Stockholm University
Department of Economics

Course name:	**Game Theory**
Course code:	**EC7112**
Type of exam:	Written exam
Examiner:	**Mathias Herzing**
Number of credits:	**7,5 credits**
Date of exam:	**Sunday 5 May 2019**
Examination time:	**3 hours [09:00-12:00]**
Aids:	No aids are allowed.

Write your identification number on each answer sheet (the number stated in the upper right hand corner on your exam cover).

Start each new question on a new answer sheet.

Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked.

The exam consists of 3 questions. The maximum score of questions 1 and 2 is 35 points each. The maximum score of question 3 is 30 points. The maximum score on the exam is 100 points. For grade E 40 points are required, for D 50 points, for C 60 points, for B 75 points and for A 90 points.

Your results will be made available on your Ladok account (www.student.ladok.se) within 15 working days from the date of the examination.

Good luck!

Department of Economics, Stockholm University
Mathias Herzing
Exam for EC7112, Game theory, 5 May 2019

1. (35 points) Consider two players (1 and 2) that participate in a second-price sealed-bid auction. The object on sale is valued at $v_1 = 3$ and $v_2 = 2$ by the two players. Let b_1 and b_2 be the bids that are submitted by players 1 and 2, respectively. Bids are made in discrete numbers, and the highest bid that either person can submit is 4, i.e. the set of bids is given by $B = \{0, 1, 2, 3, 4\}$ for both players. The person who has submitted the highest bid wins and pays the bid submitted by the loser. If there is a tie, a coin is tossed such that there is a probability of $\frac{1}{2}$ of winning for each player. The expected utility of player i is $EU_i = v_i - b_{-i}$ if $b_i > b_{-i}$, $EU_i = \frac{1}{2}(v_i - b_i)$ if $b_i = b_{-i}$, and $EU_i = 0$ if $b_i < b_{-i}$.

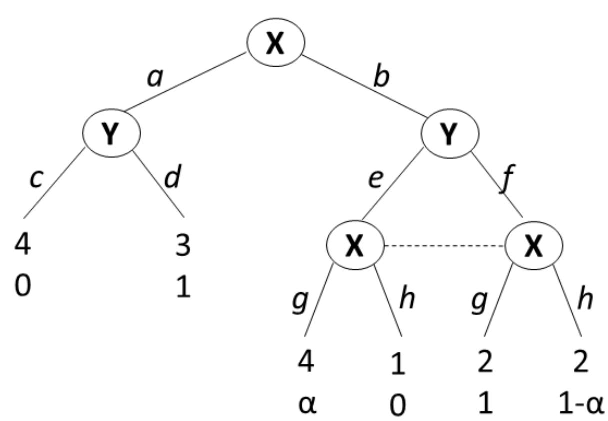
   a) Calculate the expected utilities of player 1 for all combinations of b_1 and b_2. State the best response function of player 1.

   b) Calculate the expected utilities of player 2 for all combinations of b_1 and b_2. State the best response function of player 2.

   c) Illustrate the best response functions in a figure, with $b_1 \in B$.on the horizontal axis and $b_2 \in B$.on the vertical axis.

   d) Identify the Nash equilibrium where both 1 and 2 play weakly dominant strategies.

2. (35 points) Consider the following extensive form game between players **X** and **Y**, where payoffs are presented in the following order: u_X, u_Y.



   a) Define the strategy sets, the player function and the set of terminal histories of this game. Identify all subgames of the game.

   b) Determine the pure strategy subgame perfect Nash equilibria for all $\alpha \in \mathbb{R}$.

3. (30 points) Consider two criminals (Ann and Barbara) who both can choose between committing a crime (strategy C) and not committing a crime (strategy N). When no crime is committed the utility of the criminal is 0. If a crime is committed the criminal's utility is 2 if she does not get caught and -1 if she gets caught and punished. The likelihood of getting caught depends on overall criminal activity - if a criminal is the only offender, she will be caught for sure, whereas the probability of being caught is 0.5 when both criminals commit a crime. Ann and Barbara take their decisions independently and simultaneously.

   a) Illustrate the interaction between the two criminals on normal form, i.e. in a payoff matrix with the expected utilities of Ann and Barbara.

   b) Derive the best response functions of Ann and Barbara. Illustrate the best response functions in a figure. Use the figure to identify all Nash equilibria of this game.

   c) Use you results in b) to provide an intuitive explanation for why crime rates might differ across regions, even if conditions are similar everywhere. (In other words, why can either an equilibrium with no criminal activity or an equilibrium with a high crime rate emerge?)

   Now assume that decisions are taken sequentially. That is, first Ann chooses between committing and not committing a crime, whereupon Barbara who is able to perfectly observe Ann's choice takes her decision.

   d) Illustrate the interaction between the Ann and Barbara on extensive form.

   e) Identify the subgame perfect Nash equilibrium/equilibria of this game.