Macro I, Spring 2018, Final Exam
March 22, 2018

Directions

The exam yields a total of 100 points. Provide **brief and concise** answers. Keep auxiliary computations **separate** from your main results. Write **legibly** (**pen** $\succ$ **pencil**). Thank you and good luck.

Problem 1: Solow model and the long-run capital-output ratio (18 points)

We have the following Solow setup: The capital stock, K_t, accumulates according to

$$K_{t+1} = K_t(1-\delta) + I_t, \tag{1}$$

where $0 < \delta < 1$ is the depreciation rate and I_t denotes investment. This equation can be rewritten as

$$K_{t+1} = K_t + \tilde{I}_t, \tag{2}$$

where $\tilde{I}_t \equiv I_t - \delta K_t$ denotes investment net of depreciation. We assume that the investment net of depreciation is given by a constant fraction $\tilde{s}$ of output net of depreciation or formally

$$\tilde{I}_t = \tilde{s}\left[K_t^{\alpha}\left(\eta^t\gamma^t\right)^{1-\alpha} - \delta K_t\right]. \tag{3}$$

We have $\gamma > 1$ and $\eta > 1$. In the following we define capital per-efficiency units as $k_t \equiv \frac{K_t}{\eta^t\gamma^t}$.

(a) **(5 points)** Derive a first-order difference equation in k_t. Is there a steady state $k^\star$? If yes, calculate the steady state value of k.

(b) **(3 points)** Calculate the steady state ratio between capital and income net of depreciation, i.e., $\tilde{\beta}_t \equiv \frac{K_t}{K_t^{\alpha}(\eta^t\gamma^t)^{1-\alpha} - \delta K_t}$.

(c) **(5 points)** Calculate the approximate speed of convergence $z \equiv \frac{\partial \log[k_{t+1}/k_t]}{\partial \log[k_t]}$ around the steady state $k^\star$. To establish non-oscilating (local) stability, what value range do you expect z to take? Show that z indeed is within this range.

(d) **(5 points)** Calculate the golden rule capital stock in this economy. What saving rate $\tilde{s}$ makes the steady state capital stock correspond to the golden rule capital stock?

Problem 2: Choosing to grow? (22 points)

Consider an economy with the following resource constraint

$$K_{t+1} = AK_t - c_t. \tag{4}$$

There is no population growth and there is a representative household with the following preferences

$$\mathcal{U}_0 = \sum_{t=0}^{\infty} \beta^t \left[\phi\, c_t - \frac{\kappa}{2}\,(c_t)^2\right], \tag{5}$$

with $\phi > 0$ and $\kappa > 0$. K_0 is exogenously given. In the following we assume that this K_0 is "small enough" such that the marginal utility of consumption remains positive, i.e., such that the implied consumption fulfills $\phi > \kappa c_t$. We have $0 < \beta < 1$ and $\beta A > 1$.

(a) **(4 points)** State the planner's problem and solve it to arrive at the consumption Euler equation.

(b) **(5 points)** Guess and verify that there is a closed form solution for c_t as a function of K_t. (A promising guess is $c_t = z_1 K_t + z_2$ where z_1 and z_2 are some constants to be solved for.)

(c) **(5 points)** Illustrate the dynamics in K in a figure. Do the dynamics lead to a sustained growth in K? Or does K converge to an asymptotic levle? Either way, calculate the asymptotic capital growth rate or the asymptotic capital level. Analyze whether this asymptotic point is locally/globally stable.

(d) **(2 points)** Do the preferences in (5) allow for Gorman aggregation, i.e., do they allow for a representative agent? Why?

(e) **(2 points)** Does the production side of this economy, (4), make sustained growth feasible, i.e., is it possible to chose a path of steady output growth? (Hint: this question is just about feasibility not about whether such a path is actually chosen.)

(f) **(4 points)** Suppose that preferences are instead of (5) given by

$$\mathcal{U}_0 = \sum_{t=0}^{\infty} \beta^t \log(c_t). \tag{6}$$

Solve for the planner's solution in this case and show that there is a closed form solution too. Characterize the equilibrium path. How do the dynamics differ between the two preference specifications? What is the most salient difference? What is the intuition?

Problem 3: Return on capital (10 points)

How does the average return on capital change in the long run in the data and what is roughly its level? How would you measure the long-run average return to capital? What are different approaches? Suppose the economy is characterized by a *neoclassical growth model* and suppose the population growth rate will fall in future persistently to a lower level (whereas the rate of technical change, technology and preferences remain unchanged). Would you predict that this decrease in the population growth rate increases or decreases the return on capital in the long-run (or is it unaffected)? Why? How would the interest rate in a *Solow setup* respond in the long run to a drop in the population growth rate?

Exercise 4: A Competitive Equilibrium (20 points)

A pure endowment economy consists of two consumers $i = \{1, 2\}$. Consumers order consumption streams according to,

$$U_i = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(C_t^i\right)^{1-\gamma},$$

where $C_t^i \geq 0$ and $\beta \in (0,1)$ is a common discount factor. The consumption good is tradable but non-storable. The consumer of type 1 is endowed with the consumption sequence

$$y_t^1 = \mu > 0 \quad \forall t > 0.$$

The consumer of type 2 is, by contrast, endowed with the consumption sequence

$$y_t^2 = \left\{ \begin{array}{cc} 0 & \text{if t is even} \\ \alpha & \text{if t is odd} \end{array} \right\},$$

where $\alpha = \mu \left(1 + \beta^{-1}\right)$.

(a) [**4 points**] Show competitive equilibrium prices with time-zero trading are constant multiples of β^t in even and odd periods.

(b) [**4 points**] Compute the competitive equilibrium with time-zero trading.

(c) [**4 points**] Compute the competitive equilibrium with sequential trading.

(d) [**4 points**] Suppose now that α increases such that $\alpha > \mu \left(1 + \beta^{-1}\right)$. Are one period interest rates higher or lower than those you computed in part c?

(e) [**4 points**] Compute the time-zero wealth of consumers using the equilibrium prices.

Exercise 5: Two-sided Altruism (20 points)

Assume a Cobb-Douglas production function, with share of labor α, and the simplest two-period-lived overlapping generations model. The population grows at rate n. Individuals supply inelastically one unit of labor in the first period of their lives. Each generation cares about the utility of the next generation (its children) and the previous (its parents).

Denote by W_t the *direct utility* of the tth generation,

$$W_t = u\left(c_t^t\right) + \frac{1}{1+\rho} u\left(c_{t+1}^t\right).$$

With two-sided altruism, the parents' utility is affected not only by the utility of their children but also by the utility of their own parents. It is thus natural to write the *overall utility* for the t*th* generation as

$$\mathcal{V}_t = W_t + (1+R)^{-1} V_{t+1} + (1+\phi)^{-1} V_{t-1}, \tag{7}$$

where $(1+R)^{-1}$ is the weight the parent places on her childrens' utility, and $(1+\phi)^{-1}$ is the corresponding weight she places on her parents'.

Budget constraints modified to take into bequests and gifts are

$$\begin{aligned} C_t^t + S_t^t &= w_t + b_t - g_t \quad (8) \\ C_{t+1}^t + (1+n)\, b_{t+1} &= (1+r_{t+1})\, S_t^t + (1+n)\, g_{t+1}. \quad (9) \end{aligned}$$

Here g_t is the gift made by a young person to her parents in period t and b_{t+1} is the bequest that she leaves over to her children.

(a) [**4 points**] Suppose that (7) reduces to

$$\mathcal{V}_t = \sum_{i=-\infty}^{\infty} \gamma_i W_{t+i}, \tag{10}$$

in which $\gamma_i \in \mathbb{R}$. What restrictions should be satisfied on the sequence of utility weights $\{\gamma_i\}_i$ for the solution of maximizing overall utility (10) subject to budget constraints (8) and (9) to exist? What choices do the tth generation make?

(b) [**7 points**] Suppose the conditions are met. Show that the first-order conditions of tth generation's maximization problem entail three relationships that link: *(1)* C_t^t to C_{t+1}^t, *(2)* C_t^t to C_t^{t-1}, and last *(3)* C_{t+1}^t to C_{t+1}^{t+1}. Discuss and interpret these relationships.

(c) [**5 points**] Show that the first-order conditions imply that the steady-state interest rate is equal to $a(1+n) = 1 + r^{ss} = b(1+n)$.[1] What is a and b?

(d) [**4 points**] What is the requirement for the golden rule interest rate in this economy? Does the inclusion of two-sided altruism ensure Pareto optimality in this overlapping generations model?

[1] Notice that in steady state the consumption of young will be constant. The same holds true for the consumption of old.

Exercise 6: Growth and the Savings Rate (10 points)

Consider the simplest two-period overlapping generations model with exogenous endowments, log-utility, and no population growth. The first-period endowment of an individual born at time t is equal to e_t, and the second-period endowment of the same individual to $e_t(1+g)$, where $g > 0$. How does an increase in the growth rate of income expected by one individual g affect his saving rate? How does an increase in g affect the aggregate saving rate. Discuss these questions in light of comments about that high growth is responsible for the high Chinese savings rate.