Part I: Linear Algebra

Outline:
Linear algebra is the study of vector spaces, matrices and linear transformations. It provides a common framework for analyzing and solving such seemingly unrelated problems as a system of linear equations or a differential equation. It has many applications in economics and econometrics.

We will begin by describing and finding solutions to systems of linear equations, \( A \mathbf{x} = \mathbf{b} \). To do this, we will learn about vector spaces and their subspaces, the concept of linear independence and basis and dimension. This part culminates in the Fundamental Theorem of Linear Algebra, which finds the dimensions of the four subspaces attached to a matrix and shows that they are orthogonal. Along the way, we will learn how to find the complete solution to a system of linear equations. Finally, we see that every matrix can be represented by a linear transformation and vice versa.

In the second part, we will find the ‘best’ solution to a linear system, when \( A \mathbf{x} \neq \mathbf{b} \). This arises frequently when there are many more equations than unknowns and is the quintessential problem to be solved in econometrics and regression analysis. We will learn how to solve it via projection and least squares.

In the third part, we focus on the algebra of square matrices and examine determinants and eigenvalues. These are numbers that are attached to a matrix and that encode an astonishing amount of information about the matrix. Determinants tell us, for example, whether a matrix is invertible. For eigenvalues (and eigenvectors), the key equation is \( A \mathbf{x} = \lambda \mathbf{x} \). They are used in the diagonalization of matrices and contain information about the long-term behavior of systems of difference and differential equations.

The final topic brings together the previous ones in the study of symmetric matrices, which arise frequently in optimization and econometrics. We look at the property of positive/negative definiteness of a matrix, which is important, for example, in multivariable optimization. This property also reveals deep connections between pivots, determinants and eigenvalues.

Readings

Main text
Gilbert Strang, *Introduction to linear algebra*

Additional materials
Jörgen Weibull, Lecture Notes
R. B. J. T. Allenby, *Linear algebra*
Charles W. Curtis, *Linear algebra: An Introductory approach*

Review of matrix algebra and systems of linear equations
Part II: Probability Theory

Outline:

In the remainder of the course, we will study probability theory, which provides a mathematical framework to study chance and uncertainty.

In the first part, we use set theory to define the sample space, outcomes and events and give an axiomatic foundation of probability. We also define independence, conditional probability and Bayes theorem.

We will then study random variables, which are functions on the sample space, and their distribution functions. We will encounter discrete and continuous random variables and their attendant probability mass functions and densities.

Next, we will learn how to summarize and describe distributions of random variables by their moments, such as the mean, the variance and higher order moments. We will see how the moment generating function can be used to calculate the moments of a distribution.

We will then learn how to find the distribution of a transformed random variable.

Finally, we will also briefly review a number of common discrete and continuous probability distributions and the circumstances under which they arise.

In the second part, we will prepare for the situation we face in actual data, namely, many random variables, which vary together. We will introduce the joint distribution and density functions and how to find marginal and conditional densities from them. We also introduce the covariance and correlation, a measure of how much two random variables move together, as well as the conditional expectation and variance.

We also learn how to find the distribution of sums, differences and ratios of random variables using the bivariate change of variable formula or moment generating functions.

In the final part, we show what can be learned about a distribution (of a random variable) by repeatedly sampling from it. Several sample statistics, such as the sample mean and sample variance, and their distributions will be presented. We will then investigate what happens to certain sample statistics and their distributions as the sample size approaches infinity. The convergence concepts we will present here, such as convergence in probability, convergence in distribution, the law of large numbers and the central limit theorem, form the basis for inference in much of econometric analysis.

Readings
G. Grimmett and D. Stirzaker, Probability and Random Processes
G. Casella and R.L. Berger, Statistical Inference
Joseph Blitzstein and Jessica Hwang, Introduction to Probability