Stockholm University
Department of Economics

Course name:	The Climate & the Economy
Course code:	EC7104
Type of exam:	Written
Examiner:	John Hassler, Per Krusell and Conny Olovsson
Number of credits:	7.5
Date of exam:	June 5, 2019
Examination time:	13.00-17.00
Aids:	No aids are allowed.

Write your identification number on each answer sheet (the number stated in the upper right hand corner on your exam cover).

Start each new question/section/part on a new answer sheet.

Explain notions/concepts and symbols. If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked.

The exam consists of 9 questions. 100 points in total. Each question is worth 15 or 8 points, 100 points in total. For the grade E 45 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Your results will be made available on your Ladok account (www.student.ladok.se) within 15 working days from the date of the examination.

Good luck!

EC7104 The Climate & the Economy
Spring 2019
June 2019

Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions (15 points each)

1. **A Carbon Circulation Model (15)**
   Consider the following simple example of a linear carbon circulation model with two sinks (reservoirs), S (atmosphere) and S^L (ocean):

$$\begin{aligned} S_t - S_{t-1} &= -\phi_1 S_{t-1} + \phi_2 S^L_{t-1} + M_{t-1}, \\ S^L_t - S^L_{t-1} &= \phi_1 S_{t-1} - \phi_2 S^L_{t-1}. \end{aligned}$$

   where ϕ_1 and ϕ_2 are parameters and M_{t-1} represents emissions in period $t-1$.

   (a) (10) Suppose at time 0 (before any human emissions have begun), the system is in a pre-industrial steady state, i.e., $S_0 = S_{-1} = 500\ GtC$, $S^L_0 = S^L_{-1} = 50000$ and $M_{-1} = 0$.

      i. Suppose that the flow of carbon from the atmosphere to the ocean in the pre-industrial steady state is 100 GtC per period. Use this to calibrate ϕ_1.

      ii. How large is the flow of carbon from the ocean to the atmosphere in the pre-industrial steady state? Use this to calibrate ϕ_2.

      iii. We have now emitted around 505 GtC into the atmosphere (this exact value simplifies the calculations in the question). Suppose we now stop emitting and never emit again. After (a long) transition period, the system will reach a new post-industrial steady state, where S and S^L are both constant.

      How large will S and S^L be in the post-industrial steady state according to the model? (Hint: Denote by x the amount of the emitted 505 GtC that remain in the atmosphere. Then, the remainder 505-x is in the ocean. The ratio between $\frac{S}{S^L}$ is then $\frac{500+x}{50000+505-x}$. What is this ratio equal to in terms of the pre-industrial stocks, S_0 and S^L_0? Solve for x.

      iv. During the transition period to the post-industrial steady state, will S be larger or smaller than its value in the post-industrial steady state? Will S^L be larger or smaller than in the post-industrial steady state?

   (b) (5) Suppose that there is a non-linearity in the system. As the temperature in the oceans increases, its ability to store carbon falls. We can model this by assuming that the flow to the ocean falls when its temperature increases.

      i. Describe how we could change the model to incorporate this feature.

ii. Illustrate qualitatively how the non-linearity would change the dynamics of the atmospheric carbon concentration for the scenario in question 1.a.iii. Do this in a graph with the atmospheric carbon concentration on the y-axis and time on the x-axis. Draw two curves, one for the linear and one for the non-linear case. Assume they start at the same point and illustrate the qualitative difference between the two curves. No numbers are required.

2. Assume that there are three countries in the world: two countries (labeled 1 and 2) that both are using oil and labor to produce output Y, and one country that owns and sells the oil to country 1 and 2. The world exists for one period and the total supply of fossil fuel is fixed at the amount E_{tot}. The cost of providing a marginal unit of oil for the oil exporting country is equal to zero. The production functions for output in country 1 and 2 are given by

$$Y_i \equiv f\left(\overline{E}, E_i, L_i\right) = \overline{E}^{-\beta} E_i^{\alpha} L_i^{1-\alpha}, \quad i = \{1, 2\},$$

with $0 < \beta < \alpha < 1$, and $L_1 = L_2 = 1$. The term $\overline{E}^{-\beta}$ captures the damages that are caused by the use of oil. In equilibrium it must be true that $\overline{E} = E_1 + E_2$ but both countries consider $\overline{E}$ to be exogenous (think of the representative firm in each region as consisting of many small identical firms). The oil producing country is not affected by climate change. Assume now that country 1 imposes the tax τ on oil use in order to reduce the adverse effects associated with its use, but that country 2 does not implement any taxes. The total price that firm 1 has to pay for a marginal unit of oil is then $(p + \tau)$, where p is the market price of oil.

(a) Explain how the how the quantity of oil supplied by the oil exporting country depends on the the price of oil (p). What does this supply curve imply for the possibility of the tax τ to reduce the total amount of oil use?

(b) Formulate the profit maximization problems for the firms in country 1 and 2, and derive the first order conditions with respect to E_i and L_i for $i = \{1, 2\}$.

(c) Assume now that the tax in country 1 is set to $\tau = p$. Use this fact, the two first order conditions and the fact that $E_1 + E_2 = E_{tot}$ to solve for E_1 in terms of E_{tot}.

(d) Solve for the relationship between E_1 and E_2 if country 1 sets $\tau = 0$.

3. **Solow's growth model with an energy input**

Consider a slightly altered version of Solow's model, described with the following equations:

$$k_{t+1} = sy_t,$$

i.e., we have set the rate of depreciation of capital to 100%; and

$$y_t = A_t k_t^{\alpha} e_t^{\nu}$$

where e_t is energy. I.e., there is technological change, captured by A_t depending on t, but the energy input might also change over time. We will assume that α and $\alpha + \nu$ are less than one (there is a labor input too that is normalized to 1).

(a) First consider a pre-industrial era where $A_t = \underline{A}$ and where e_t consists of a small amount of biofuel: $e_t = \underline{e}$. Starting from an initial capital stock close to zero, illustrate diagrammatically (with time on the x axis) how capital and output grow over time. Show diagrammatically, with k_t on the x axis and k_{t+1} on the y axis, that the economy converge toward a steady state. Calculate the value of the capital stock in such a steady state (described in terms of primitives).

(b) The industrial revolution is characterized by growth in A, but suppose also that energy input begins growing as a result of overall economic growth. For simplicity, assume that e_t grows at a constant net rate g_e, while A grows at a constant net rate g_A. That is,

$$\frac{A_{t+1}}{A_t} = 1 + g_A \quad \text{and} \quad \frac{e_{t+1}}{e_t} = 1 + g_e.$$

Notice that this situation is identical to labor-augmenting growth at rate g, which is usually expressed as a production function $y_t = k_t^{\alpha}\left((1+g)^t\right)^{1-\alpha}$, so long as g satisfies $\left((1+g)^t\right)^{1-\alpha} = (1+g_A)^t\left((1+g_e)^t\right)^{\nu}$. (It is a matter of algebra to write g as an explicit function of g_A and g_e but you do not have to do this.)

Describe how capital and output will evolve during the industrial era. Use the same kinds of diagrams as in the previous question, though not necessarily with k and y on the axis but some transformations thereof. Is there some sense of convergence here as well? If so, calculate an expression for capital-output ratio toward which the economy converges. You do not need to use g_A or g_e in these calculations: just use g.

(c) Suppose, finally, that at time T there is a sudden insight—after a long industrial period—that the primary source of energy, fossil fuel, hurts the climate. As a result, the world's leaders decide to immediately stop the growth of energy and instead keep it at its current level, i.e., $e_t = e_T$ forever after. Assume that technology growth continues as before, i.e., at rate g_A.

Describe how capital and output growth behave in the short and the long run as a result of this policy change. More precisely, describe the evolution of these variables with time on the x axis, both before and after T. What will happen to the long-run growth rate of the economy? Only a qualitative answer is needed—you do not need to do any algebra.

4. **An IAM with Two Consumers**

Consider a static IAM with the following structure:

- There are two consumers, R and P (rich and poor). Their preferences are $\log c_R$ and $\log c_P$, respectively, where the cs represent the consumption level per person. The group of R consumers is of size λ and the group of P consumers is of size $1-\lambda$. Hence total consumption is $c = \lambda c_R + (1-\lambda)c_P$.
- The production of consumption is given by

$$c = Dn^{1-\nu-\mu}e^{\nu},$$

where D is TFP, n is total labor use, and e is energy use.
- Energy is produced for free—think of it as oil—and there is a total of R units available: $e \leq R$.
- Each of the consumers works and they each have 1 unit of labor available. Thus the total labor available in the economy is $\lambda \cdot 1 + (1-\lambda)\cdot 1 = 1$.
- The carbon cycle is given by $S = \varphi e$.
- TFP is given by $D = Ae^{-\gamma S}$.

(a) Suppose a social planner of this economy cares differently about the two consumers: he places weight ρ on the utility of the R consumers and $1-\rho$ on the utility of the P consumers. I.e., the planner objective function is

$$\rho \log c_R + (1-\rho)\log c_P.$$

(E.g., if $\rho > \lambda$ the planner gives the R consumer a bigger weight than the P consumer.) State the planning problem for this economy and, assuming that all oil is used up, solve it to obtain c_R and c_P in terms of primitives (exogenous parameters).

(b) Under what condition will the planner use up all the oil? Does the answer depend on the value of ρ?

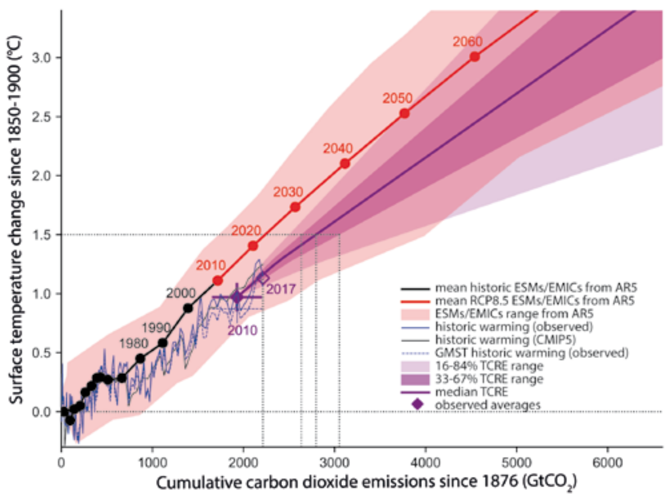
(c) Now consider a competitive market equilibrium for this economy and let us assume that the R consumer owns the oil whereas the P consumer only has labor income (and the R consumer has labor income too). Assume that no taxes on fossil fuel are raised. Define a competitive market equilibrium: list all the quantities and prices and specify the conditions they need to satisfy.

(d) Solve for a market equilibrium: specify what c_R and c_P will equal in terms of primitives.

(e) Suppose that the planner wants to arrive at the optimal allocation, from the respective of the planner's objective function. To this end, the planner can use a carbon tax (levied on the seller of the oil) τ and a transfer of any carbon-tax revenues back to the consumers in equal amounts per person. I.e., each person obtains T, with $T_R = T_P = T$, so that $\lambda T_R + (1-\lambda)T_P = T$, with the government budget simply reading $\tau e = T$. Using these tax-transfer instruments, can the planner achieve its objective? Distinguish two cases: the parameters are such that the planner wants to use up all the oil and the parameters are such that the planner does not want to use up all the oil. (There is no need to use math here; merely try to answer the question based on your intuitive understanding of the problem.)

B. Short questions (8 points each)

1. Matthews et al. (2009) introduced the concept of a constant Climate-Carbon Response (CCR). The following graph from IPCC depicts the relation between the global mean temperature and accumulated emissions measured in $GtCO_2$. Use it to estimate the CCR in degrees Celsius per Gt Carbon($°C/\text{GtC}$) rather than per $GtCO_2$. (Hint: Use the red line which has a slope of approximately $0.65°C$ per 1000 $GtCO_2$ and convert this to $°C/\text{GtC}$ by noting that one GtC equals 3.67 $GtCO_2$. If you don't have a calculator, provide an expression and a rough approximation.)



2. Consider the simplest model of the energy budget of earth. The system is initially in balance – the energy inflow equals the outflow so the energy budget is zero. Suppose the outflow suddenly decreases while the inflow remains constant. An energy budget surplus thus arises. Show qualitatively in a graph what happens over time to the **temperature, the energy outflow and the energy budget**. No numbers are required but it must be seen whether the variables increase, decrease or are constant and whether they settle down to a constant or not.

3. Under what conditions are price and quantity regulations equivalent, in the sense that they will lead to the same outcome? Under what circumstances are they not equivalent? Give one argument in favour of a tax to reduce global warming and one argument in favour of a quantity restriction.

4. In class, it was argued that a global carbon tax at the right level would be a good way to deal with the problem of global warming. However, many worry about the revenue side of the tax. Critically evaluate the following statements: (i) if all the revenues are given back to consumers/firms, they will simply generate as much emissions as without the tax, so the tax will not have any effect; (ii) if the revenues are used to lower other taxes, we can eliminate all taxes on labor and capital income.

5. Consider a utility function of a dynasty to equal $u(c_1) + \frac{1}{2}u(c_2) + \frac{1}{3}u(c_3) + \frac{1}{4}u(c_4) + \ldots$, where c_t is consumption of generation t and $u(c_t)$ is the utility to the dynasty deriving from this consumption. That is, the discount rate between time 1 and time t, β_t, satisfies $\beta_t = \frac{1}{t}$. (This means that discounting is not constant, which it was in the model we studied in class.) As time approaches infinity, what is the discount rate between two consecutive periods t and $t+1$? How does this kind of discounting change our conclusions about the optimal level of the carbon tax?