# EC7104 The Climate & the Economy
## Spring 2018
May 2018

**Instructions**. The exam consists of 9 questions that should all be completed. The total maximum score is 100 points. The final course grade will be given based on the problem sets and the exam. If the score on the problem sets is higher than the exam score, the final score is the weighted average of the exam and the problem sets, with weights $\frac{4}{5}$ and $\frac{1}{5}$, respectively. If not, the final score is the exam score. Grades will be given using the standard scale from A to F.

There will be two types of questions. We call the first type analytical, where you are supposed to provide a formal analysis motivating your answer. There are 4 of these questions, each giving a maximal score of 15 points. The second type are short questions, where shorter answers without formal proofs are sufficient. There are 5 short questions, each giving a maximal score of 8 points.

The core of your answers should be based on what you have learned during the course. Make sure you specify your definitions and assumptions clearly.

A. Analytical questions

1. **The Solow model**

The following involves a sequence of questions on the Solow model and its use in understanding differences in output over time and across countries.

(a) Given a Solow growth model with a saving rate $s$, an output growth rate $g$, no population growth, and a depreciation rate $\delta$, define balanced growth and show how to compute the balanced-growth value for the capital-output ratio.

(b) Setting $g = 0$ in addition and assuming a production function $y = A\sqrt{k}$, where $A$ is a constant, $y$ is output, and $k$ is capital, derive the steady-state value for the capital stock as a function of $A$, $s$, and $\delta$.

(c) Suppose, with the information just given, that the economy has reached a steady state and that $A$ suddenly rises to a new permanent level. Describe the implied growth path for capital and output graphically: draw a stylized graph with time on the x axis and $k$ and $y$ on the y axis.

(d) Long-run differences in output per capita can be accounted for by differences in basic model parameters. Suppose we use the setup mentioned here and that there are differences in $A$ and in $s$ across the two countries 1 and 2; in particular, $A$ is 10% larger in country 1 but $s$ 10% larger in country 2. Moreover, country 1 has twice as high an initial capital stock as that of country 2. What is the implied long-run ratio of output across the two countries?

2. **A climate model**

Consider the following simple climate model:

$$\begin{aligned} F_t &= \eta \frac{\ln\left(\frac{S_t}{\bar{S}}\right)}{\ln(2)}, \\ T_t - T_{t-1} &= \sigma_1 \left(F_{t-1} - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T_{t-1}^L\right)\right), \\ T_t^L - T_{t-1}^L &= \sigma_3 \left(T_{t-1} - T_{t-1}^L\right), \end{aligned} \tag{1}$$

where $F_t$ is forcing, $S_t$ measures the stock of carbon in the atmosphere, $\bar{S}$, preindustrial carbon, $T_t$ is the atmospheric global mean temperature, $T_t^L$ the mean temperature in the ocean (both measured in excess of their pre-industrial levels), and $\eta$, $\sigma_1$, $\sigma_2$, $\sigma_3$, and $\kappa$ are all constant parameters. Subscript $t$ indicates time period.

(a) Suppose we want the global mean atmospheric temperature not to go above 2 degrees. Provide an expression for the highest **permanent** forcing $F$ that can be allowed.

(b) Suppose it is discovered that the arctic ice sheets are melting faster than expected as the global temperature increases. This implies that the direct reflection of sunlight falls as the temperature rises. Which parameter in the model needs to be updated and in which direction?

(c) In the model above, the only driver of climate change is atmospheric carbon. Suppose we include another driver, e.g., methane emissions $M_t$. Assume that that there is a linear relation between $M_t$ and the energy budget, i.e., each unit of $M_t$ adds to forcing by an amount $\psi M_t$. Show how the model should be changed to take methane emissions into account.

3. **The world oil market**

Assume that there are three countries in the world: two countries (labelled 1 and 2) that both are using oil and labor to produce output $Y$, and one country that owns and sells the oil to country 1 and 2. The world exists for one period and the total supply of oil is fixed at the amount $O_{\max}$. The cost of providing a marginal unit of oil for the oil exporting country is equal to zero. Denoting oil by $O$ and labor by $N$, the production functions for output in country 1 and 2 are given by

$$Y_i \equiv f\left(\overline{E}, E_i, L_i\right) = \overline{O}^{-\gamma} O_i^{\xi} N_i^{1-\xi}, \quad i = \{1, 2\}$$

with $0 < \gamma < \xi < 1$, and $N_1 = N_2 = 1$. The term $\overline{O}^{-\beta}$ captures the damages that are caused by the use of oil. In equilibrium it must be true that $\overline{O} = O_1 + O_2$ but both countries consider $\overline{O}$ to be exogenous (think of the representative firm in each region as consisting of many small identical firms). The oil producing country is not affected by climate change. Assume now that country 1 imposes the tax $\tau$ on oil use in order to reduce the adverse effects associated with its use, but that country 2 does not implement any taxes. The total price that firm 1 has to pay for a marginal unit of oil is then $p+\tau$, where $p$ is the market price of oil.

(a) Explain how the how the quantity of oil supplied by the oil exporting country depends on the price of oil ($p$). What does this supply curve imply for the possibility of the tax $\tau$ to reduce the total amount of oil use?

(b) Formulate the profit maximization problems for the firms in country 1 and 2.

(c) Derive the first order conditions with respect to $O_i$ and $N_i$ for $i = \{1, 2\}$.

(d) Assume now that the tax in country 1 is set to $\tau = p$. Use this fact, the two first-order conditions, and the fact that $O_1 + O_2 = O_{\max}$ to solve for $O_1$ in terms of $O_{\max}$.

4. **A static IAM**

Consider a static IAM where the consumer has utility function

$$\log c + \nu \log e - D$$

where $c$ is consumption and $e$ energy used for heating; $\nu$ is an exogenous constant. The coefficient $D$ is endogenous and captures an effect of the climate on the economy: when $S$, the atmospheric carbon content is higher, it causes changes in the climate that make people appreciate their leisure less. We model this by assuming $D = -\gamma S$; here, $\gamma$ is an exogenous constant, capturing the damages from climate change.

Energy is produced from coal and the production of coal uses labor, $n_e$: $e = \chi n_e$, where $\chi$ is an exogenous productivity parameter. The production of consumption good also uses labor only, $n_c$:

$c = An_c$, where $A$ is an exogenous productivity parameter. There is one unit of labor and it can be allocated in any way society decides across the two sectors: $n_c + n_e = 1$.

Finally, the atmospheric carbon content, $S$, is given by $S = \phi e$, where $\phi$ is an exogenous constant describing the carbon cycle.

(a) State the planning problem and simplify it by writing it in terms of a choice of only one variable, $e$.

(b) Characterize the solution to the planning problem as far as you are able. You do not need to obtain a closed-form solution but you should be able to derive one equation in the unknown $e$.

(c) In a market equilibrium, consumers supply their unit of labor inelastically and can work in any sector, so the wage across the two sector therefore has to be equal. Consumers therefore buy the consumption good and buy energy for heating; their income is simply the wage income. Denote the price of energy by $p$. Describe the consumer's problem and the firm's problem. We assume that there are no taxes and that there is perfect competition in all markets, i.e., all consumers and firms take prices as given.

(d) Derive the first-order conditions for consumers and for firms. Using them, try to solve for the level of $e$ in the market equilibrium.

(e) What is the optimal carbon tax in this environment, measured in units of utility? What is the optimal tax measured in units of consumption?

B. Short questions

1. In class we discussed the exponential damage function $e^{-\gamma S}$ defining GDP net of damages as $e^{-\gamma S}Y$. $S$ is the excess atmospheric carbon stock measured in GtC and $Y$ is GDP before damages. Suppose $\gamma = 4 * 10^{-5}$. Currently, the excess carbon carbon stock is about 250 GtC. How large are damages as a percent of GDP net of damages? (You may provide an approximation or the formula for the exact number.)

2. The three-variable linear carbon circulation model we discussed in class has the property that the size ratios of three sinks (the atmosphere, the surface ocean, and the deep oceans) are eventually restored if emissions permanently stop. We now have about 250 GtC extra in the atmosphere. What will happen with that carbon in the long run in the model if emissions permamently stop? Use qualitative words like "most of it", "all of it" or "none of it" when you answer in a sentence or two.

3. Describe what the "Green Paradox" is. Provide intuition for the paradox. Is the paradox equally relevant both oil and coal?

4. State the Hotelling rule when the marginal cost of producing oil is $mc_t$ at time $t$; let $p_t$ be the price of oil at $t$ and $r > 0$ the (constant) real rate of interest. Suppose $mc_t$ grows at a constant rate $g$. Will the price of oil grow faster or more slowly than at rate $r$?

5. The optimal carbon tax, as a proportion of world output, is approximately a function of only three parameters. What kinds of parameters are they and how can their values be assigned? Do these values involve ethical consideration or just "hard measurement"?