

1. Short questions:

- a) Do the following elementary utility functions represent risk averse, risk neutral or risk loving preferences? Motivate your answers.
- (i)  $v(c) = \ln(c^4)$
  - (ii)  $v(c) = \sqrt{c} + c$
  - (iii)  $v(c) = \sqrt{1 + c^2}$
  - (iv)  $v(c) = e^{1+\ln c}$
- b) Consider a factory owner who needs to hire staff. The value of output is given by  $S(q) = \ln(1 + q)$ , where  $q$  is the amount produced by a worker. The factory owner pays a wage  $t$  to workers. There are two types of workers who differ with respect to their cost of production. Workers of type  $\underline{\Theta} = \frac{1}{4}$  incur cost  $C(q, \underline{\Theta}) = \frac{1}{4}q$ , while workers of type  $\bar{\Theta} = \frac{1}{2}$  incur cost  $C(q, \bar{\Theta}) = \frac{1}{2}q$  when producing quantity  $q$ . Workers' utilities are given by  $u_{\Theta}(q, t) = t - C(q, \Theta)$ . Determine the first-best quantities of the two types of workers. State the participation constraints of the two types of workers and illustrate these in a figure. Assuming that the factory owner has all bargaining power, identify the menu of first-best contracts in the figure (i.e. use the figure to show where the contracts that are offered under complete information regarding the efficiency of workers are located).
- c) What is "nonresponsiveness"? Explain under which type of informational asymmetry it may occur and how it arises. What type of contract will a principal offer to deal with "nonresponsiveness"? (Note: no formulas, just words!)
- d) In lecture 6 you were presented a model built on the assumption that attending university does not have any positive effect on productivity. In the context of this model, provide an intuitive explanation for why you have chosen to waste your time on attending this course (and all other courses). What is the purpose of education in this model?

2. There are two states of the world, state 1 and state 2. The probability for state 1 occurring is  $\pi = \frac{1}{2}$  (and the probability for state 2 occurring is  $1 - \pi = \frac{1}{2}$ ). It is not possible to directly trade in state claims. However, there exists a complete asset market, where two assets, asset  $A_1$  and asset  $A_2$ , can be traded.

a) State the conditions that have to be satisfied for a complete asset market.

The price of  $A_1$  is given by  $P_1^A = 2$ , and the price of  $A_2$  is given by  $P_2^A = 1$ . The following yield matrix indicates how much each asset yields in each state (e.g.  $A_1$  yields  $z_{11} = 2$  in state 1):

	State 1	State 2
Asset $A_1$	$z_{11} = 2$	$z_{12} = 4$
Asset $A_2$	$z_{21} = 3$	$z_{22} = 1$

Consider an individual whose preference-scaling function is given by  $v(c) = \ln c$  and who is endowed with  $\bar{q}_1 = 4$  units of  $A_1$  and  $\bar{q}_2 = 4$  units of  $A_2$ .

- b) What are the implicit prices of state claim 1 ( $P_1$ ) and state claim 2 ( $P_2$ )? (Hint:  $P_1^A = z_{11}P_1 + z_{12}P_2$  and  $P_2^A = z_{21}P_1 + z_{22}P_2$ .)
- c) What will the individual's portfolio of assets (i.e. the endowment  $\bar{q}_1$  and  $\bar{q}_2$ ) yield in the two different states? (That is, what is the individual's implicit endowment of state claims  $\bar{c}_1$  and  $\bar{c}_2$ ?)
- d) State the von-Neumann-Morgenstern expected utility function.
- e) To obtain the optimal amounts of implicit state claims, two conditions need to be satisfied. State these two conditions. Calculate the optimal amounts of state claims.
- f) Given the optimal amounts of state claims, what are the optimal amounts of assets  $A_1$  and  $A_2$ ?
3. Hen owns a bicycle which is worth 81 SEK. By undertaking precautionary measures at cost  $\Psi = 1$  Hen can increase the likelihood of the bike not being stolen within the next year to  $\pi_1 = \frac{2}{3}$ . Hen's elementary utility function is given by  $v(c, \Psi) = \sqrt{c} - \Psi$ , where  $c$  is the value of Hen's wealth which consists of the bike, i.e.  $c = 81$  if the bike is not stolen and  $c = 0$  if it is stolen.

a) What is the expected value of Hen's wealth in one year's time? What is Hen's expected utility?

Hen has an option of purchasing insurance at a premium of 32 SEK, in which case Hen receives 81 SEK if the bike is stolen. The insurance company is able to perfectly verify that Hen undertakes precautionary measures to keep the probability of the bike not being stolen at  $\pi_1 = \frac{2}{3}$  after having bought insurance. Hence, the insurance contract is offered under complete information.

- b) If Hen buys insurance, what is the expected value of Hen's wealth in one year's time? What is Hen's expected utility?
- c) Explain why Hen will buy insurance although expected wealth is lower when insurance is purchased. (Hint: Relate your answer to Hen's attitude to risk.)
- d) Is the insurance premium actuarially fair? If not, what is the actuarially fair insurance premium?

Now assume that the insurance contract is offered under asymmetric information. Thus, Hen becomes less careful after having bought insurance, thereby saving the cost for undertaking precautionary measures, which reduces the probability of the bike not being stolen to  $\pi_0 = \frac{1}{3}$ .

- e) Explain in words how the insurance contract has to be designed to induce Hen to undertake precautionary measures. State the constraints that need to be satisfied.
- f) What contract will a monopolistic insurance company offer in the presence of hidden action?