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1. Short questions:

   a) Provide four reasons for why we observe non-actuarial prices in the insurance market. What is the consequence of non-actuarial prices?

   b) Consider a factory owner who needs to hire staff. The value of output produced is given by $S(q) = 3q^{\frac{1}{3}}$, where q is the amount being produced by a worker. The factory owner pays a wage t to workers. There are two types of workers who differ with respect to their cost of effort. Workers of type $\underline{\Theta} = 1$ incur cost $C(q, \underline{\Theta}) = q$, while workers of type $\overline{\Theta} = 2$ incur cost $C(q, \overline{\Theta}) = 2q$ when producing q. Workers' utilities are given by $u_\Theta(q, t) = t - C(q, \Theta)$. The factory owner has no information regarding the efficiency of workers, but knows that the share of efficient workers is ν. State the factory owner's optimization problem and all constraints that need to be satisfied. Which constraints are relevant? Explain why the other constraints are not relevant.

   c) Consider a market where there are two types of workers. Type 0 has marginal product $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = 1$. Type 1 has marginal product $\Theta_1 = 3$ and an outside opportunity wage of $w_0(\Theta_1) = 2$. The cost of education z is given by $C(z, \Theta_0) = \frac{z}{\Theta_0} = z$ for type 0 and $C(z, \Theta_1) = \frac{z}{\Theta_1} = \frac{z}{3}$ for type 1. A worker's utility function is given by $U(w, z, \Theta) = w - C(z, \Theta)$. Workers know their own type but employers cannot tell the high from the low productivity workers. Illustrate in a figure, with wage on the y-axis and the amount of education on the x-axis, which contracts will make it possible to separate type 1 from type 0 workers. Assume that, if two contracts yield the same level of utility, a type 0 worker prefers the one which requires less education. Use the figure to identify the contract that will be offered to type 1 workers by a monopsonist employer. Identify the contract that will be offered if there is perfect competition among employers. (Note that you do not have to derive the contracts mathematically; just show where these two contracts are located in your figure.)

   d) Consider an entrepreneur (the agent) who needs to borrow money to realize a project. The entrepreneur turns to a bank (the principal) which provides a loan of size I (at cost I). The return from the project is random, either high ($\overline{R} = 6$) or low ($\underline{R} = 2$). However, the probability of a high return π_e also depends on the effort $e \in \{0, 1\}$ that the entrepreneur exerts, such that $\pi_0 = \frac{1}{4}$ and $\pi_1 = \frac{3}{4}$. The cost that the entrepreneur incurs from exerting effort e is $\Psi(e) = e$. The bank is a profit maximizing monopolist. Since it is unable to observe how much effort the entrepreneur exerts, it offers a contract, which specifies the repayment $\underline{z}$ that the entrepreneur has to make if the return from the project is low and the repayment $\overline{z}$ that the entrepreneur has to make if the return from the project is high. The entrepreneur's expected utility is given by $EU_e = \pi_e(\overline{R} - \overline{z}) + (1 - \pi_e)(\underline{R} - \underline{z}) - \Psi(e)$. When designing the contract the bank also has to take into account that the entrepreneur's liability is limited such that he/she cannot incur any losses. State the bank's optimization problem and all the constraints that need to be satisfied.

2. There are two states of the world, state 1 and state 2. The probability for state 1 occurring is $\pi = \frac{1}{3}$ (and the probability for state 2 occurring is $1 - \pi = \frac{2}{3}$). It is not possible to directly trade in state claims. However, there exists a complete asset market, where two assets, asset A_1 and asset A_2, can be traded.

   a) State the conditions that have to be satisfied for a complete asset market.

   The price of A_1 is given by $P_1^A = 6$, and the price of A_2 is given by $P_2^A = 6$. The following yield matrix indicates how much each asset yields in each state (e.g. A_1 yields $z_{11} = 4$ in state 1):

	State 1	State 2
Asset A_1	$z_{11} = 4$	$z_{12} = 1$
Asset A_2	$z_{21} = 2$	$z_{22} = 2$

   The individual's preference-scaling function is given by $v(c) = \sqrt{c}$, and the individual is endowed with $\overline{q_1} = 10$ units of A_1 and $\overline{q_2} = 5$ units of A_2.

   b) What are the implicit prices of state claim 1 (P_1) and state claim 2 (P_2)? (Hint: $P_1^A = z_{11}P_1 + z_{12}P_2$ and $P_2^A = z_{21}P_1 + z_{22}P_2$.)

   c) What will the individual's portfolio of assets (i.e. the endowment $\overline{q_1}$ and $\overline{q_2}$) yield in the two different states? (That is, what is the individual's implicit endowment of state claims $\overline{c_1}$ and $\overline{c_2}$?)

   d) State the von-Neumann-Morgenstern expected utility function.

   e) To obtain the optimal amounts of implicit state claims, two conditions need to be satisfied. State these two conditions. Then calculate the optimal amounts of state claims.

   f) Given the optimal amounts of state claims, what are the optimal amounts of assets A_1 and A_2?

3. Consider an insurance market where the type of insurees is hidden knowledge. Each individual owns a bike worth 81. An individual of type $\underline{\Theta}$ has a probability $\underline{\Theta} = \frac{1}{9}$ of his/her bike being stolen (low-risk type), while an individual of type $\overline{\Theta}$ has a probability $\overline{\Theta} = \frac{1}{3}$ of his/her bike being stolen (high-risk type). The share of type $\underline{\Theta}$ agents is given by $\nu = \frac{1}{2}$. The utility of each individual is given by $u = \sqrt{w}$, where w represents the individual's wealth, which is given by the value of the bike (0 if stolen, 81 else). By purchasing full insurance at premium P, $w = 81 - P$ irrespective of the bike being stolen or not.

   a) For type $\underline{\Theta}$, what is the expected utility of not buying insurance? Determine the highest premium $\widehat{P}_{\underline{\Theta}}$ that a type $\underline{\Theta}$ individual is willing to pay for full insurance.

   b) For type $\overline{\Theta}$, what is the expected utility of not buying insurance? Determine the highest premium $\widehat{P}_{\overline{\Theta}}$ that a type $\overline{\Theta}$ individual is willing to pay for full insurance.

   c) Determine the range of premia P, for which both types buy insurance. What is the expected loss per insuree in this case?

   d) Determine the range of premia P, for which only one type buys insurance. What is the expected loss per insuree in this case?

   e) Determine the range of premia P, for which no one buys insurance.

   f) Which premium will be offered in a perfectly competitive insurance market?

   g) Which premium will be offered by a monopolistic supplier of insurance? Calculate the expected profit of the monopolistic insurer.

   h) Will the market outcome be adverse selection (i.e. one type leaving the market)? Provide an intuitive explanaition for your answer. (No calculations, just words!)

   i) Explain how insurance contracts can be designed to separate low-risk from high-risk types. (No calculations, just words!)