

Course name:	The Economics of Uncertainty and Asymmetric Information
Course code:	EC2110
Type of exam:	Written exam
Examiner:	Mathias Herzing
Number of credits:	7,5 credits
Date of exam:	Sunday 17 February 2019
Examination time:	3 hours [09:00-12:00]

Write your identification number on each answer sheet (the number stated in the upper right hand corner on your exam cover).

Start each new question on a new answer sheet.

Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked.

No aids are allowed.

The exam consists of 3 questions. The maximum score of question 1 is 32 points. The maximum score of questions 2 and 3 is 34 points each. The maximum score on the exam is 100 points in total. For the grade E 45 points are required, for D 50 points, for C 60 points, for B 75 points and for A 90 points.

Your results will be made available on your Ladok account (www.student.ladok.se) within 15 working days from the date of the examination.

Good luck!

Department of Economics, Stockholm University Mathias Herzing Exam for EC 2110, Uncertainty and asymmetric information, 17 February 2019

- 1. Short questions:
 - a) Consider a farmer (the agent) who needs to borrow money to grow crops. The farmer turns to a bank (the principal) which provides a loan of size k (at cost k). The repayment of the farmer is given by t. The bank's profit is thus given by V = t k. The value of the farmer's output P is determined by the size of the loan and his/her type: $P(k, \Theta) = 2\Theta k^{\frac{1}{2}}$, where $\Theta = \Theta = 1$ if the farmer is inefficient and $\Theta = \overline{\Theta} = 4$ if the farmer is efficient. (Note: a higher Θ implies higher efficiency!) The farmer's profit is given by $U_{\Theta} = P(k, \Theta) t$. The bank has no information regarding the efficiency of the farmer, but knows that the share of efficient farmers is ν . State the bank's optimization problem and the two relevant constraints that are binding given that it has all bargaining power. Use the two binding constraints to simplify the bank's optimization problem (i.e. express the bank's expected payoff in terms of \underline{k} and \overline{k} only).
 - **b)** In the case of hidden-knowledge type of asymmetric information the principal faces the problem of incentive compatibility when designing contracts for different types of agents. Explain why the principal might have an incentive to renegotiate after an agent has chosen a contract. Is renegotiating contracts an equilibrium outcome?
 - c) The owner of a farm hires a worker to grow crops. The crop yield is random (depending on e.g. weather conditions), either high or low. However, the probability of the crop yield being high π_e also depends on the effort $e \in \{0, 1\}$ that the worker exerts, such that $\pi_0 = \frac{1}{4}$ and $\pi_1 = \frac{1}{2}$. The cost that the worker incurs from exerting effort e is $\Psi(e) = e$. The farm owner, who is the only employer, offers a contract $(\underline{t}, \overline{t})$ that induces the worker to exert high effort. The worker's expected utility is given by $EU_e = \pi_e \overline{t} + (1 \pi_e)\underline{t} \Psi(e)$. Unfortunately it is not possible for the farmer to observe how much effort has been exerted. Moreover, the worker's liability is limited such that losses from transfers cannot exceed 2. State all the constraints that need to be satisfied for a contract $(\underline{t}, \overline{t})$ offered by the farm owner. Illustrate these constraints graphically in a figure, with \underline{t} on the x-axis and \overline{t} on the y-axis.
 - d) Explain briefly how deductibles can be used to deal with different types of asymmetric information in the insurance market.

- 2. Chris has the following elementary utility function: $v(c) = \ln(c+1)$, where c is consumption. Assume that there are only two possible states of the world, 1 and 2, where the probability of state 1 being realized is π . Consumption in state 1 is denoted by c_1 , and consumption in state 2 is denoted by c_2 .
 - a) State Chris' von Neumann-Morgenstern utlility function.
 - b) Derive a mathematical expression for Chris' marginal rate of substitution (MRS) between consumption in the two possible states of the world. (Hint: $v'(c) = \frac{1}{c+1}$.)

Chris' endowment of state claims is given by $\overline{c}_1 = 7$ and $\overline{c}_2 = 3$, and $\pi = \frac{1}{4}$. It is possible to trade in state claims at prices $p_1 = 1$ and $p_2 = 4$.

- c) Which two conditions need to be satisfied to solve Chris' optimization problem?
- d) Use these two conditions to determine Chris' optimal amounts of state claim 1 and state claim 2.
- e) What is the actuarily fair market price ratio?
- **f)** What would Chris' optimal amounts of claims in state 1 and state 2 be if prices were actuarily fair?

Now assume that trading takes place in an economy which only consists of Chris and another person called Kim, i.e. the market equilibrium is determined by the interaction between Chris and Kim. Kim has the utility function $v(c) = 4c^{\frac{1}{4}}$ and is endowed with $\bar{c}_1^K = 1$ and $\bar{c}_2^K = 5$. Kim also attributes the probability $\pi = \frac{1}{4}$ to state 1 occurring.

g) Let (c_1^{C*}, c_2^{C*}) and (c_1^{K*}, c_2^{K*}) denote Chris' and Kim's market equilibrium amounts of state claims. State all six conditions that need to be satisfied to obtain the market equilibrium. (Note: you are not supposed to calculate the equilibrium outcome - just state the conditions that have to be satisfied.)

- 3. Consider a perfectly competitive labor market. There are two types of workers. Type 0 has a marginal productivity of $\Theta_0 = 1$ and an outside opportunity wage of $w_0(\Theta_0) = 1$. Type 1 has a marginal productivity of $\Theta_1 = 2$ and an outside opportunity wage of $w_0(\Theta_1) = 1.25$. The cost of education z is given by $C(z, \Theta_0) = \frac{z}{\Theta_0} = z$ for type 0 and $C(z, \Theta_1) = \frac{z}{\Theta_1} = \frac{z}{2}$ for type 1. A worker's utility function is given by $U(w, z, \Theta) = w C(z, \Theta)$. The share of type 0 workers is given by 0.9. Workers know their own type but the employer cannot tell the high from the low productivity workers.
 - a) In the absence of any educational screening, will there be adverse selection in the market?
 - b) When educational screening is possible, what contract will be offered to type 1 workers if there is just one monopsonist employer? For simplicity assume that, if two contracts yield the same level of utility, a type 0 worker prefers the one with less education. Illustrate your answer in a figure with wage on the y-axis and the amount of education on the x-axis.
 - c) What contract will be offered to type 1 workers if there are many employers competing for workers? Illustrate your answer in the same figure. Calculate the utility of a type 1 worker under this contract.

Now assume that type 2 workers enter the labor market. Type 2 has a marginal productivity of $\Theta_2 = 3$ and an outside opportunity wage of $w_0(\Theta_2) = 1.5$. The cost of education z is given by $C(z, \Theta_2) = \frac{z}{\Theta_2} = \frac{z}{3}$ for type 2 workers, and the utility function is given by $U(w, z, \Theta) = w - C(z, \Theta)$ as above.

- d) Given that a separating equilibrium is feasible, what contract will be offered to type 2 workers when there are many employers? For simplicity assume that, if two contracts yield the same level of utility, a type 1 worker prefers the one with less education. Illustrate your answer in the same figure as above. Calculate the utility of a type 2 worker under this contract.
- e) Determine the highest possible value for the average productivity of type 1 and type 2 workers to ensure that the contracts that you have identified constitute a separating Nash equilibrium.