EC7104 The Climate & the Economy
Spring 2019
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Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. **A climate model**

Consider the following simple climate model:

$$\begin{aligned} F_t &= \eta \frac{\ln\left(\frac{S_t}{\bar{S}}\right)}{\ln(2)} \\ T_t - T_{t-1} &= \sigma_1 \left(F_{t-1} - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T_{t-1}^L\right)\right), \\ T_t^L - T_{t-1}^L &= \sigma_3 \left(T_{t-1} - T_{t-1}^L\right) \end{aligned} \tag{1}$$

where F_t is forcing, S_t measures the stock of carbon in the atmosphere, $\bar{S}$, preindustrial carbon, T_t is the atmospheric global mean temperature, T_t^L the mean temperature in the ocean (both measured in excess of their pre-industrial levels), and η, σ_1, σ_2, σ_3 and κ are all constant parameters. Subscript t indicates time period.

(a) Suppose we want the global mean atmospheric temperature not to go above 2 degrees. Provide an expression for the highest **permanent** forcing F that can be allowed.

(b) In the model above, the only driver of climate change is atmospheric carbon. Suppose we include another driver, e.g., atmospheric methane concentration M_t. Assume that that there is a linear relation between M_t and the energy budget, i.e., each unit of M_t adds to forcing by an amount $\psi_1 M_t$. Also assume that emissions of methane increases in temperature in a way that makes the methane concentration a convex function of the temperature, specifically, assume

$$M_t = \psi_2 T_t^2$$

Show how the model needs to be changed to account for endogenous methane emissions.

(c) With the convex methane concentration, the model will have a tipping point such that at a particular temperature, its dynamics becomes explosive. Explain why this happens and compute at which temperature the tipping point occurs. (Hint; Higher temperature has a negative (stabilizing) effect on the energy budget through the term $-\kappa T_{t-1}$. There is now a counteracting effect from methane release with an effect on the energy budget that increases in temperature. When are the two effects equal?)

2. It has been observed for most developed countries that, over long periods of time, the capital-output ratio, i.e., the ratio of the (value of the) capital stock to annual GDP is stable around a value of 3. The following questions aim to try to make sense of this observation.

   (a) Make use of the following assumptions to derive an analytical expression for the capital-output ratio:

   - both capital and output grow at a constant net rate $g > 0$
   - capital depreciates from one year to the next by a fraction δ (i.e., δ is between 0 and 1)
   - gross investment into new capital is a constant fraction s of output (i.e., s is also between 0 and 1).

   Your answer should express a k/y as a function of g, s, and δ.

   (b) Make numerical assumptions about the values of g, s, and δ that are realistic and consistent with a $k/y = 3$.

   (c) The analysis just undertaken relied on the assumption that k and y both grow at the rate g, and it is not obvious why they would. Now let us try to prove that if y grows at rate g, i.e., $y_t = (1+g)^t y_0$, where y_0 is the value of output at time 0, k must eventually grow at that same rate, no matter what k's initial value (k_0) is.

   For this, begin with the equation you used to derive an expression for k/y. Define the variable $\tilde{k}_t$ to be equal to $k_t/(1+g)^t$ and show that you can obtain the equation

   $$\tilde{k}_{t+1}(1+g) = (1-\delta)\tilde{k}_t + sy_0.$$

   Plot this equation with $\tilde{k}_t$ on the x axis and $\tilde{k}_{t+1}$ on the y axis and explain how you can use the graph to show that $\tilde{k}_t$ must converge to a constant (which means that capital will grow at rate g). Show that this constant equals $sy_0/(g+\delta)$.

   (d) Having shown that a constant growth of y gives us the desired result under the assumptions entertained, we finally need to add assumptions and argue why (on earth) output grows at a constant rate g. This is what Solow did in his famous 1956 paper. Explain what assumptions he added and go through the logic of his reasoning.

3. Consider an economy where final output is produced with the following production function

   $$Y \equiv F\left(A, L_1, E, \overline{E}\right) = AL_1^{1-\alpha}E^{\alpha}\overline{E}^{\phi},$$

   where L_1 denotes labor used to produce final output, E is fossil fuel and $\overline{E}$ is an externality from E, which is taken as exogenous by the representative firm. In equilibrium, however, it must be that $\overline{E} = E$. Fossil energy is produced with the use of labor according to the following technology

   $$E = BL_2,$$

   where L_2 is the amount of labor used to produce fossil energy. Labor is supplied inelastically and the total supply of labor is normalized to 1, which implies that

   $$L_1 + L_2 = 1.$$

   The price of fossil fuel is denoted by p, and the wage rate is denoted by w. The price of final output can be normalized to one. There are no savings, so all final output has to be consumed, i.e.,

   $$C = AL_1^{1-\alpha}E^{\alpha}\overline{E}^{\phi}.$$

   (a) Set up the profit maximization problem for the representative firm that produces final output and derive the necessary first order conditions.

(b) Set up the maximization problem for the firm that produces fossil energy and derive the necessary first order conditions.

(c) Combine the first-order conditions and other constraints that you may need to derive an equation that pins down L_1. Solve for L_1 in terms of the parameters.

4. Consider an infinite-horizon IAM where the representative world consumer has utility

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi n_t - \gamma G_{t-1}\right),$$

where c is consumption, n is labor effort, and G is the atmospheric greenhouse gas concentration. The parameter ψ is positive because it is costly for people to work. The damage coefficient γ is also positive, so there is a direct, and linear, utility loss from higher greenhouse gas concentration. The interpretation is that higher greenhouse gas concentration causes warming and warming is damaging to people. β, the discount factor, is positive and less than one, capturing impatience.

Notice that the loss incurred at time t derives from the greenhouse gas concentration at time $t-1$. This is because we assume here that there is a one-period lag in the climate dynamics: with higher greenhouse gas concentration at time t, it is not until time $t+1$ that global warming has occurred. This also means that damages from greenhouse gas emissions never hurt people immediately in this model formulation: you add greenhouse gases to the atmosphere today and only one period later will it become a problem. We will think of one time period as roughly 20 years.

We interpret consumption here as beef and assume the production function

$$c_t = An_t.$$

I.e., one unit of labor effort gives A units of beef. There is a pure externality also, however, of beef production: the emission of methane, a greenhouse gas. In particular, the methane emission at time t, e_t, is equal to ρc, where ρ is a constant.

We will completely abstract from carbon dioxide emission and other greenhouse gases. We will also make the empirically reasonable assumption that methane only stays in the atmosphere for one period (unlike carbon dioxide, it leaves from the atmosphere very quickly). So

$$M_t = e_t = \rho c_t,$$

independently of past emissions. An important feature of this formulation is thus that, with methane only, the stock of greenhouse gases in the atmosphere in period t is only a function of production activities at t.

(a) State the planning problem for this economy: a maximization problem which involves choosing how much to work in every time period t. In this formulation, you should write both c and M as a function of the work choice.

(b) If you have specified the problem correctly, you will notice that it boils down to a sequence of separate maximization problems of the following sort:

$$\max_{n_t} \log n_t - \psi n_t - \beta\gamma\rho A n_t.$$

The first term comes from the utility of consumption, the second from the disutility of working, and the third from the emissions ($\rho A n_t$), which is equal to the greenhouse gas concentration at time t, times the damage coefficient γ, times β, since there is a one-period delay in how the climate is affected by the greenhouse gas concentration.

Solve this maximization problem: solve for n_t as a function of the parameters in the problem. Does discounting affect how much the planner assigns people to work and, if so, how? Interpret.

(c) In a market economy without taxes, consumers and firms will ignore the fact that beef breeding warms the climate and causes damages. Show that in a competitive market equilibrium, the working choice will be given by $n_t = 1/\psi$ at all t.

(d) If a tax on beef meet is adopted, how should it be set in order to obtain maximal consumer utility? You may answer verbally but it is also possible to obtain a simple analytical answer.

B. Short questions

1. In class we discussed the exponential damage function $e^{-\gamma S}$ defining GDP net of damages as $e^{-\gamma S} Y$. S is the excess atmospheric carbon stock measured in GtC and Y is GDP gross of damages. In Golosov et, al., γ is set to $2.4 * 10^{-5}$. Currently, the excess carbon carbon stock is about 250 GtC. How large are damages as a percent of GDP? (You may provide an approximation or the formula for the exact number.)

2. Consider the climate model given by the equations in question A.1. Suppose the system is initially in a pre-industrial balance. F_0 is thus 0 as is T_0 and T_0^L. Suppose the outflow suddenly decreases and we model this as a permanent increase in forcing, i.e., F_t is constant at positive value for all t larger than zero . Show in one or separate graphs what happens over time to: i) the temperature in the atmosphere, ii) the temperature in the ocean, and iii) the atmosphere's energy budget (i.e., equation 2 in the model).
   No numbers are required but it must be seen whether the variables increase, decrease or are constant, if some adjustments are faster than others, and whether they settle down to a constant or not.

3. In the public debate, you sometimes hear the argument that the financial system can mitigate global warming with so-called green investments. The idea is that the financial sector should direct investments to projects with a low carbon footprint rather than to projects with a high such footprint. Discuss the pros and cons of this strategy, and explain what the expected effects on global warming are.

4. Critically evaluate the following two statements (true or false, plus a brief argument): (i) "A general carbon tax doesn't help the climate. Take gasoline as example: people's gasoline demand is very price insensitive, they buy gasoline for their cars even if the price goes up."; (ii) "Given perfect knowledge about how the economy, the climate, and the carbon cycle work, it is equally effective to base optimal carbon policy on a price (a carbon tax) or a quantity (a cap on carbon emissions) intervention."

5. Suppose the real rate of interest is 2%. The marginal cost of producing kerosene is mc_t and its market price p_t in year t. If you have a choice between producing at t or at $t + 1$, under what condition do you strictly prefer to produce in year t?