EC7104 The Climate & the Economy

Spring 2018

August 2018

Instructions. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are enough. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. **The Solow model**

The following involves a sequence of questions on the Solow model and its use in understanding differences in output over time and across countries.

(a) Consider a Solow growth model with a saving rate s, no population growth, a depreciation rate δ, and a production function at time t equal to $Ak^{\alpha}(1+\gamma)^{(1-\alpha)t}$, where $\gamma > 0$ and $0 < \alpha < 1$. Suppose, moreover, that at an initial date the capital stock of the economy equals k_0. Write an expression for k_1 (as a function of all k_0 and the parameters given).

(b) Given the information in the previous sub-question, derive an expression for the long-run value of k_t/y_t.

(c) Given the same information above except for the fact that γ now equals 0, suppose that the economy initially is in steady state. Now suddenly the value of δ drops to a lower level and stays there forever. Describe, first, in a stylized graph with k_t on the x axis and k_{t+1} on the y axis, how the mapping between k_t and k_{t+1} will change as a result of the lower value of δ. Second, describe the implied growth path for capital over time: draw a stylized graph with time on the x axis and k on the y axis.

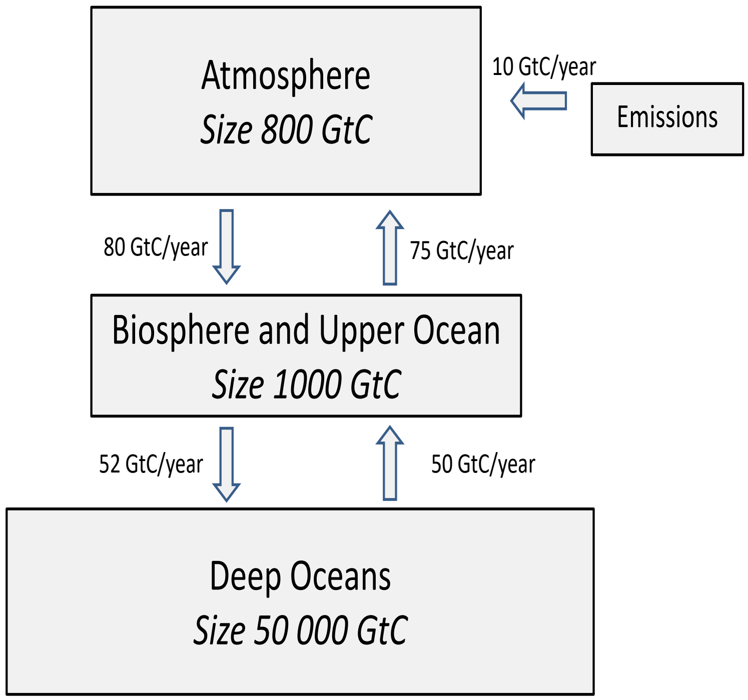
(d) Long-run differences in output per capita can be accounted for by differences in basic model parameters. Suppose we use the setup mentioned here and that there are differences only in A and s across the two countries 1 and 2; in particular, A is 5% larger in country 1 but s 5% larger in country 2. What is the implied long-run ratio of output across the two countries?

2. **A carbon circulation model**

Consider the following simple carbon circulation model:

$$\begin{aligned}
S_t - S_{t-1} &= -\phi_{12}S_{t-1} + \phi_{21}S^U_{t-1} + E_{t-1} \\
S^U_t - S^U_{t-1} &= \phi_{12}S_{t-1} - (\phi_{21}+\phi_{23})\,S^U_{t-1} + \phi_{32}S^L_{t-1} \\
S^L_t - S^L_{t-1} &= \phi_{23}S^U_{t-1} - \phi_{32}S^L_{t-1},
\end{aligned} \tag{1}$$

where S_t represents the atmosphere in period t, S^U_t the surface ocean and biosphere and S^L_t the deep oceans. Emissions in period $t-1$ is denoted E_{t-1}. $\phi_{12}, \phi_{21}, \phi_{23}$, and ϕ_{32} are constant parameters. In the picture below, the system is described with the sizes of flows and the three carbon reservoirs. (In order to make calculations easier, the sizes are substantially rounded but they are still at the right order of magnitude.)



(a) Use the numbers in the figure to calibrate the parameters $\phi_{12}, \phi_{21}, \phi_{23}$, and ϕ_{32}.

(b) Is the system in a steady state in the sense that the sizes of the carbon reservoirs are constant? If not, which are growing and by how much?

(c) Suppose emissions stop immediately. Provide an expression for the long-run relative sizes of the three reservoirs. Do this in two steps. First write a system of three equations that is satisfied in a steady state of the system (1). Second, show that these equations imply two ratios S/S^U and S/S^L that are determined by the parameters $\phi_{12}, \phi_{21}, \phi_{23}$, and ϕ_{32}. You do not need to compute the specific numerical values; the expressions in terms of $\phi_{12}, \phi_{21}, \phi_{23}$, and ϕ_{32} are sufficient.

3. **An externality on production from fossil energy**

Consider an economy with two sectors, one for the production of final output and one for the production of fossil fuel. A firm producing final output has the production function

$$Y \equiv F\left(A, H_1, E, \overline{E}\right) = AH_1^{1-\alpha}E^{\alpha}\overline{E}^{\phi},$$

where H_1 denotes labor used to produce final output, E is fossil fuel, and $\overline{E}$ is an externality, which is taken as exogenous by the representative firm. Fossil energy is produced by firms using labor according to the technology

$$E = BH_2,$$

where H_2 is the amount of labor used to produce fossil energy. Labor is supplied inelastically and the total supply of labor is normalized to 1, which implies that in equilibrium,

$$H_1 + H_2 = 1.$$

The price of fossil fuel is denoted by p and the wage rate is denoted by w. The price of final output can be normalized to one. There are no savings, so all final output has to be consumed in equilibrium, i.e.,

$$C = AH_1^{1-\alpha} E^{\alpha} \overline{E}^{\phi}.$$

(a) Set up the profit maximization problem for the representative firm that produces final output.

(b) Derive the necessary first-order conditions.

(c) Set up the maximization problem for the firm that produces fossil energy.

(d) Derive the necessary first-order condition.

(e) Combine the relevant first-order conditions and additional equations into one equation that pins down the equilibrium value of H_1. Solve for H_1 in terms of the parameters. How does the value of ϕ influence H_1? Interpret.

4. **An IAM with a "dirty" and a "clean" good**

Consider a static IAM where the consumer has utility function over two consumption goods, c_1 and c_2, given by

$$\log c_1 + \theta \log c_2 - D,$$

where the coefficient D is endogenous and to be described later.

c_1 is produced from labor only according to

$$c_1 = n_1$$

and c_2 is produced from labor and oil according to

$$c_2 = \sqrt{n_2}\sqrt{e},$$

where n_i is labor used to produce good i and e is the amount of oil used. There is one unit of labor available, so $n_1 + n_2 = 1$, and there is an amount R of oil available at zero production cost, so $e \leq R$.

Finally, getting back to the utility function, D captures an effect of the climate on the economy: when S, the atmospheric carbon content, is higher, it causes changes in the climate that make people appreciate their leisure less. We model this by assuming $D = -\gamma S$. Here, γ is an exogenous constant, capturing the damages from climate change, and S is the atmospheric carbon concentration. S, in turn, is given by $S = \phi e$, where ϕ is an exogenous constant describing the carbon cycle.

(a) State the planning problem and simplify it by writing it in terms of a choice of two variables, n_1 and e (i.e., let $n_2 = 1 - n_1$).

(b) Characterize the solution to the planning problem as far as you are able. In particular determine under what conditions, if any, all the oil is used up.

(c) Define a market equilibrium. In such a market equilibrium, consumers supply their unit of labor inelastically and can work in any sector (i.e., the sector producing good 1 and the sector producing good 2), so the wages in the two sectors therefore have to be equal—label this wage w. Consumers also receive income from selling oil to firms in sector 2; oil sells for a price we denote p. Consumers buy the two consumption goods, where we let the price of good 1 be normalized to unity and the price of good 2 be q. Firms in both sectors behave perfectly competitively both in selling their output and in buying their inputs. Neither firms nor consumers recognize the impact of their actions on D: D is a pure externality.

(d) Find the equilibrium quantities of n_1 and e.

(e) Is the equilibrium ever optimal in this economy? If it is not optimal, how could the government implement the optimal allocation with a quota system?

B. Short questions

1. Nordhaus recently assumed a simple global damage function given by

$$D(T) = 0.023\left(\frac{T}{3}\right)^2,$$

where T is the global mean temperature. With a quadratic damage function, marginal damages increase linearly with the temperature. Some climate scientists argue that marginal damages increase much faster than this. How would you implement their argument by changing the damage function?

2. Describe in a sentence or two the method “statistical downscaling”. This method has been used to analyze the sensitivity of the temperature at different latitudes (distance from the equator) to changes in the global mean temperature. What has been found? (Answer in one sentence.)

3. Explain the concept of *discounting* and its relevance in the climate-economy debate. Provide one argument in favor of using the market-determined real interest rate as a discount rate in climate calculations. Then provide one argument against using the market-determined real interest rate for this purpose.

4. Suppose the annual interest rate, at which you can both borrow and lend freely, is 2 percent and that the price of a barrel of oil today is \$70 per barrel and \$72 in one year. You are an oil producer and your marginal cost of producing a barrel is \$9 this year and \$10 next year. Your oil well can produce a total of 100 barrels—after that there is no oil left—and you can choose any production mix between today and next year, so long as the total does not exceed 100 barrels. How much will you produce now and how much will you produce one year from now?

5. From a global optimality perspective, should the carbon tax—per unit of carbon—be (i) different for different kinds of fossil fuel, to the extent their market prices differ, and (ii) different in different countries, to the extent the fossil-fuel production costs differ across countries? Answer yes or no on each of (i) and (ii) and defend your answers very briefly.