Department of Economics, Stockholm University
Mathias Herzing
Exam for EC 2110, Uncertainty and asymmetric information, 18 August 2017

1. Short questions:

a) What is a "complete asset market"? Which conditions need to be satisfied for a "complete asset market"?

b) Consider a market where there are two types of workers. Type 0 has marginal product $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = \frac{3}{2}$. Type 1 has marginal product $\Theta_1 = 4$ and an outside opportunity wage of $w_0(\Theta_1) = 3$. The cost of education z is given by $C(z, \Theta_0) = \frac{z}{\Theta_0} = z$ for type 0 and $C(z, \Theta_1) = \frac{z}{\Theta_1} = \frac{z}{4}$ for type 1. A worker's utility function is given by $U(w, z, \Theta) = w - C(z, \Theta)$. Workers know their own type but employers cannot tell the high from the low productivity workers. Illustrate in a figure, with wage on the y-axis and the amount of education on the x-axis, which contracts will make it possible to separate type 1 from type 0 workers. Assume that, if two contracts yield the same level of utility, a type 0 worker prefers the one which requires less education. Use the figure to identify the contract that will be offered to type 1 workers by a monopsonist employer. Identify the contract that will be offered if there is perfect competition among employers. (Note that you do not have to derive the contracts mathematically; just show where these two contracts are located in your figure.)

c) Consider a farmer (the agent) who needs to borrow money to grow crops. The farmer turns to a bank (the principal) which provides a loan of size k (at cost k). The repayment of the farmer is given by t. The bank's profit is thus given by $V = t - k$. The value of the farmer's output P is determined by the size of the loan and his/her type: $P(k, \Theta) = 2\Theta k^{\frac{1}{2}}$, where $\Theta = \underline{\Theta} = 1$ if the farmer is inefficient and $\Theta = \overline{\Theta} = 4$ if the farmer is efficient. (Note: a higher Θ implies higher efficiency!) The farmer's profit is given by $U_\Theta = P(k, \Theta) - t$. The bank has no information regarding the efficiency of the farmer, but knows that the share of efficient farmers is ν. State the bank's optimization problem and the two relevant constraints that are binding given that it has all bargaining power. Use the two binding constraints to simplify the bank's optimization problem (i.e. express the bank's expected payoff in terms of $\underline{k}$ and $\overline{k}$ only).

d) Explain briefly how deductibles can be used to deal with different types of asymmetric information in the insurance market.

2. Kim has the following elementary utility function: $v(c) = 3c^{\frac{1}{3}}$, where c is consumption. Assume that there are only two possible states of the world, 1 and 2, where the probability of state 1 being realized is π. Consumption in state 1 is denoted by c_1, and consumption in state 2 is denoted by c_2.

   a) State Kim' von Neumann-Morgenstern utlility function.

   b) Derive a mathematical expression for Kim's marginal rate of substitution (MRS) between consumption in the two possible states of the world.

   Kim's endowment of state claims is given by $\overline{c}_1^K = 5$ and $\overline{c}_2^K = 2$, and $\pi = \frac{1}{3}$. It is possible to trade in state claims at prices $p_1 = 1$ and $p_2 = 2$.

   c) Which two conditions need to be satisfied to solve Kim's optimization problem?

   d) Use these two conditions to determine Kim's optimal amounts of state claim 1 and state claim 2.

   e) Are market prices actuarily fair? Use your answer in d) to provide an intuitive explanation.

   Now assume that trading takes place in an economy which only consists of Kim and another person called John, i.e. the market equilibrium is determined by the interaction between Kim and John. John has the utility function $v(c) = 4c^{\frac{1}{4}}$, and John's endowment is given by $\overline{c}_1^J = 2$ and $\overline{c}_2^J = 4$. Both attribute the same probability $\pi = \frac{1}{3}$ to state 1 occurring.

   f) Let (c_1^{K*}, c_2^{K*}) and (c_1^{J*}, c_2^{J*}) denote Kim's and John's market equilibrium amounts of state claims. State all six conditions that need to be satisfied to obtain the market equilibrium. (Note: you are not supposed to calculate the equilibrium outcome - just state the conditions that have to be satsified.)

   g) Explain in words why the equilibrium price ratio cannot be actuarily fair in this market consisting of only Kim and John. What other reasons are there for non-actuarial prices?

3. The owner of a production facility hires a worker. The value of output is random, either high ($\overline{S} = 60$) or low ($\underline{S} = 4$). However, the probability of the output value being high π_e also depends on the effort $e \in \{0, 1\}$ that the worker exerts, such that $\pi_0 = \frac{1}{4}$ and $\pi_1 = \frac{1}{2}$. The cost that the worker incurs from exerting effort $e = 0$ is $\Psi(0) = 0$, and the cost of exerting effort $e = 1$ is $\Psi(1) = \ln 3$. The employer (who is the only one hiring workers) offers a contract $(\underline{t}, \overline{t})$ that specifies the transfers to be paid to the worker, depending on the output value. The employer's expected profit is given by $EV_e = \pi_e(\overline{S} - \overline{t}) + (1-\pi_e)(\underline{S} - \underline{t})$, and the worker's expected utility is given by $EU_e = \pi_e \ln(1+\overline{t}) + (1-\pi_e)\ln(1+\underline{t}) - \Psi(e)$.

   a) State the participation constraint of the worker for contracts that specify effort $e = 1$.

   b) State the employer's optimization problem under complete information.

   c) Given that the employer has all bargaining power, what high-effort-inducing contract(s) will be offered under complete information? (Recall the logarithm rules: $a \ln b = \ln b^a$, $\ln a + \ln b = \ln ab$ and $\ln a - \ln b = \ln \frac{a}{b}$.)

   Assume now that information regarding the worker's actions is hidden to the employer.

   d) Which constraints need to be satisfied?

   e) What high-effort-inducing contract(s) will be offered?

   f) Calculate the agency cost.

   g) Provide an intuitive explanation for why hidden action and the worker's attitude to risk lead to a decrease in the employer's profit. (No formulas, just words!)