EC7104 The Climate & the Economy
Spring 2017

Exam month: August 2017

Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly. You may use a calculator. If you don't have one, provide unsolved numerical expressions if you need to.

A. Analytical questions

1. **A Carbon Circulation Model (15)**
   Consider the following simple example of a carbon circulation model with two sinks (reservoirs), S (atmosphere) and S^L (ocean):

$$\begin{aligned} S_t - S_{t-1} &= -\phi_1 S_{t-1} + \phi_2 S^L_{t-1} + M_{t-1}, \\ S^L_t - S^L_{t-1} &= \phi_1 S_{t-1} - \phi_2 S^L_{t-1}. \end{aligned}$$

   where ϕ_1 and ϕ_2 are parameters and M_{t-1} represents emissions in period $t-1$.

   (a) (10) Suppose that at time 0 (before any human emissions have begun) the system is in a pre-industrial steady state, i.e., $S_0 = S_{-1} = 500\ GtC$, $S^L_0 = S^L_{-1} = 50000$, and $M_{-1} = 0$.

      i. Suppose that the flow of carbon from the atmosphere to the ocean in the pre-industrial steady state is 100 GtC per period. Use this to calibrate ϕ_1.

      ii. How large is the flow of carbon from the ocean to the atmosphere in the pre-industrial steady state? Use this to calibrate ϕ_2.

      iii. Suppose that during a period we can call the fossil episode, 1010 GtC is emitted into the atmosphere. After this period, emissions are zero. After a (long) transition period, the system will reach a new post-industrial steady state, where S and S^L are both constant. How large will S and S^L be in the post-industrial steady state according to the model? (Hint: 1010 is 2% of 500+50000, the total pre-industrial amount of carbon in the system.)

      iv. During the transition period to the post-industrial steady state, will S be larger or smaller than its value in the post-industrial steady state? Will S^L be larger or smaller than in the post-industrial steady state?

   (b) (5) Suppose that there is a non-linearity in the system such that the more carbon is absorbed by the ocean, the lower is its ability to absorb additional carbon.

      i. Describe how we could change the model to incorporate this feature by allowing parameters to depend on state variables.

      ii. How (in qualitative terms; no numbers required) would the non-linearity change the answer to question (a)iii?

2. **Carbon Leakage (15)**
Assume that there are two identical countries that both produce a final good (the same good in the two countries). In countries 1 and 2, the production function of a representative firm is given by

$$Y_1 = AE_1^{\upsilon},$$
$$Y_2 = AE_2^{\upsilon},$$

respectively, where A is a measure of the technological level and E is energy use. Assume first that the cost functions, in terms of the final good, in countries 1 and 2 are given by $C(E_1) = \mu E_1$ and $C(E_2) = \mu E_2$, respectively.

   (a) (3) Solve for energy use in both countries.

   (b) (4) Assume now that country 1 decides to reduce its energy use by 50% in order to reduce global warming. Country 2, on the other hand, does not care about global warming. What will happen to energy use in country 2? Will there be carbon leakage? Defend your answer with formal calculations.

   (c) (4) Assume now instead that the cost functions in countries 1 and 2 are both given by $C(E_1 + E_2) = (E_1 + E_2)^{\varphi}$. Derive the first-order conditions with respect to energy in both countries.

   (d) (4) Country 1 now again decides to reduce its energy use by 50% in order to reduce global warming (and country 2 still does not care about global warming). Set $\upsilon = 1$ and analyze formally whether there will be carbon leakage in this case.

3. **A Static IAM (15)**
Consider a static IAM with oil, where oil is costless to produce but only available in the amount R. The following equations describe the physical environment:

   - Preferences: $\log c$.
   - Production of consumption: $c = De$.
   - Carbon cycle: $S = \varphi e$.
   - TFP net of damages: $D = Ae^{-\gamma S}$.

   Notice that the model is a bit simpler than that in class: neither capital nor labor is used in production, and hence they can be abstracted from.

   (a) (3) Solve for a competitive equilibrium without taxes.

   (b) (5) State the planning problem and solve it. Here pay attention to the condition under which all the oil will be used: state "If ... then all of the oil will be used; otherwise, the amount of oil use will be ...", with the both of the "..."s clearly spelled out.

   (c) (3) What is the optimal carbon tax as a fraction of (optimal) output in this framework?

   (d) (4) Suppose there is now a second damage from climate change: one that affects utility directly (we can think of it as loss of life quality due to global warming). In particular, utility is now $\log c - \hat{\gamma} S$, i.e., a term proportional to the atmospheric carbon concentration is subtracted from the utility of consumption. This effect, like the effect of S on D, is a pure externality.

      i. How is the competitive equilibrium affected by the presence of the direct utility damage?

      ii. How is the optimal carbon tax affected?

4. **A Growth Model (15)**
Consider the following simple growth model: output is given by

$$y_t = Dk_t,$$

where k is capital and D is a TFP parameter, possibly influenced by the climate. Capital is accumulated as follows:

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where δ is the depreciation rate and i_t is investment. Investment, finally, is given by

$$i_t = sy_t,$$

where s is the rate of saving.

(a) (3) Derive an equation for how capital accumulation occurs in this economy, i.e., derive k_{t+1} as a function of k_t.

(b) (3) What is the growth rate of GDP in this economy from t to $t+1$? How is it affected by an increase in the saving rate?

(c) (3) Does this economy exhibit *convergence*? (You need to also define the concept.) If it does, explain the key reason for it; if not, explain why not.

(d) (3) Suppose each unit of output produced requires χ units of carbon, e. Moreover, carbon—via the carbon cycle—is costless to produce and use (similarly to conventional oil) and we can assume that it is abundant so that we can abstract from its finiteness in what follows. Carbon use, however, affects TFP (D) negatively, as in our typical IAMs ($D_t = e^{-\gamma S_t}$): the higher is the total carbon concentration in the atmosphere, the lower is D, via global warming (you may assume the linear carbon depreciation structure described in class, where $1 - d_j$ units remain j periods after emission. Your task is now to describe, in *qualitative* terms only, the growth process—in particular the rate of output growth—over time in this framework.

(e) (3) Could the information given be used to compute an optimal carbon tax (as a fraction of GDP)? If so, what would the computation be?

B. Short questions

1. Explain briefly the differences between the two general ways to estimate climate damages, namely, the bottom-up and reduced-form approaches.

2. Consider the simplest model of the energy budget of the earth. The system is initially in balance: the energy inflow equals the outflow. Suppose the outflow suddenly decreases while the inflow remains constant. An energy budget surplus thus arises. Explain in simple words what then takes place over time. Disregard any feedbacks (except the Planck feedback). Be sure to mention the effects over time on the temperature, the energy outflow, and the energy budget.

3. When solving a utility maximization subject to a budget constraint, a typical first-order condition appears which sets the marginal rate of substitution between two goods equal to their relative price. In the context of the decision of how much to save, what are the two goods and how can we identify the relevant relative price in the data? Conversely, suppose we have a value for this relative price, how can it be used to estimate any parameters in preferences relevant to the climate context? Only a qualitative answer is needed.

4. The distribution of GDP across countries evolves over time. Describe this evolution along the following dimensions: whether or not it is spreading out over time, whether or not its mean is increasing over time, and whether or not countries move significantly within the distribution. When answering these questions be clear on whether you are referring to the levels or logarithms of GDP.

5. Suppose the utility function of a dynasty is given as follows: (i) each cohort k has a utility function over its own consumption that is given by $u_k = u(c_{1k}, c_{2k}, c_{3k}, \ldots) = \log c_{1k} + \beta \log c_{2k} + \beta^2 \log c_{3k} + \ldots$, where c_{ik} is cohort ks consumption in its ith period of life and (ii) the overall utility of a cohort k, taking into account altruism toward other cohorts, is given by $U_k = u_k + \hat{\beta} u_{k+1} + \hat{\beta}^2 u_{k+2} + \ldots$. Assuming that both β and $\hat{\beta}$ are strictly between 0 and 1, describe which of these parameters—one of these mainly or both roughly equally—are key for climate policy. Moreover, would a larger value for this parameter/these parameters increase or decrease the optimal carbon tax?