ is of the same size N and its members are also endowed with w_o goods, and their utility is strictly increasing in c_o^0. For simplicity, you can proceed with $N = 1$, i.e., a representative agent in each cohort.

In each period $t \geq 1$, agents who are alive can trade in one-period bonds. Let R_t be the gross interest rate between periods t and $t + 1$.

Starting in the first period $t = 1$, the government pays a constant benefit $\epsilon \geq 0$ to every old agent for the indefinite future, which is financed by levying lump-sum taxes and by issuing one-period bonds. Specifically, each agent of the generation born in period $t \geq 1$ pays taxes $\tau_y^t \geq 0$ and $\tau_o^t \geq 0$ in young and old age, respectively (implying that the initial old generation receives benefits but pays no taxes.) Let B_t be one-period government bonds issued in period t. Initially, there is no government indebtedness, $B_0 = 0$.

(a) [**4 points**] Define a competitive equilibrium for this economy.

(b) [**8 points**] Suppose that the government runs a pay-as-you-go system, i.e., the benefits to the old in period $t \geq 1$ are financed with a tax on the current young, $N\epsilon = N\tau_y^t$. For any permissible value of ϵ, compute the competitive equilibrium, including an expression for R_t.

(c) [**5 points**] Derive the Pareto-optimal policy in question b, for all possible values of $w_y > 0$ and $w_o \geq 0$.

(d) [**8 points**] Suppose that the government finances the benefits in period $t \geq 1$ by issuing bonds, $N\epsilon = B_t$, and pays off those bonds next period $t + 1$ by taxing the old in that period, $R_t B_t = N\tau_o^t$. Compute the competitive equilibrium, and provide an economic interpretation of how it compares to outcomes in question b.

(e) [**10 points**] Revisit the bond-finance scheme in question d, $N\epsilon = B_t$, but suppose that the government pays off the bonds next period $t+1$ by taxing the young in that period, $R_t B_t = N\tau_y^{t+1}$. Derive formulas for recursively computing the competitive equilibrium from primitives.

(f) [**5 points**] For any permissible value of ϵ, examine how the interest rate R_1 and the welfare of an agent born at time 1, differ across the two bond-financed schemes in questions d and e. Provide economic interpretations.

(g) **[5 points]** For any permissible value of ϵ in question e, derive the steady state to which the equilibrium converges. Then, use your formula to compute the steady state for the parameterization $\epsilon = w_y/4$ and $w_o = 0$.

(h) **[5 points]** Based on your above analyses, go as far as you can to describe and interpret how the Pareto-optimal policies differ across the two bond-financed schemes in questions d and e. Into which economy would you like to be born?

Hint: A quadratic equation in the unknown x,

$$x^2 + \beta x + \alpha = 0\,,$$

has the solutions

$$x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha}}{2}\,.$$