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Exam for EC 2110, Uncertainty and asymmetric information, 2 June 2017

1. Short questions:

a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
(i) $v(c) = c + \ln c$
(ii) $v(c) = e^{\ln(c^2)}$
(iii) $v(c) = \frac{c}{1+c}$
(iv) $v(c) = 5(c+1)$

b) Consider an individual who may end up in two states. Unfortunately it is not possible to directly trade in state claims. However, there exists an asset market where it is possible to buy and sell assets. The following matrix illustrates how much available assets yield in the two states:

	State 1	State 2
Asset A_1	$z_{11} = 3$	$z_{12} = 1$
Asset A_2	$z_{21} = 2$	$z_{22} = 0$

It is possible to hold a negative amount of an asset, but not to go bankrupt. State the four conditions that need to be satisfied for an asset market to be complete. Verify that each of these conditions is satsified for this asset market. (Note: you should demonstrate this by using the available information.)

c) Chris has the following elementary utility function: $v(c) = \sqrt{c}$, where c is the value of Chris' fortune which consists of a bike worth 64. The likelihood of the bike being stolen is 50%. Chris can choose between two different insurance contracts. Under insurance contract A the premium is $P_A = 15$ and there is a deductible of $D_A = 33$. Insurance contract B specifies a premium of $P_B = 28$ and no deductible. Which contract does Chris prefer?

d) Consider a market with a monopsonist employer. There are two types of workers. Type 0 has marginal product $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = 2$. Type 1 has marginal product $\Theta_1 = 5$ and an outside opportunity wage of $w_0(\Theta_1) = 4$. Workers know their own type but the employer cannot tell the high from the low productivity workers. When educational screening is possible, what contract will be offered to type 1 workers? (Assume that, if two contracts yield the same level of utility, a type 0 worker prefers the one which requires less education.)

2. Consider a factory owner who needs to hire staff. The factory owner offers contracts that specify the amount q to be produced and the transfer t to be paid to employees. The value of output produced is given by $S(q) = q$. The factory owner's payoff from a contract is thus given by $V = S(q) - t$.

   An employee of type Θ incurs cost $C(q, \Theta) = \Theta q^2$ when producing q. There are two types of employees: efficient employees are of type $\underline{\Theta} = 1$, while ineffcient employees are of type $\overline{\Theta} = 2$. Employees' utilities are given by $u_\Theta(q, t) = t - C(q, \Theta)$. The share of type $\underline{\Theta}$ employees is $\nu = \frac{1}{2}$.

   a) Determine the socially optimal output levels $\underline{q}^*$ and $\overline{q}^*$ of the two employee types.

   b) Determine the transfers that will be paid to the two types of employees given that the factory owner has all bargaining power. State the first-best menu of contracts.

   c) Calculate the information rent that efficient employees can extract by mimicking inefficient employees.

   Assume now that information regarding employees' types is hidden to the factory owner.

   d) State the factory owner's optimization problem and all constraints that need to be satisfied. Which constraints are relevant? Explain why the other constraints are not relevant.

   e) Simplify the optimization problem by taking into consideration that the factory owner has all bargaining power. Solve the optimization problem to determine the second-best menu of contracts.

   f) Provide an intuitive explanation for how the second-best menu of contracts is affected by an increase in the share of efficient employees. Explain how a higher share of efficient employees impacts on the factory owner's payoffs from the second-best contracts. (Just words, no calculations!)

3. Consider an entrepreneur (the agent) who plans to set up a production facility. However, the entrepreneur needs to borrow an amount of I from a bank (the principal) to be able to carry out the project. If the bank offers a loan, the return from production equals $\overline{V} = 5$ with probability π_e and $\underline{V} = 1$ with probability $1 - \pi_e$, where $e \in \{0, 1\}$ represents the entrepreneur's effort level, and $\pi_1 = \frac{1}{2}$ and $\pi_0 = \frac{1}{3}$. The cost of exerting effort $e = 1$ is $\Psi_1 = 1$ (the cost of exerting no effort is zero).

The loan contract specifies how much the entrepreneur has to pay back to the bank. If the return is high ($V = \overline{V}$), the entrepreneur has to make the repayment $\overline{z}$, and if the return is low ($V = \underline{V}$), the entrepreneur has to make the repayment $\underline{z}$. Hence, the entrepreneur will end up with $\overline{t} = \overline{V} - \overline{z}$ if the return is high and with $\underline{t} = \underline{V} - \underline{z}$ if the return is low. The risk neutral entrepreneur's expected profit is thus given by $EU_1 = \pi_1\overline{t} + (1 - \pi_1)\underline{t} - \Psi_1$ if $e = 1$ and $EU_0 = \pi_0\overline{t} + (1 - \pi_0)\underline{t}$ if $e = 0$. The bank's expected profit for offering a high-effort inducing loan contract is given by

$$EV_1 = \pi_1\overline{z} + (1 - \pi_1)\underline{z} - I = \pi_1(\overline{V} - \overline{t}) + (1 - \pi_1)(\underline{V} - \underline{t}) - I.$$

a) State the participation constraint of the entrepreneur.

b) State the bank's optimization problem under complete information.

c) Given that the bank is the only money lender, which contract(s) will be offered under complete information? Illustrate your answer in a figure, with $\underline{t}$ on the horizontal axis and $\overline{t}$ on the vertical axis.

d) What will the bank's expected payoff for implementing the contract(s) be? For what range of I is it optimal to offer the high-effort inducing contract(s)?

Now assume that information regarding the entrepreneur's actions is hidden to the bank.

e) Which constraints need to be satisfied? Which contract(s) will be offered? Illustrate your answer in the same figure as above.

f) What will the bank's expected payoff for implementing the contract(s) be? For what range of I is it optimal to offer the high-effort inducing contract(s)?

Now assume that the entrepreneur's liability is limited such that losses cannot exceed 1.

g) Which constraints need to be satisfied? Which contract(s) will be offered? Illustrate your answer in the same figure as above.

h) What will the bank's expected payoff for implementing the contract(s) be? For what range of I is it optimal to offer the high-effort inducing contract(s)?

i) Provide an intuitive explanation for why limited liability leads to a lower expected payoff for the bank. Assuming that there are many entrepreneurs who require different loan amounts I, explain how limited liability and moral hazard lead to an economically inefficient outcome.